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Extreme Rainfall Frequency Analysis for Meteorological Sub-Division 4 of India Using L-Moments

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***Abstract*—**Extreme rainfall frequency analysis for Meteorological Sub-Division 4 of India was analyzed using L-moments approach. Serial Correlation and Mann Kendall tests were conducted for checking serially independent and stationarity of the observations. The discordancy measure for the sites was conducted to detect the discordant sites. The regional homogeneity was tested by comparing with 500 generated homogeneous regions using a 4 parameter Kappa distribution. The best fit distribution was selected based on Z^{DIST} statistics and L-moments ratio diagram from the five extreme value distributions GPD, GLO, GEV, P3 and LP3. The LN3 distribution was selected and regional rainfall frequency relationship was established using index-rainfall procedure. A regional mean rainfall relationship was developed using multiple linear regression with latitude and longitude of the sites as variables.

***Keywords*—**L-moments, Z^{DIST} statistics, Serial correlation, Mann Kendall test.

I. Introduction

THE extreme rainfall frequency analysis for the sites can be reasonably carried out using conventional methods where the data are available as compare to the desired return periods. For proper planning of water resources the knowledge of frequency of recurrence of extreme rainfall events are very important. Regional based frequency analysis approach is a very effective tool where the availability of the data are in sparse. It is the basic assumptions in most of the frequency analysis methods that data are serially independent and identically distributed. So, the testing of the data for serial correlation and stationary are very essential for a meaningful frequency analysis. In this study L-moments based regional extreme rainfall frequency analysis is carried out by testing serial correlation and stationary of the data using autocorrelation check and Mann Kendall trend test prior to the frequency analysis.

The whole of India has been divided into 36 meteorological sub-divisions (National Climatic Centre, India Meteorological Department, Pune).This study area lies in the Meteorological Sub-Division 4 and covers the states of Nagaland, Manipur, Mizoram and Tripura. Most of the study area are hilly regions and receives its rainfall every year from the south-west monsoon from June to September. A study to evaluate extreme rainfall events for this sub-division is conducted to give reliable information for the recurrence intervals of extreme rainfall events by conducting regional frequency analysis using L-moments approach with the testing of serial correlation check and Mann Kendall trend test.

The literatures for regional rainfall frequency analysis with different regionalization techniques have been found everywhere. Sveinsson Oli in Northeastern Colorado [12], Kadri Yurekli in Tokat [8], Jan Kysely et al. in Czech Republic [9], B.P. Parida et al. in Botswana [10] and Cosmo S. Ngongondo et al. in southern Malawi [6] conducted regional rainfall frequency analysis using heirarical clustering methods like single linkage, double linkage, average linkage and Ward's method for regionalization of the study area. Tao Yang et al. in China [13] conducted regional frequency analysis and spatio-temporal pattern characterization of rainfall extremes in the Pearl River Basin.

In this study annual maximum daily rainfall data from 6 nos. of gauged sites (Table I) are collected from Regional Meteorological Centre, Gauwahati. The collected data ranges from 1990 to 2010. The objectives of this study are (1) To conduct autocorrelation or serial correlation check and Mann Kendall trend test to identify the serially independent and stationarity of the observations. (2) To conduct regional extreme rainfall frequency analysis for Meteorological sub-division 4 of India using L- moments. (3) To develop regional mean rainfall relationship with latitude and longitude of the sites.

II. Study Area And Data Collection

This study area (Fig. 1) lies within 22^0 N to 27^0 N and 91^0E to 96^0 E which covers the states of Nagaland, Manipur, Mizoram and Tripura. Most of the study area are hilly regions and receives its rainfall every year from the south-west monsoon from June to September. Annual maximum daily rainfall data from 6nos. of gauged sites (Table I) are collected from Regional Meteorological Centre, Gauwahati. The collected data ranges from 1990 to 2010.

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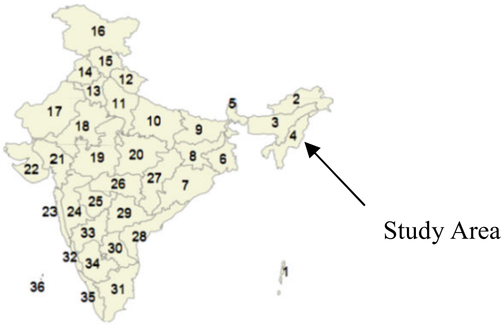

Fig. 1 Location of Meteorological Sub-Divisions 4 of India

TABLE I
NAME AND LOCATION OF THE SITES

Sl. No.	Name of Station	Lat.	Lon.	observations (year)
1	Kohima	$25^0 38$N	$94^0 10'$E	21
2	Imphal	$24^0 46$ N	$93^0 54'$E	21
3	Aizwal	$23^0 44'$N	$92^0 43'$E	21
4	Lengpui	$23^0 50'$N	$92^0 37'$E	10
5	Agartala	$23^0 53'$N	$91^0 15'$E	21
6	Kailashahar	$24^0 19'$N	$92^0 00'$E	20

Lat. = Latitude; Lon = Longitude

III. AUTOCORRELATION OR SERIAL CORRELATION

If the observations at different sites are serially independent the bias and standard error of the estimated quantiles can be reduced. The coefficient of autocorrelation r_k of a discrete time series for lag-k is projected as

$$r_k = \frac{\sum_t^{n-k}(x_t - \bar{x}_t)(x_{t+k} - \bar{x}_{t+k})}{\sqrt{\sum_t^{n-k}(x_t - \bar{x}_t)^2 . \sum_t^{n-k}(x_{t+k} - \bar{x}_{t+k})^2}}$$

$\bar{x}_t$ = the sample mean for the first (n-k)
$\bar{x}_{t+k}$ = the mean for the last (n-k) terms.

If a time series is completely random and the sample size is large, the lagged-correlation coefficient is approximately normally distributed with mean 0 and variance $\frac{1}{N}$ Chatfield [14]. It follows that the approximate threshold or critical level of correlation for 95% significance (α = 0.05) is $r_k = 0 \pm \frac{2}{\sqrt{N}}$ where, N is the sample size. The calculated autocorrelation coefficients of lag 1, lag 5 and lag 8 for each of the annual series is given in Table II which are all less than the critical values and the observations in the series can be accepted being independent.

TABLE II
AUTOCORRELATION COEFFICIENTS OF LAG 1, 5 AND 8

Station	r_1	r_5	r_8
Kohima	0.157	-0.392	0.225
Imphal	0.247	-0.27	0.184
Aizwal	-0.136	-0.287	0.067
Lengpui	0.367	-0.349	0.157
Agartala	-0.399	0.031	0.124
Kailashahar	-0.154	-0.065	-0.071

Autocorrelation Coefficients r_1= lag1, r_2= lag2, r_8= lag8,

IV. MANN KENDALL TEST

In a frequency analysis it is the basic assumption that the events observed in the past are typical of what would be expected in the future that is observations at any given site are required to be identically distributed. This assumption may be invalid if nonstationarity are present in the observations. The Mann Kendall statistics S is given as:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sign(x_j - x_i)$$

The application of trend test is done to a time series x_i that is ranked from $i = 1, 2, 3 \ldots\ldots\ldots.(n-1)$ and x_i which is ranked from $j = (i+1), (i+2), (i+3) \ldots\ldots\ldots.n$. Each of the data point x_i is taken as reference point which is compared with the rest of the data points x_j so that when $n \geq 8$, the statistic S is approximately normally distributed with the mean $E(S) = 0$.

$$Var(s) = \frac{n(n-1)(2n+5) - \sum_{i=1}^{m} t_i\ (i)(i-1)(2i+5)}{18}$$

where t_i is considered as the number of ties up to sample i. The test statistics is computed as

$$Z_c = \frac{S-1}{\sqrt{Var\ (S)}}$$
$$= 0, \text{ when } S = 0$$
$$= \frac{S+1}{\sqrt{Var\ (S)}}\ \ when\ S < 0$$

Z_c here follows a standard normal distribution. A positive value of Z_c signifies an upward trend and a negative value signifies a downward trend. A significance level α is also used for testing either an upward or downward monotone trend. If Z_c appears greater than $Z_{\alpha/2}$ where α depicts the significance level, the trend is considered as significant.

All the p – values for the stations are all greater than Z_c there is no significance trend in the time series (Table III).

TABLE III
P- VALUES OF MANN KENDALL TEST

Station	P- Value
Kohima	0.974
Imphal	0.924
Aizwal	0.677
Lengpui	0.119
Agartala	0.209
Kailashahar	0.629

V. METHODOLOGY

The analysis consists of the following steps:

A. L-moment Approach

L-moments [2] are linear combinations of probability weighted moments (PWM). Greenwood et al [1] summarizes the theory of probability weighted moments and defined them as

$$\beta_r = \frac{1}{N} \sum_{j=i+1}^{N} (x_j) \frac{(j-1)(j-2)(j-3)\ldots\ldots\ldots(j-1)}{(N-1)(N-2)(N-3)\ldots\ldots\ldots(N-r)}$$

The first four L-moments are

$$\lambda_1 = \beta_0$$
$$\lambda_2 = 2\beta_1 - \beta_0$$
$$\lambda_3 = 6\beta_2 - 6\,\beta_1 + \beta_0$$
$$\lambda_4 = 20\beta_3 - 30\,\beta_2 + 12\beta_1 - \beta_0$$

where λ_1 is the L-mean which measures the central tendency, λ_4 is the L-standard deviation which measures the dispersion. Again, Hosking [2] defined the dimensionless L-moment ratios as

$$\tau = L - coeffcient\ of\ variance = \frac{\lambda_2}{\lambda_1}$$
$$\tau_3 = L - Skewness = \frac{\lambda_3}{\lambda_2}$$
$$_4 = L - Kurtosis = \frac{\lambda_4}{\lambda_2}$$

The λ_1, λ_2, λ_3 , λ_4, τ, τ_3, τ_4 and discordancy for D_i for the sites are given in Tables IV and V.

TABLE IV
L – MOMENTS FOR THE SITES

Station	λ_1	λ_2	λ_3	λ_4
Kohima	80.1142	16.6580	4.0309	1.0979
Imphal	78.2095	13.4242	1.7406	1.7627
Aizwal	102.6857	12.9100	1.4132	2.9058
Lengpui	109.1000	10.7977	2.8116	-0.2478
Agartala	148.3667	27.9290	5.2442	1.1078
Kailashahar	137.7050	24.3071	12.2557	11.7187

λ_1, λ_2, λ_3 and λ_4 = Linear-moments. λ_1= Linear-mean; λ_4= Linear-standard deviation

λ_1, λ_2, λ_3 and λ_4 are the four L-moments where λ_1= L-mean; λ_4= L-standard deviation

TABLE V
L –MOMENTS RATIOS AND DISCORDANCY FOR THE SITES

Station	n_i	τ	τ_3	τ_4	D_i
Kohima	21	0.20792	0.24198	0.0659	0.73623
Imphal	21	0.17164	0.12966	0.13131	0.36882
Aizwal	21	0.12572	0.10947	0.22508	1.22106
Lengpui	10	0.09897	0.26039	-0.02290	1.63704
Agartala	21	0.18824	0.18777	0.03966	0.39101
Kailashahar	20	0.17651	0.50420	0.48211	1.64580

n_i = No. of observations, τ = Linear coefficient of variance, τ_3 = Linear-skewness, τ_4 = Linear- kurtosis
D_i= Discordancy measure.

The regional weighted values of ${\lambda_1}^R$ = 109.14035, ${\lambda_2}^R$ = 18.276052, $\tau^R =$ 0.167406 , ${\tau_3}^R$ = 0.234515 and ${\tau_4}^R$ = 0.167667.

B. Discordancy Measure (Hosking and Wallis)

Hosking & Wallis [3], [7] defined discordancy measure of sites to detect the discordance sites among other sites as

$$D_i = \tfrac{1}{3}(u_i - \bar{u})^T A^{-1} (u_i - \bar{u})$$

where $u_i = (\tau, \tau_3, \tau_3)^T$ is a vector containing, τ, τ_3 *and* τ_3 values of site , the superscript T denotes transpose of a matrix or vector.

$$\bar{u} = \tfrac{1}{N} \sum u_i = \text{ the (unweighted) group average}$$

A^{-1} is the inverse of the covariance matrix A of u_i.The elements of A^{-1} are given by the relation

$$A = \frac{1}{N} \sum_{i=1}^{N} (u_i - \bar{u})((u_i - \bar{u})^T$$

where N is the number of sites in the region.

The dicordancy D_i of the 6 sites are all less than the critical value of 1.648 for 6 numbers of sites and hence there are no discordance sites.

C. Heterogeneity Measure (Hosking and Wallis)

A statistic based on the weighted variance of the L-coefficient of variation (L-cv) is derived Hoskings & Wallis [3]. The heterogeneity measure is defined as

$$H = \frac{V - \mu_v}{\sigma_v}$$

where V = weighted standard deviation of L-cv values

$$= \left[\frac{\sum_{i=1}^{N}(n_i)(\tau - \tau^R)}{\sum_{i=1}^{N}(n_i)}\right]^{1/2}$$

n_i = record length for the site; τ = the L-Cv for the site $i = 1, 2, 3 \ldots\ldots\ldots\ldots N$

τ^R = the weighted group mean L-Cv; N= no. of sites in the region and $\mu_v$and $\sigma_v$are the mean and standard deviation respectively for the weighted standard deviations of 500 generated homogeneous regions by Monte Carlo simulation using a 4-parameter Kappa distribution.

The quantile function of this distribution is

$$x(F) = \xi + \frac{\alpha}{K}\left\{1 - \left[\frac{(1 - F^h}{h}\right]^K\right\}$$

Using the regional L-moments and their ratios ${\lambda_1}^R$ = 109.14035, ${\lambda_2}^R$ = 18.276052, ${\tau_3}^R$ = 0.234515 and ${\tau_4}^R$ = 0.167667 the parameters of this distribution are calculated as ξ = 0.110454, α = 0.025889, K = 0.506099 & h= - 0.338699. A computer program written in JAVA is used for iteration and simulation purposes. A region is considered, Hoskings and Wallis [7] as –

(i) homogeneous if H < 1
(ii) possibly heterogeneous if $1 \leq H \leq 2$
(iii) definitely heterogeneous if $H \geq 2$

For this Sub-Zone the values of V = 0.033633, μ_v= 0.034792 and σ_v = 0.006235

$$H = \frac{V - \mu_v}{\sigma_v} = -0.185886 < 1.0$$

This shows the sub-division is homogeneous.

D. Goodness of Fit Using Z^{DIST} Statistics

For each of the candidate distributions, the goodness-of-fit measure is given by

$$Z^{DIST} = \frac{\tau_4{}^{DIST} - \tau_4{}^{R} + \beta_4}{\sigma_4}$$

where $\tau_4{}^{DIST}$ = L- kurtosis of the fitted distribution.

$$\beta_4 = \frac{\sum_{i=1}^{N_{sim}} (\tau_{4Sim}{}^{R} - \tau_4{}^{R})}{N_{Sim}}$$

= bias of weighted $\tau_4{}^R$ with weighted $\tau_{4Sim}{}^R$ of the simulated regions.

$$\sigma_4 = \left[\frac{1}{(N_{Sim} - 1)} \left\{ \sum_{m=1}^{N_{sim}} (\tau_{4Sim}{}^{R} - \tau_4{}^{R})^2 - N_{Sim}\beta_4{}^2 \right\} \right]^{1/2}$$

= standard deviation of $\tau_4{}^R$ and N_{Sim} is number of simulations.

If $|Z^{DIST}|$ is close to zero, then the fit is considered adequate or reasonable. If all the distributions have their $|Z^{DIST}| \leq 1.64$, the one which is closest to zero is considered to be the best fit distribution. For this sub-division β_4= - 0.002618 and σ_4 = 0.192400, the Z^{DIST} of the distributions are given in Table VI.

All the distributions have their Z^{DIST} values $\leq$ 1.64 and hence all are reasonable but LN3 has the lowest value of Z^{DIST} = 0.056236 followed by GEV = 0.118881

TABLE VI
Z^{DIST} VALUES FOR THE DISTRIBUTIONS

Distributions	Z^{DIST}	$\lvert Z^{DIST} \rvert$
GPD	- 0.302030	0.302030
GLO	0.298166	0.298166
GEV	0.118881	0.118881
P3	- 0.079333	0.079333
LN3	0.056236	0.056236

Z^{DIST} = Z-distribution statistics; GPD = Generalised Pareto Distribution; GLO = Generalised Logistic Distribution; GEV = Generalised Extreme Value Distribution; P3 = Pearson Type III Distribution and LN3 = Log- Normal Type III Distribution

E. L-Moments Ratio Diagram

L-Moment ratio diagram [15], [16] is used for distribution selection by comparing its closeness to the regional L-skewness and L-kurtosis. For this τ_4 of the distributions are derived from their relationship of τ_3 for the sites. The (τ_3, τ_4) sets for each of the distribution are plotted and their closeness to regional values of ($\tau_3{}^R$, $\tau_4{}^R$) are compared. In L-moment ratio diagram (Fig. 2), the regional $\tau_3{}^R$ and $\tau_4{}^R$ lie closest to LN3 as shown by a red dot.

Based on Z^{DIST} statistics and L-moment ratio diagram LN3has been selected as the best fit distribution for this sub-division.

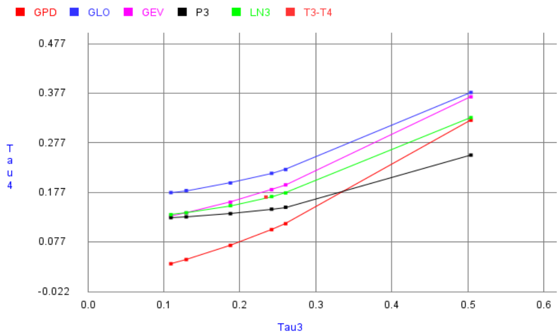

Fig. 2 L-moments ratio diagram

F. Index Rainfall Procedure

In this method for a homogeneous region we have,

$$Q_i\,(F) = \mu_i\, q(F)$$

where, $i = 1, 2, 3 \ldots\ldots\ldots\ldots N$ and μ_i is the site-dependent scale factor, the index rainfall usually at site mean and $q(F)$ is the regional growth factor Dalrymple [4]. Let $\bar{\mu}_i$ be the estimate of the scale factor at site i

The dimensionless rescaled data are

$$q_{ij} = \frac{Q_{ij}}{\bar{\mu}_i}$$

where $j = 1, 2, 3 \ldots\ldots\ldots\ldots N = i = 1, 2, 3 \ldots\ldots\ldots\ldots N$ Q_{ij} = observed data of i^{th} site at j^{th} rank and q_{ij} = rescaled data of i^{th} site at j^{th} rank.

The regional growth factor

$$\frac{Q_T}{\bar{\mu}} = Q_i(F) = q(F)$$

where, $Q_i(F)$ is the quantile function of the selected distribution and its parameters are estimated using their relationships with L-moments of the rescaled data and $q(F)$ is the corresponding growth factor for the different return periods.

1. Fitting LN3 Distribution

The quantile function of LN3 is given by

$$Q_T = \xi + \frac{\alpha}{K}\,(1 - e^{-kz})$$

The parameters of the LN3 distribution are estimated using their relationships with the regional values of $\lambda_1{}^R$, $\lambda_2{}^R$ and $\tau_3{}^R$ of the rescaled data obtained from index-rainfall procedure. Here, $\lambda_1{}^R$= 1.0; $\lambda_2{}^R$ = τ^R = 0.167406 and τ_3 = 0.234515. The parameter of LN3 are ξ = 1.061648, α = 0.268826 and K = - 0.486303 and the regional growth factor and regional rainfall frequency relationship are given in (1), (2) and (3) respectively.

$$q(F) = \frac{Q_T}{\bar{\mu}} = \xi + \frac{\alpha}{K}\left(1 - e^{-kz}\right) \quad (1)$$

$$\frac{Q_T}{\bar{\mu}} = 0.508853 + 0.552795e^{0.486303z} \quad (2)$$

$$Q_T = [\,0.508853 + 0.552795e^{0.486303z}\,].\bar{\mu} \quad (3)$$

The regional growth factor and estimated rainfall quantiles for different return periods of the sub-division are given in Tables VII and VIII respectively.

TABLE VII
REGIONAL GROWTH FACTOR FOR DIFFERENT RETURN PERIODS

Q_T	1	2	5	10	20	50	100
$\bar{\mu}$	0.670	1.060	1.341	1.539	1.739	2.009	2.222

$\frac{Q_T}{\bar{\mu}}$ = Regional Growth Factors

TABLE VIII
ESTIMATED RAINFALL FOR DIFFERENT RETURN PERIODS

Station	1	2	5	10	20	50
Kohima	53.8	85.0	107.4	122.8	139.3	161.0
Imphal	52.5	83.0	104.9	119.9	136.0	157.2
Aizwal	68.9	109.0	137.7	157.4	178.6	206.4
Lengpui	73.2	115.8	146.3	167.2	189.7	219.3
Agartala	99.6	157.5	199.0	227.4	258.0	298.2
Kailashahar	92.6	146.4	184.9	211.4	239.8	277.2

VI. DEVELOPMENT OF REGIONAL MEAN RAINFALL RELATIONSHIP

In this study the regional mean rainfall relationship with latitude and longitude of the sites is established using multiple linear regression [5] and [11] by relating rainfall with their latitude and longitude of the sites as in (4). The estimated mean and their Chi-Square values are given in Table IX. It is seen that the Chi-Square value is only 0.9416 against the critical value of 11.070 at 5% significance level with 5 degrees of freedom which shows that there is no significance difference between the observed and estimated mean.

$$\overline{X} = 2758.5 + 12.5\,(\text{Lat.}) - 31.8\,(\text{Lon.}) \quad (4)$$

TABLE IX
COMPARISON OF ESTIMATED RAINFALL MEANS AND OBSERVED MEANS

Station	Observed mean (Q)	Lat.	Lon.	Estimated Mean (E)	$\frac{\sum(Q-E)^2}{E}$
Kohima	80.11	25.63	94.16	84.58	0.2362
Imphal	78.20	24.76	93.90	81.98	0.1743
Aizwal	102.68	23.73	92.71	106.94	0.1697
Lengpui	109.10	23.83	92.61	111.37	0.0463
Agartala	148.36	23.88	91.25	155.25	0.3058
Kailashahar	137.90	24.31	92.00	136.77	0.0093
χ^2 Chi-Square					0.9416

Lat. = Latitude; Lon. = Longitude

VII. RESULTS

The maximum annual daily rainfall data for the 6 numbers of stations in the sub-division were conducted autocorrelation check and Mann Kendall trend test. The autocorrelation coefficients of lag 1, lag 5 and lag 10 for each of the annual series were all less than the threshold or critical levels at 95% significance and the observations were independent. In the Mann Kendall trend test there were no significance trend in the data and the data were stationary. The discordancy measures (Di) for the 6 sites were all less than the critical value of 1.648 for 6 numbers of sites and there was no discordant site in the sub-division. The heterogeneity measure for regional homogeneity was conducted by comparing with 500 simulated homogeneous regions using a 4-parameter Kappa distribution in a simulation program written in JAVA and indicated that H value was -0.185886 which was less than the critical value of 1.0, so the sub-division was homogeneous. In the Z^{DIST} statistics, all the distributions had their $|Z^{DIST}|$ values very close to zero and all were reasonable but LN3 had the lowest value of Z^{DIST} as 0.056236 followed by P3 as Z^{DIST} = 0.079333. Based on Z^{DIST} statistics and L-moments ratio diagram LN3 was selected as the best fit distribution for the sub-division. The parameters for the selected distribution were estimated using index-rainfall procedure and regional growth factor was established with the development of regional rainfall frequency relationship. The rainfall quantiles for different return periods were estimated. The regional mean rainfall relationship with latitude and longitude of the sites was established by multiple linear regression with latitude and longitude of the sites as attributes. The Chi-Square value between the observed mean and estimated means was only 0.9416 against the critical value of 11.070 at 5% significance level with 5 degrees of freedom which indicated that there was no significance difference between the observed and estimated mean. The recurrence intervals for the real data of annual daily maximum rainfall for the sites were examined and it was found that all the sites had their return periods less than 20 years.

VIII. DISCUSSION

Serial correlation and Mann Kendall trend test were conducted prior to frequency analysis to fulfill the assumptions made in the index rainfall procedure that data are serially independent and identically distributed. Serial correlation or autocorrelation test indicated that data are serially independent. No trends of the observations were detected by Mann Kendall test. Most of the stations had their real data with return periods of 1, 2, 5 and 20 years, except 450 years return periods for the site Kailashahar in the year 1993.

IX. CONCLUSION

The annual extreme daily rainfall data for meteorological sub-division 4 of India was conducted using L-moments approach with autocorrelation or serial correlation check and Mann Kendall trend test. The return periods for the sites data in this sub-division are almost at the range of 1, 2, 5, 10 and 20 years. So, the design return period for the extreme rainfall for this sub-division may be taken as having 20 years return periods.

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