A Robust Method for Finding Nearest-Neighbor using Hexagon Cells
Ahmad Attiq Al-Ogaibi, Ahmad Sharieh, Moh’d Belal Al-Zoubi, R. Bremananth

Abstract—In pattern clustering, nearest neighborhood point computation is a challenging issue for many applications in the area of research such as Remote Sensing, Computer Vision, Pattern Recognition and Statistical Imaging. Nearest neighborhood computation is an essential computation for providing sufficient classification among the volume of pixels (voxels) in order to localize the active-region-of-interests (AROI). Furthermore, it is needed to compute spatial metric relationships of diverse area of imaging based on the applications of pattern recognition. In this paper, we propose a new methodology for finding the nearest neighbor point, depending on making a virtually grid of a hexagon cells, then locate every point beneath them. An algorithm is suggested for minimizing the computation and increasing the turnaround time of the process. The nearest neighbor query points \( \Phi \) are fetched by seeking fashion of hexagon holistic. Seeking will be repeated until an AROI \( \Phi \) is to be expected. If any point \( \Upsilon \) is located then searching starts in the nearest hexagons in a circular way. The First hexagon is considered be level 0 \( (L_0) \) and the surrounded hexagons is level 1 \( (L_1) \). If \( \Upsilon \) is located in \( L_1 \), then search starts in the next level \( (L_2) \) to ensure that \( \Upsilon \) is the nearest neighbor for \( \Phi \). Based on the result and experimental results, we found that the proposed method has an advantage over the traditional methods in terms of minimizing the time complexity required for searching the neighbors, in turn, efficiency of classification will be improved sufficiently.

Keywords—Hexagon cells, \( k \)-nearest neighbors, Nearest Neighbor, Pattern recognition, Query pattern, Virtually grid.

I. INTRODUCTION

Finding nearest neighbor points among volume-of-pixels (voxels) is pertinent computationally challenge issue in many statistical classification applications. In the searching of regions-of-interests, segmentation decays an image into ingredient regions, acquaintance of boundaries, or objects to be seeking. The level to which decomposition is performed based on the problem being solved in applications such as automation inspection using images and interest lies in analyzing objects’ outlines. These applications currently suffers due to deplorable computation turnaround time of traditional nearest neighbor method such as \( k \)-nearest neighbors (\( k \)-NN). In \( k \)-NN, \( k \) depends on the number of nearest neighbors utilized in the classification while seeking for the objects. Especially, in remote sensing, user control over imaging is inadequately restricted due to the choice of image sensors, for these applications, seeking of boundaries mainly depends on segmentation. However, efficacy of classification mainly depends on accuracy of segmentation accuracy.

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Fig. 1. \( \Upsilon \) is the closet to query point \( \Phi \) in \( n \) points 2-D space. In that, \( \Phi \) is represented by the small circle with cross inside it, while data points are represented by the small filled circles.

Optimize the accuracy of \( k \)-nearest neighbor classification based on the weighted distance was presented in [8]. A selective incremental approach for transductive Nearest Neighbour Classification was proposed by Viswanath et al. [10]. In this paper, we provide a solution to classify the patterns by using hexagonal searching of neighbors.

We can treat a set of \( \Upsilon \) number of \( n \) points in two-dimensional (2-D) spaces, in that, a point \( \Upsilon \) is a nearest point to a query point, \( \Phi \). This is represented by a cross encircle enclosed by a circle in Fig. 1. [4]

The proposed method starts in preprocessing phase by making a virtually grid of a hexagon cells, then locate every point beneath them. In order to find the nearest neighbor to a query point \( \Phi \), a search on the same hexagon is performed until \( \Phi \) is expected to be found. However, if any point \( \Upsilon \) is found while searching, then circular way of seeking is required for the nearest hexagons. An initial hexagon is called level 0 \( L_0 \) and the surrounded hexagons is referred as level \( L_1 \). If any point \( \Upsilon \) is not found in \( L_1 \) then search is enforced to the next level \( L_2 \) in order to ensure that \( \Upsilon \) is the nearest neighbor for \( \Phi \), as shown in Fig. 2.

The mathematical model of the proposed method can be summarized as follows: A given set \( \Upsilon \) has \( n \) number of points in 2-D space. It is represented by small filled circles as shown in Fig. 1, and a query point \( \Phi \) which is denoted by small circle with cross inside. The problem is to find the nearest \( \Phi \), from the points \( \Upsilon \) \( (i = 1, 2, 3 \ldots n) \) by starting from initial point \( \omega \).

In preprocessing phase, a virtually grid of a hexagon cells over \( \omega \) is drawn as shown in Fig. 3a. The located points are underneath on every hexagon and store them in a linked list while implementation. The list shows every hexagon with the points beneath on it and every point which hexagon belongs
to. After that, locate Φ in a hexagon at level \( L_0 \), then searching is continued inside boundary of the hexagon. If any point Υ is found, then searching in surrounded hexagons \( L_1 \) is carried out for nearest point to Φ rather than found in the same hexagon. This is shown in Fig. 3b. If Φ is considered as matching of query point, then the initial point found in one of the surrounded hexagons, for example, \( L_1 \), as shown in Fig. 3c. A search in the remaining hexagons in the same level is repeated until the next level \( L_2 \) is made. Our contribution is to search in two levels of hexagons in order to find the nearest point for the query point Φ in the set \( \Phi \) of data. This is a easier way than drawing circles and closer to the circle shape than square, and it is easier way to search than exhaustive method and speed up the process.

The remainder of this paper is organized as follows. Section II describes state-of-art techniques related with neighbor classification and nearest neighbor seeking problems. Proposed methodology is given in Section III. Section IV depicts the result and analysis based on the benchmark of data set in term of time and space complexities. Concluding remarks and future enhancement are suggested in Section V.

II. LITERATURE REVIEW

There are many state-of-art methods which are related with neighborhood searching and classification problems. Based on the review of previous literature, we can faction the neighborhood operation issues by two extensive categories such as seeking of query points related to its neighbors and neighborhood operation issues by two extensive categories such as seeking of query points related to its neighbors and classification rules based on weighted distance. These weights fixation specific for each unambiguous class, feature set for the classification and characteristics of the archetype. The proposed learning algorithms are resulted by minimizing the Leaving-One-Out-Cross Validation-classification error (LOOCVCE) of the given training set.

A transductive method was proposed for nearest neighbor classification through non-spectral or spectral graph minimum cuts for the rouge patterns [10]. Solution to these issues is achieved based on the LOOCVCE reverence to base classifiers such as support vectors machine (SVM), \( k \)-NN and others. A zero LOOCVCE can be achieved through a clustering method, however it has a less classification efficacy for diverse noise patterns which are involved in the training and testing processes. Lawrence et al. proposed a learning framework for fast retrieval of nearest neighbor points based \( k \)-dimensional tree space-partitioning data structure [4]. In [6], Nearest neighbor imputation of species-level was proposed for analyzing ground and remotely sensed data measured comprehensively across the same forested landscape.

In [12], \( k \)-nearest neighbor text classification algorithm based on fuzzy integral was proposed. The proposed method avoided independence demand of Dempster-shafter theory (D-S theory) and enhanced the performance of text classification.

David et al. [7] described a method to find the closest match within a database of other such items, which was a task performed in numerous domains. Image matching, data mining, and electroencephalogram data analysis are a few varied examples. They have used the extension of the concept of Euclidean distance in 2D and 3D space to higher dimensional space provides an effective comparison of items in these sorts of domains.

In [11], a method was suggested to find nearest neighbors using \( k \)-nearest neighbors algorithm (\( k \)-NN). It adopted in the development of a general methodology, neighborhood counting, for devising similarity functions. In order to measure the similarity between two data points, all neighborhoods are considered to cover both data points. Neighborhood can be defined for different types of data for multivariate data and derived a formula for such similarity, called neighborhood counting measure (NCM). The NCM was tested experimentally in the framework of \( k \)-NN. In the experiments, the NCM consistently outperforms a mixture of Euclidean and Hamming distances (HEOM). The NCM has a computational complexity in the same order as the standard Euclidean distance function. The NCM was proven the sound for multivariate data experimentally.

In [5], a method proposed for computing the given set \( S \) of points in a metric space with distance function \( D \). In order to solve the nearest-neighbor searching problem, a data structure was built for \( S \). So that for an input query point \( q \), the point \( s \) to \( S \) that minimizes \( D(s, q) \) can be found rapidly. Several measures of dimension can be estimated using nearest-neighbor searching, while others can be used to estimate the cost of the searching with low-dimensional spaces.

In [9], Sun, et al. proposed a way to incorporate the nearest neighbor search problem based on analysis of algorithms course. The problem of searching the elements of a set that are close to a given query element under some similarity criterion has a vast number of applications in many branches such

![Fig. 2. Representation of hexagonal space of points to be searched for the given query point Φ.](image-url)
as pattern recognition to textual and multimedia information retrieval. Many solutions have been proposed in different areas, in many cases without cross-knowledge. Because of this, the same ideas have been preconceived several times, and very different presentations have been given for the same approaches. They presented some basic results that explain the intrinsic difficulty of the search problem. Sunil Arya et al. suggested an optimal algorithm for approximating nearest neighbor searching in fixed dimension in [2].

In [1], they proposed the idea of using the hexagonal search space.

From previous review and from our thesis [1], it can be noticed that all methods used to find the nearest neighbors are traditional methods depending on either probability principals or analytical methods, while the proposed method, in this research will be forming a new paradigm, depending on drawings, geometry and computationally a simple calculations. Based on the result comparisons, we found that the proposed method will make searching query point in a robust way and around 20-30% faster than existing methods.

III. THEORETICAL BACKGROUND AND PROBLEM FORMULATION

The problem of finding the neighborhood center is involved on image data arbitrary dimensionality. It doesn’t represent in terms of color or intensity of pixels instead, it is a structure of related volume-of-pixels (voxels). Region of voxels dependents on which search is made by the operation. That is, voxels is either a fixed size square or a cube depending on the dimensionality of the image data. So that, neighborhood searching is needed of arbitrary dimension of data structure to accomplish the center point searching among the neighbor points. In our paper, we have utilized \( k \)-dimensional tree (kd-trees) data structure for performing exact and approximate nearest neighbor searching. Bentley J. L. introduced the kd-trees as a generalization of the binary search tree in higher dimensions [3]. Each node of the tree is implicitly associated with a \( d \)-dimensional rectangle that is called a cell. The root node is associated with the bounding rectangle, which encloses all of the data points. Each node is also implicitly associated with the subset of data points that lie within this rectangle. If the number of points associated with a node falls below a given threshold, called the bucket size, then this node is a leaf, and these points are stored with the leaf.

Based on the fundamental of kd-trees, we contribute the method of hexagonal data structure in order to decrease the turn around time of the each query point over the \( d \)-dimensional space. In our experiments, we utilized a bucket size of one. Our splitting algorithm is based on the hexagonal points as stated in Fig. 3. Queries are answered by a recursive algorithm. In the basis case, when the algorithm arrives at a leaf of the tree, it computes the distance from the query point to each of the data points associated with this node. The smallest such distance is saved. When arriving at an internal node, it first determines the side of the associated hexagonal on which the query point lies. On returning from the seeking, it is to determine that whether the cell is associated with the other siblings which is closer to the query point or not. If it is closer to the query point, then the seeking is repeated. If existence of query point, then this sibling and leafs are also visited recursively. When the search returns from the root, the closest point is
An important observation is that for each query point, every leaf whose distance from the query point is less than the nearest neighbor will be visited by the algorithm. In order to generalization, seeking algorithm is approximated to the nearest neighbor queries. Let $\Psi$ denote the allowed error bound. In the processing of an internal node, the further child is visited only if its distance from the query point is less than the distance to the closest point so far, divided by $1+\Psi$.

A. Construction of Hexagons and Algorithm

In order to construct hexagonal formulation, we model the neighbors of pixels as illustrated in Fig. 4. It is started by constructing a hexagonal ABCDEF and specifying the center of the hexagon.

**Step 1.** The initial step starts by constructing the hexagon on 2-D space, as shown in Fig. 4

**Step 2.** The second step is the preprocessing i.e. going to the center of the hexagon, and the heads of this hexagon using equations: (1)-(6).

\[
A = (x + L, y), \quad (1) \\
B = (x + L\cos(\frac{\pi}{3}), y + L\sin(\frac{\pi}{3})), \quad (2) \\
C = (x - L\cos(\frac{\pi}{3}), y + L\sin(\frac{\pi}{3})), \quad (3) \\
D = (x - L, y), \quad (4) \\
E = (x - L\cos(\frac{\pi}{3}), y - L\sin(\frac{\pi}{3})), \quad (5) \\
F = (x + L\cos(\frac{\pi}{3}), y - L\sin(\frac{\pi}{3})). \quad (6)
\]

**Step 3.** Then, the hexagon center coordinates is accumulated to the hexagons list in order to make a search phase. To read the points under the hexagon, to add it to the points list, we need to determine the region belong to the hexagon. A rectangle with $(X, Y)$ dimensions, where $X = 2L$ and $Y = 2LSin(\frac{\pi}{3})$, is created as shown in Fig.5.

**Step 4.** After the hexagon region is determined, then test the points beneath it for the black points to add it to the points list. Create pointers to the hexagon list to determine which hexagon the points belong and which points belong to the hexagon. Then, redo these processes every time upon creating a hexagon, using equations (7)-(10).

\[
R_t = y - L\sin(\frac{\pi}{3}) \quad (7) \\
R_b = y + L\sin(\frac{\pi}{3}) \quad (8) \\
R_l = x - L \quad (9) \\
R_r = x + L \quad (10)
\]

**Step 5.** To create the rest of the hexagon cells, the hexagon center is determined in the first level according the following equations and the entire process of finding first hexagon level is represented in Fig.6.

\[
L_x = x \quad (11) \\
L_y = y - \text{Level}(2LSin(\frac{\pi}{3})) \quad (12)
\]

The next step is constructing the other hexagons, locating the centers and then constructing the hexagons as the following:

- points from top to right
  \[
  \sum_{i=0}^{\text{Level}} L_x = L_x + \frac{1}{2}L, \quad L_y = L_y + L\sin(\frac{\pi}{3})
  \quad (13)
  \]

- points in the right side
  \[
  L_x = x \quad (14) \\
  \sum_{i=0}^{\text{Level}} L_y = L_y + 2L\sin(\frac{\pi}{3})
  \quad (15)
  \]

- points from right to bottom
Fig. 6. Representation of first hexagon levels.

\[ \sum_{i=0}^{\text{Level}} L_x = L_x - \frac{1}{2} L, L_y = L_y + L \sin \left( \frac{\pi}{3} \right) \] (16)

• points from bottom to left

\[ \sum_{i=0}^{\text{Level}} L_x = L_x - \frac{1}{2} L, L_y = L_y - L \sin \left( \frac{\pi}{3} \right) \] (17)

• points in the left side

\[ L_x = x \]

\[ \sum_{i=0}^{\text{Level}} L_x = L_x - \frac{1}{2} L, L_y = L_y - 2L \sin \left( \frac{\pi}{3} \right) \] (18)

• points from left to top

\[ \sum_{i=0}^{\text{Level}} L_x = L_x - \frac{1}{2} L, L_y = L_y - L \sin \left( \frac{\pi}{3} \right) \] (20)

Based on the proposed method, in the search phase, a point is chosen from the collected points in 2D space. Then, the initial hexagon center point is determined which belongs to the search space boundary. Initial hexagonal is as level 0 by calculating the location of the hexagons centers, and comparing them with other hexagons, and determine the closest other hexagons, and their levels. The goal is to specify the nearest neighbor for a given point through searching on such levels. By searching through level 0, and level 1, if there is no nearest point, then it will go to the next highest levels of the hexagonal in the space. If nearest point is found and this is a required one, then, mark the points and continue the search space until the end-of-search space is expected. Fig. 7 shows a complete searching on 2-D space.

Fig. 7. A complete hexagonal searching of 2-D space.

Fig. 8. A sample searching for the comparison of Euclidean, city block, chess board, and hexagonal on 3-D space.

Based on searching behavior of these four methods, a minimum of different center points are calculated and in the Euclidean, square root of the square of sum of the distance has been computed whereas in the city block method sum of the absolute distance is computed. The distance is computed with the condition as \( |x_1 - x_2| + |y_1 - y_2| \) and Hexagonal space distance computed as in (21).

\[ D_h = \begin{cases} |x_1 - x_2| + |z_1 - z_2| + \sqrt{3} - 1|y_1 - y_2|, & \text{if } \chi > \eta \\ \sqrt{3} - 1|x_1 - x_2| + |z_1 - z_2| + |y_1 - y_2|, & \text{otherwise} \end{cases} \] (21)

Based on searching behavior of these four methods, hexagonal searching has covered an optimal regions of voxels.
when an active-region-of-interest (AROI) is localized as compare to other three approaches.

IV. RESULTS ANALYSIS

The proposed method using hexagonal cells has been implemented in C++ and results analysis were done in Matlab 7. Three different phases of experiments were conducted. During the first phase, for the same data set, Brute and Circular methods were tested. The time complexity of the search space was measured and recorded in the repository. In the second phase, 2-D Fourier space coefficients were searched and its distances errors were computed. Discrete cosine space has been searched using our proposed methodology in the third phase of experiments.

A. Comparison of Searching Time Complexities

A real number space is formed for searching the points. In our experiments, multiple of 10 points were searched with three different methods such as brute force, circular and hexagonal neighbor searching methods. The turn around time for entire search space has been observed and depicted in Fig.9. It reveals that brute force searching requires around 40% less turn around time for 10 number of points whereas circular method requires around five more times as compare with hexagonal methods. However, if more points are required to search as shown in Fig. 9, around 490, then, hexagonal outperforms other two methods. The search points are a multiple of tens for each ticks on x-axis and zeros entries in y-axis mean turn around time less than one second in Fig. 9.

B. Distance error in Fourier space

In this experiment, we have done searching of 2-D fourier coefficients based on the hexagonal approach. Since this space is a combination of real and complex numbers, in that particularly locating the center point of the query point is a cumbersome process. However, hexagonal method was produced ±4 pixels distance errors among $F_x$ coefficients and ±3 pixels of distance errors in $F_y$ coefficients. A sample hexagonal searching based on the center point is illustrated in Fig. 10. This sample real-space was transformed into Fourier space and then searching was initiated to find the nearest points of the AROI.

Fig. 11a shows a 2-D Fourier space of the searching points and their distance errors based on the searching of AROI is shown in Fig. 11b.

C. Distance error in Discrete Cosine space

Discrete Cosine Transform (DCT) is an important process of certain kinds of applications in remote sensing, compression and computer vision hence we have done the experiments.
their distance errors were studied. The distance errors were frequencies and a set of query points have searched. Fig. 12a collection of sample data set has been transformed into Cosine of hexagonal computation.

Fig. 12. Representation of DCT space and its distance errors (a). DCT space of a sample search space (b) Error based on the query point and the distance of hexagonal computation.

on this space of domain that how our proposed method will perform while seeking the query points. For that, a collection of sample data set has been transformed into Cosine frequencies and a set of query points have searched. Fig. 12a shows a sample DCT search space. 200 Cosine frequencies points have been taken for searching the query points and their distance errors were studied. The distance errors were ±0.03 ranges in the Cosine space and shown in Fig. 12b.

V. CONCLUSION

The nearest neighbor searching is an important problem and deserves more attention because it has wide range of applications. In this paper, a robust searching method is proposed based on the hexagonal portrayed. The method was tested on sample sets with diverse data sizes of images. Searching of query points was done on three different spaces as real-space, Fourier space and DCT space. Based on the experimental results, we found that the Hexagonal-cells method outperforms other methods in term of turn around time.

The following conclusions can be made:

Time for both Brute method and circular methods for searching about the nearest neighbors is increasing as the number of searched point’s increases.

The time needed in searching about the nearest neighbor using the Hexagonal Cells method is almost constant, for the tested samples, and it can be said that it doesn’t depend strongly on number of points searched.

The time needed in searching about the nearest neighbor for searched points less than (nearly) 400 is similar in the three methods.

Future enhancement: It is recommended to create another method depending on creating another form of searching may be the pentagon. It is expected that the searching time will be reduced. Because the time for circular is more, in hexagons cells it is reduced. In pentagon it is expected to be less, i.e. as the number of sides of polygon increase the time increase, if it is known that the circle can be represented as a polygon with infinite sides.

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REFERENCES


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