World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:8, No:1, 2014

System Reduction by Eigen Permutation Algorithm and Improved Pade Approximations

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Abstract**—**A mixed method by combining a Eigen algorithm and improved pade approximations is proposed for reducing the order of the large-scale dynamic systems. The most dominant Eigen value of both original and reduced order systems remain same in this method. The proposed method guarantees stability of the reduced model if the original high-order system is stable and is comparable in quality with the other well known existing order reduction methods. The superiority of the proposed method is shown through examples taken from the literature.

Keywords**—**Eigen algorithm, Order reduction, improved pade approximations, Stability, Transfer function.

I. Introduction

EVERY physical system can be translated into a mathematical model. The mathematical procedure of system modelling often leads to comprehensive description of a process in the form of high order differential equations which are difficult to use either for analysis or controller synthesis. Hence, it is useful and sometimes necessary to find the possibility of finding some equation of the same type but of lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration. Numerous methods are available in the literature for order reduction of linear continuous systems in time domain as well as in frequency domain [1]-[4]. Further, several methods have also been suggested by combining the features of two different methods [5]-[7]. Pal et al. proposed [8] pole-clustering using Inverse Distance Criterion and time moment matching. Vishwakarma [9] modified the pole clustering by an iterative method, the complexity with these methods is in selecting poles for the clusters. A method based on Eigen Spectrum Analysis suggested by Mukherjee [10], in which both the pole centroid and system stiffness of the original and reduced order systems are kept exactly same to obtain the reduced order system.

Parmar et al. [11] proposed a mixed method using Eigen Spectrum Analysis with Factor division algorithm to determine the numerator of the reduced model with known denominator. In some cases, The difficulty with these methods [10]-[11] are tendency to become non-minimum phase due to equalization of system stiffness. Every methods have their advantages and disadvantages when tried on a particular system. Major number of methods available in the literature but no approach constantly gives the best results for all systems. In the present work, authors have directly taken poles from the proposed Eigen algorithm while the zeros are taken from the improved pade approximations to obtain the reduced order system. Proposed method reduces the problem of non-minimum phase in the reduced models. The reduction procedure is simple and computer oriented.

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II. Statement of the Problem

Let the transfer function of high order original system of the order $'n'$ be

$$G(s)=\frac{N(s)}{D(s)}=\frac{a_0+a_1s+a_2s^2+.......+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+......b_ns^n} \tag{1}$$

where $a_i(i=0,1,2...,n-1)$ and $b_i(i=0,1,2...,n)$ are known as scalar constants.

Let the transfer function of the reduced model of the order 'k' be

$$R_k(s)=\frac{N_k(s)}{D_k(s)}=\frac{c_0+c_1s+c_2s^2+........+c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+.......d_ks^k} \tag{2}$$

where $c_i(i=0,1,2...,k-1)$ and $d_i(i=0,1,2...,k)$ are unknown scalar constants.

The objective of this paper is to realize the reduced model in the form of (2) from the original system (1) such that it retains the significant features of the original high order system.

III. Description of the Method

The reduction procedure for getting reduced models consist of the following two steps:

Step. 1

Determination of the denominator polynomial for reduced model using Eigen algorithm. The proposed method is elaborated with a suitable computer oriented algorithm as shown in Fig. 1.

Step. 2

Determination of the numerator of the reduced model using improved Pade approximations [12].

The original high order system can be expanded in power series-

$$G(s) = \sum_{i=0}^{\infty} M_i s^{-i-1} \text{ (about } s = \infty) \quad (3)$$

$$= -\sum_{i=0}^{\infty} T_i s^i \quad \text{(about } s = 0) \quad (4)$$

where M_i and T_i are the i^{th} – Markov parameter and Time moment of $G(s)$ respectively. The k^{th} – order reduced model is taken as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k} d_i s^i} \quad (5)$$

The coefficients of the numerator $N_k(s)$ may be obtained by using the following set of equations.

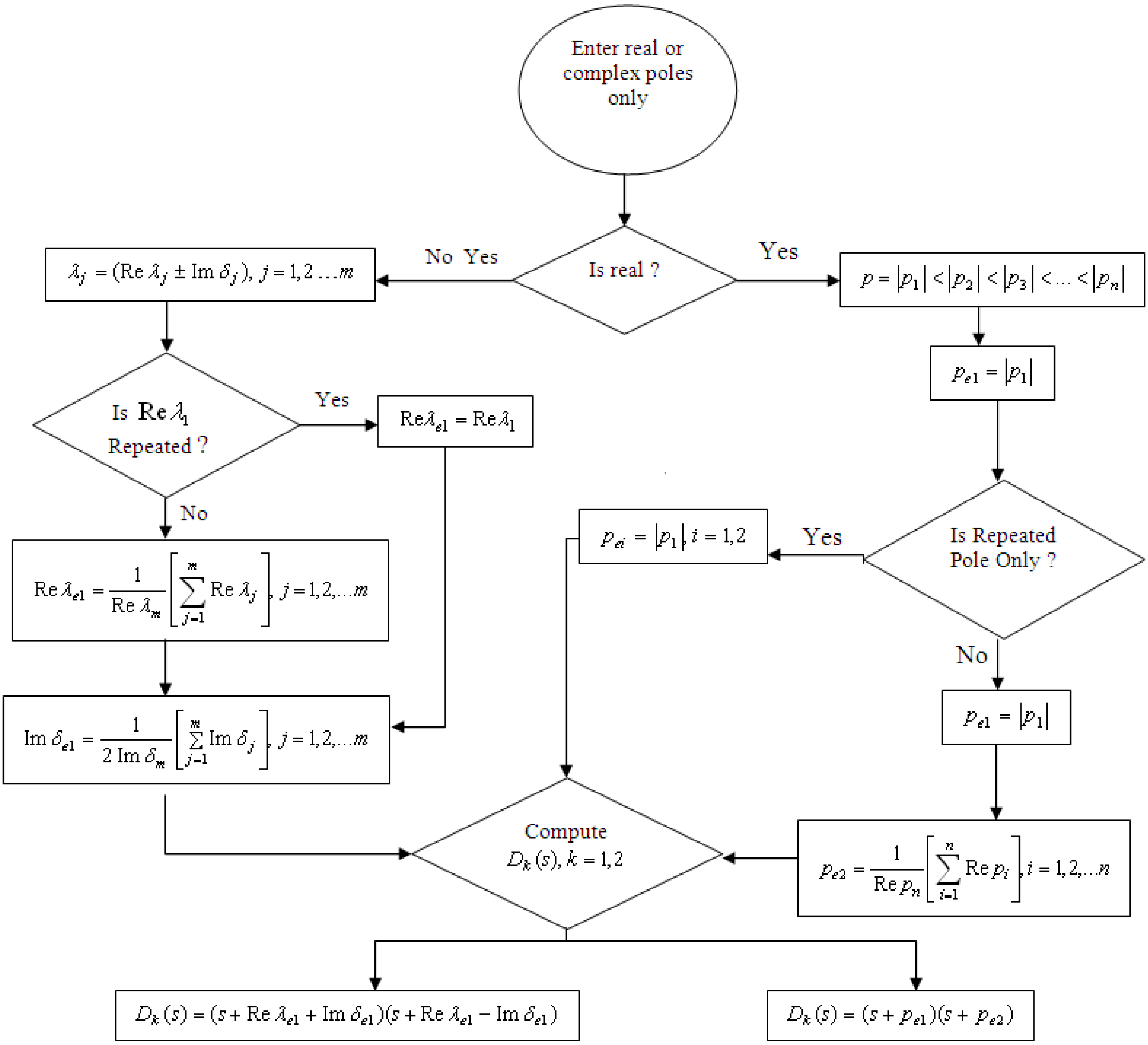

Fig. 1 Proposed Eigen Permutation Algorithm

$$\left.\begin{aligned}
&c_o = d_o T_o \\
&c_1 = d_0 T_1 + d_1 T_0 \\
&c_2 = d_0 T_2 + d_1 T_1 + d_2 T_0 \\
&\ldots \\
&\ldots \\
&c_{\alpha-1} = d_0 T_{\alpha-1} + d_1 T_{\alpha-2} + \ldots + d_{\alpha-2} T_1 + d_{\alpha-1} T_0 \\
&c_{k-\beta} = d_k M_{\beta-1} + d_{k-1} M_{\beta-2} + \ldots + d_{k-\beta+2} M_1 + d_{k-\beta+1} M_0 \\
&c_{k-\beta+1} = d_k M_{\beta-2} + d_{k-1} M_{\beta-3} + \ldots + d_{k-\beta+3} M_1 + d_{k-\beta+2} M_0 \\
&\ldots \\
&\ldots \\
&c_{k-2} = d_k M_1 + d_{k-1} M_0 \\
&c_{k-1} = d_k M_0
\end{aligned}\right\} \quad (6)$$

the coefficients $c_j; j = 1,2...(k-1)$ can be found by solving the above $'k'$ linear equations. Hence, the numerator $N_k(s)$ is obtained as

$$N_k(s) = c_0 + c_1 s + c_2 s^2 + \ldots + c_{k-1} s^{k-1} \quad (7)$$

IV. Numerical Examples

Three numerical examples have been taken from the literature to illustrate the algorithm of the proposed method. The examples are solved in details to get second order reduced model. An integral square error (ISE) and integral amplitude error (IAE) are calculated between the transient parts of the original and reduced model using MATLAB to measure the goodness of the reduced order model i.e. lower the ISE and IAE, closer the $R_k(s)$ to $G(s)$ which is given by

$$\text{ISE} = \int_0^\infty \left[g_{ij}(t) - r_{ij}(t) \right]^2 dt; \quad (8)$$

$$\text{IAE} = \int_0^\infty \left| g_{ij}(t) - r_{ij}(t) \right| dt \quad (9)$$

where, $g_{ij}(t)$ and $r_{ij}(t)$ are the unit step responses of original and reduced system respectively.

Example 1

Consider a sixth order system taken by Mahmoud and Singh [1].

$$G(s) = \frac{N(s)}{D(s)}$$

where, $N(s) = 2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1$

$$D(s) = 2s^6 + 33.6s^5 + 155.9s^4 + 209.5s^3 + 102.4s^2 + 18.3s + 1$$

Poles are : ($-0.1, -0.2, -0.5, -1, -5$ & -10)

The poles of required system i.e., second order reduced system is obtained using step-I of the proposed algorithm $p_{e1} = -0.1$ and $p_{e2} = -1.68$. Therefore

$$D_2(s) = s^2 + 1.78s + 0.168$$

The first few Time Moments and Markov Parameters are

$$T_0 = [1.000] \quad \text{and} \quad T_1 = [-10.2976]$$
$$M_0 = [1.00] \quad \text{and} \quad M_1 = [-15.300]$$

Using Step1 and Step 2 of the proposed method, the 2^{nd} – order reduced models are synthesized as-

$$R_{21}(s) = \frac{N_2(s)}{D_2(s)} = \frac{0.05s + 0.168}{s^2 + 1.78s + 0.168} \quad (T = 2, M = 0)$$

$$R_{22}(s) = \frac{N_2(s)}{D_2(s)} = \frac{1s + 0.168}{s^2 + 1.78s + 0.168} \quad (T = 1, M = 1)$$

The step response of the 2^{nd} – order reduced model i.e., $R_{21}(s)$ and original system G(s) are plotted in Fig. 2. The error index i.e., ISE, IAE are also calculated between the reduced model and original system as shown in the Table I.

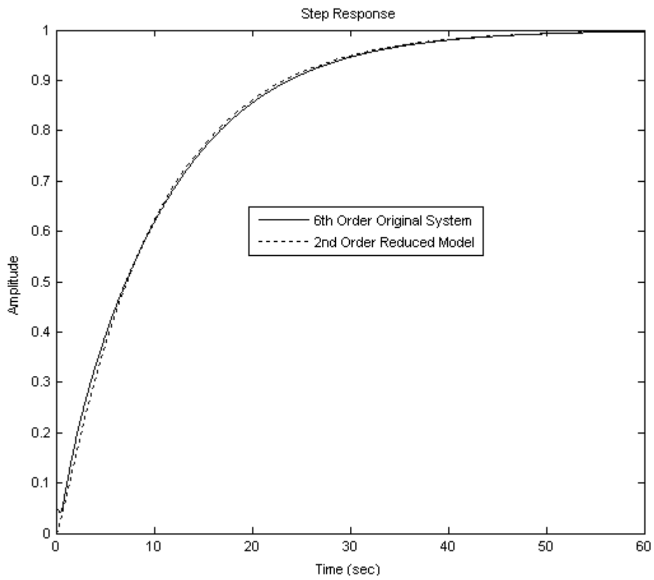

Fig. 2 Step response of original and reduced system

TABLE I
Comparison of the Proposed Method

Reduction Methods	Reduced Model $R_2(s)$	ISE	IAE
Proposed Method	$\frac{0.0496s + 0.168}{s^2 + 1.78s + 0.168}$	0.003	0.126
Vishwakarma [13]	$\frac{8s + 1}{100.805s^2 + 16.2254s + 1}$	0.843	0.223
Vishwakarma [13]	$\frac{100.8048s + 1}{100.805s^2 + 16.2254s + 1}$	4.009	22.65

Example 2

Consider a fourth order system described by its transfer function [17].

$$G(s)=\frac{28s^3+496s^2+1800s+2400}{2s^4+36s^3+204s^2+360s+240}$$

Poles are: $(-1.1967 \pm j0.6934)$ and $(-7.8033 \pm j1.3576)$

The poles of required system i.e., second order reduced system is obtained using proposed algorithm elaborated in step-I

$$\operatorname{Re}\lambda_{e1}=-1.1533 \text{ and } \operatorname{Im}\delta_{e1}=-0.7554$$

The first few Time Moments and Markov Parameters are

$$T_0=[10.0011] \quad \text{and} \quad T_1=[-7.5035]$$
$$M_0=[14.00] \quad \text{and} \quad M_1=[-3.9360]$$

Now using step 1 and step 2, Second order reduced model is obtained as

$$R_{21}(s)=\frac{8.807s+19.009}{s^2+2.3066s+1.9007} \qquad (T=2, M=0)$$
$$R_{22}(s)=\frac{14s+19.009}{s^2+2.3066s+1.9007} \qquad (T=1, M=1)$$

The step response of the 2^{nd} – order reduced model i.e., $R_{21}(s)$ and original system $G(s)$ are plotted in Fig. 3 also error index ISE and IAE are calculated between the reduced model and original system as shown in the Table II.

TABLE II
COMPARISON OF THE PROPOSED METHOD

Reduction Methods	Reduced Model $R_2(s)$	ISE	IAE
Proposed Method	$\frac{8.808s+19.0083}{s^2+2.3066s+1.9007}$	0.378	0.9689
C.B.Vishwakarma [13]	$\frac{1371.048s+2400}{201s^2+317.1498s+240}$	1.763	2.597
Seshadri V [15]	$\frac{9.046283s+13.043478}{s^2+1.701323s+1.304348}$	1.208	2.265
Prasad *et. al* [16]	$\frac{22.532255s+11.90362}{s^2+3.145997s+1.190362}$	2.743	3.371

Example 3

Consider an eight-order system described by its transfer function [14].

$$G(s)=\frac{N(s)}{D(s)}$$

where,

$$N(s)=18s^7+514s^6+5982s^5+36380s.^4+122664s^3+222088s^2+185760s+40320$$

$$D(s)=s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320$$

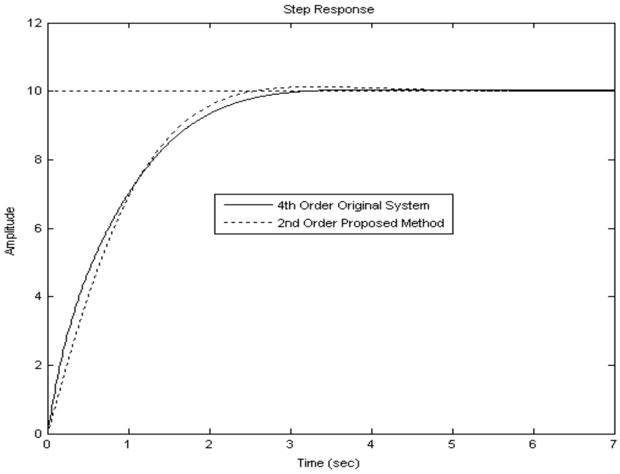

Fig. 3 Step response of original and reduced system

The poles are: $(-1,-2,-3,-4,-5,-6,-7,-8)$

Using step 1 and step 2, Second order reduced model is obtained as

$$R_{21}(s)=\frac{14.0068s+4.5}{s^2+5.5s+4.5} \qquad (T=2, M=0)$$
$$R_{22}(s)=\frac{18s+4.5}{s^2+5.5s+4.5} \qquad (T=1, M=1)$$

The step response of the 2^{nd} – order reduced model i.e., $R_{21}(s)$ and original system $G(s)$ are plotted in Fig. 4 and error index ISE & IAE are calculated between the reduced model and original system as shown in the Table III.

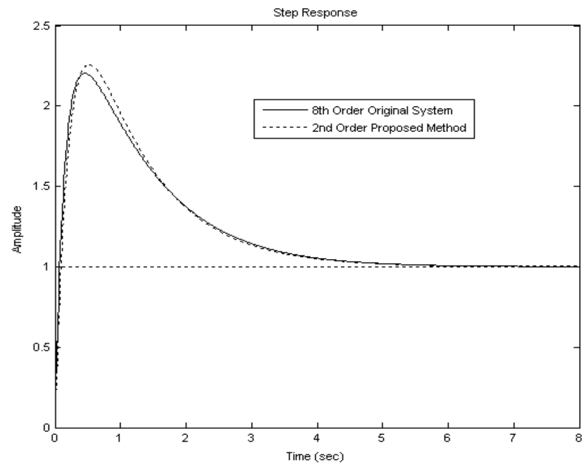

Fig. 4 Step response of the original and reduced system

TABLE II
COMPARISON OF THE PROPOSED METHOD

Reduction Methods	Reduced Model $R_2(s)$	ISE	IAE
Proposed Method	$\frac{14.0097s+4.5}{s^2+5.5s+4.5}$	1.001×10^{-2}	0.1310
G.Parmar [11]	$\frac{24.11429s+8}{s^2+9s+8}$	4.81×10^{-2}	0.3007
Mukherjee *et. Al* [3]	$\frac{11.3909s+4.4357}{s^2+4.2122s+4.4357}$	5.69×10^{-2}	0.4572
Mittal *et al.* [2]	$\frac{7.0908s+1.9906}{s^2+3s+2}$	2.689×10^{-1}	0.8054
Mishra[4]	$\frac{7.0903s+1.9907}{s^2+3s+2}$	2.689×10^{-1}	0.8054

V. CONCLUSIONS

The authors presented a mixed reduction method for reducing the order of the large scale single input single output system. In this method, the denominator polynomial is determined by using Eigen algorithm while the coefficients of the numerator are obtained by improved pade approximations. This method has been tested on three numerical examples chosen from the literature. Time response of the original and reduced systems are compared graphically and shown in Figs. 2, 3 and 4. From these comparisons, it has been concluded that the algorithm of the proposed method is simple, efficient and computer oriented. This method guarantees stability of the reduced models, if the original high-order system is stable, and also capable of making transient as well as steady state region of the original system. The proposed method is compared with some well-known order reduction methods by using performance indices, i.e. ISE and IAE.

REFERENCES

[1] M.S. Mahmoud and M.G. Singh, "Large Scale Systems Modeling", *Pergamon Press, International Series on Systems and Control* 1st ed., (1981) 3.
[2] A.K. Mittal, R. Prasad, S.P. Sharma, "Reduction of Linear Dynamic Systems Using an Error Minimization Technique", *J. Inst. Eng. India, IE (I) J. EL* 84 (March) (2004) 201–206.
[3] S. Mukherjee, Satakshi and R.C.Mittal, "Model Order Reduction Using Response-Matching Technique", *Journal of Franklin Inst.*, Vol. 342, 2005, 503-519.
[4] S. Mukherjee, R.N. Mishra, "Order Reduction of Linear Systems Using an Error Minimization Technique, *J. Franklin Inst.",* 323 (1), (1987) 23–32.
[5] C.B. Vishwakarma and R. Prasad, "Clustering Method *for* Reducing Order of Linear System Using Pade Approximation" *IETE Journal of Research*, Vol.54, No. 5, Oct. 2008, 323-327.
[6] Y. Dolgin and E. Zeheb, "On Routh-Pade Model Reduction of Interval Systems", *IEEE Trans Automat Control*, Vol. 48, No. 9, Sept. 2003, 1610-1612.
[7] V. Singh, D. Chandra and H. Kar, "Improved Routh Pade Approximants: A Computer Aided Approach", *IEEE Trans. Automat Control*, Vol. 49,No.2, Feb. 2004, 292-296.
[8] J.Pal, A.K.Sinha and N.K.Sinha, "Reduced Order Modeling Using Pole Clustering and Time Moments Matching", *Journal of the Institution of engineers (India)*, *Pt .EL*, Vol, 1995, pp.1-6.
[9] C.B.Vishwakarma "Order Reduction Using Modified Pole Clustering and Pade Approximations" *World Academy of Science, Engineering and Technology,* (2011) 56.
[10] S.Mukherjee, "Order Reduction of Linear Systems Using Eigen Spectrum Analysis", *Journal of Electrical Engineering IE(I)*, Vol 77, (1996)76-79.
[11] G.Parmar, S.Mukherjee, R.Prasad, "System Reduction Using Factor Division Algorithm and Eigen Spectrum Analysis", *Applied Mathematical Modelling Science direct*, (2007).2542-2552.
[12] J.Pal , "Improved Pade Approximamts Using Stability Equation Method" IEEE, Electronics letters 26 May 1983 vol. 19 No.11.
[13] C.B. Vishwakarma "Model Order Reduction of Linear Dynamic Systems for Control System Design" *Ph.D Thesis, Indian Institute of Technology Roorkee, Roorkee India* 2009.
[14] Shamash Y., "Linear System Reduction Using Pade Pade Approximation to Allow Retention of Dominant Modes", *International Journal of Control*, Vol. 21, (1975) 257-272.
[15] Krishnamurthy V. and Seshadri V., "Model Reduction Using the Routh Stability Criterion", *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 4, 1978, 729-731.
[16] Prasad R., Sharma S.P. and Mittal A.K.., "Linear Model Reduction Using the Advantages of Mihailov Criterion and Factor Division", *Journal of Institution of Engineers India, IE(I) Journal-EL,* Vol. 84, 2003, 7-10.
[17] Smith I.D. and Lucas T. N., "Least-Squares Moment Matching Reduction Methods", Electronics Letters, Vol. 31, (1995) 929-930.