Comparative Study on Status and Development of Transient Flow Analysis Including Simple Surge Tank

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Abstract—This paper presents the problem of modeling and simulating of transient phenomena in conveying pipeline systems based on the rigid column and full elastic methods. Transient analysis is important and one of the more challenging and complicated flow problem in the design and the operation of water pipeline systems. Transient can produce large pressure forces and rapid fluid acceleration into a water pipeline system, these disturbances may result in device failures, system fatigue or pipe ruptures, and even the dirty water intrusión. Several methods have been introduced and used to analyze transient flow, an accurate analysis and suitable protection devices should be used to protect water pipeline systems. The fourth-order Runge-Kutta method has been used to solve the dynamic and continuity equations in the rigid column method, while the characteristics method used to solve these equations in the full elastic method. The results obtained provide that the model is an efficient tool for flow transient analysis and provide approximately identical results by using these two methods. Moreover, using the simple surge tank "open surge tank" reduces the unfavorable effects of transients.

Keywords—Elastic method, Flow transient, Open surge tank, Pipeline, Protection devices, Numerical model, Rigid column method.

I. INTRODUCTION

The study of fluid transient began with the investigation of sound waves in air, the propagation of waves in shallow water and the flow of blood in arteries, and increased in recent years [1]-[3].

Hydraulic transient is the flow and pressure condition that occurs in a hydraulic system between an initial steady-state condition and a final steady-state condition, when velocity changes rapidly because a flow control component changes status (for example, a valve closing or pump turning off), the change moves through the system as a pressure wave. If the magnitude of this pressure wave is great enough and adequate transient control measures are not in place, a transient can cause system hydraulic components to fail, the general term of the transients is named as water hammer [4].

In general, any disturbance in the water generated during a change in mean flow conditions will initiate a sequence of transient pressures (waves) in the water distribution system. Potentially, these disturbances can create serious consequences for water utilities if not properly recognized and addressed by proper analysis and appropriate design and operational considerations. Hydraulic systems must be designed to accommodate both normal and abnormal operations and be safeguarded to handle adverse external events such as power failure, pipeline fracture, etc. [5]. There are several principal design tactics for mitigation of water hammer such as:

1. Alteration of pipeline properties such as profile, pressure class, type of pipe and diameter,
2. Implementation of improved valve and pump control procedures, and
3. Design and installation of surge control devices such as pressure relief valves, surge tanks, air chambers, etc.

System flow control operations are performed as part of the routine operation of a water distribution system, examples of system flow control operations include opening and closing valves, starting and stopping pumps, and discharging water in response to fire emergencies. These operations cause hydraulic transient phenomena, especially if they are performed too quickly.

Various methods of analysis were developed for the problem of transient flow in pipeline systems, they range from approximate analytical approaches whereby the nonlinear friction term in the momentum equation is either neglected or linearized, to numerical solutions of the nonlinear system. These methods can be classified as follows:

A. Arithmetic Method
This method neglects friction [6]-[8].

B. Graphical Method
This method neglects friction in its theoretical development but includes a means of accounting for it through a correction [9].

C. Method of Characteristics
This method is the most popular approach for handling hydraulic transients. Its thrust lies in its ability to convert the two partial differential equations (PDEs) of continuity and momentum into four ordinary differential equations that are solved numerically using finite difference techniques [1], [10]-[12].
D. Algebraic Method

The algebraic equations in this method are basically the two characteristic equations for waves in the positive and negative directions in a pipe reach, written such that time is an integer subscript [13].

E. Wave-Plan Analysis Method

This method uses a wave-plan analysis procedure that keeps track of reflections at the boundaries [14].

F. Implicit Method

This implicit method uses a finite difference scheme for the transient flow problem. The method is formulated such that the requirement to maintain a relationship between the length interval Δx and the time increment Δt is relaxed [15].

G. Linear Methods

By linearizing the friction term, an analytical solution to the two PDEs of continuity and momentum may be found for sine wave oscillations. The linear methods of analysis may be placed in two categories: the impedance method, which is basically steady-oscillatory fluctuations set up by some forcing function, and the method of free vibrations of a piping system, which is a method that determines the natural frequencies of the system and provides the rate of dampening of oscillations when forcing is discontinued [13].

H. Perturbation Method

With this method, the nonlinear friction term is expanded in a perturbation series to allow the explicit, analytical determination of transient velocity in the pipeline. The solutions are obtained in functional forms suitable for engineering uses such as the determination of theoretical values of velocity and pressure, their locations along the pipeline, and their times of occurrence [16].

To reduce the dangerous effects of water hammer; the surge devices have been added to the pipeline systems. Most of these protection equipments aim to protect against unfavorable large pressure fluctuations and tend to maintain the pressure at a nearly constant value at some fixed places [1], [17], [18]. Several criteria can be adopted to determine which surge devices are to be used such as the effectiveness, dependability, evaluation of cost character and frequency of maintenance requirement over an extended period [19].

Each surge device has its own characteristics: the simple surge tank is placed vertical and has larger diameter than that of pipe to avoid spillage, it is used to minimize the entrance losses [1], [18]-[20]. When the diameter of the surge tank is smaller than that of the pipe, then it is called a stand pipe. It has lower cost but provides less protection than the simple surge tank, also it is used when the spillage can be allowed [19], [20]. In the case that the size of the surge tank is to be small, then the throttled (orifice) surge tank can be used, but the penalty for that is high pressure in the pipe and the transient continues past the tank [20].

The use of digital computers for analyze hydraulic transients has been used tenths of years ago [18] and increased considerably in recent years, also sophisticated numerical methods has been introduced for such analyses [1], [2].

In this article computer software had been developed in order to simulate and design hydraulic transients in pipeline systems.

II. MATERIALS AND METHODS

Most of the formulations shown in this document are taken from [21].

The configurations considered here is shown in Figs. 1 and 2 consist of a long pipeline in which water is flowing due to effect of pump for the first case and by the effect of a reservoir for the second case.

A. Open Surge Tank at Pump Downstream “Instantaneous Pump Stopping”

Two methods (rigid column and elastic) have been used to simulate and to analyze the flow transient in pipeline systems.

1. Elastic Method

When changes in velocity, and consequently pressure, occur rapidly, both the compressibility of the liquid and the elasticity of the pipe must be included in the analysis. This procedure is often called "elastic" or "water-hammer" analysis and involves acoustic pressure waves traveling through the pipe and the solution of partial differential equations. Even though the term transient refers to all unsteady flows, it is generally used to identify the "elastic" case specifically [20].

The simplified equations that govern unsteady flow in pipeline system are motion and continuity equations which solved together ((1) and (2)).

\[
\frac{\partial H}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + \frac{V|V|}{2gD} = 0 \quad (1)
\]

\[
\frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} = 0 \quad (2)
\]

where: H is the piezometric head, V is the flow velocity, x is the distance along the pipeline, t is the time, g is the acceleration of gravity, f is the pipeline friction factor (assumed constant), D is the pipeline diameter, and a is the celerity of a pressure wave in the pipeline.

By multiplying (2) by unknown constant λ, adding it to (1), and by rearranging and taking the total derivative to obtain the compatibility equations (3) and (5):

\[
\frac{g}{a} \frac{dH}{dt} + \frac{dV}{dt} + \frac{f|V|}{2D} = 0 \quad C^+equation \quad (3)
\]

\[
For \quad \frac{dx}{dt} = +a \quad (4)
\]

\[
\frac{g}{a} \frac{dH}{dt} - \frac{dV}{dt} - \frac{f|V|}{2D} = 0 \quad C^-equation \quad (5)
\]

\[
For \quad \frac{dx}{dt} = -a \quad (6)
\]
Solution of (3) and (5) is done by using finite differences solution. The pipeline is divided to N equal sections of length Δx, the calculations were made at node 1 (Fig. 1).

A transient is generated at time t by pump failure; solution of the equations governing the transient phenomena consists of finding the values of head and flow at each node as the transient progresses. Calculations were made at each Δt time interval, where L is the pipeline length.

\[ \Delta t = \frac{L}{aN} \]  

Therefore, (9) is obtained:

\[ HP_1 - BQ P_1 = CM \]  

where: \( H_2 \) and \( Q_2 \) are the head and flow respectively at the nodes 2 at instant time \( t_0 \).

With \( B = \frac{a}{gA} \) and \( R = \frac{\Delta x}{2gD^2} \)

The equations required to calculate the head and flow at the boundary after introducing a simple surge tank at node 1, are illustrated through the following system of equations. In addition to \( C^-equation\) that derived above, the unknown variables in these equations which identified in Fig. 1 are: \( Q P_C, Q P_D, H P_1 \) and \( X L P_C \).

\[ HP_1 = CM + BQ P_D \]  

\[ X L P_C = X L_C + (Q P_C + Q_C) \frac{\Delta t}{2A_C} \]  

\[ HP_1 = X L P_C + K_C Q P_C |Q P_C| + z \]  

where: \( Q P_1 = 0 \) in the sudden pump stopping

\[ Q P_D = -Q P_C \]  

where: \( Q P_C \) is the exchanged discharge between the pipe and the open surge tank, \( Q P_D \) is the downstream discharge, \( Q P_1 \) is the pump discharge, \( H P_1 \) is the piezometric head at node 1 and \( X L P_C \) is the water level inside the surge tank. These equations combined to obtain a single nonlinear equation in \( Q P_C \).

\[ K Q P_C |Q P_C| + C_1 Q P_C + C_2 = 0 \]  

where:

\[ C_1 = \frac{\Delta t}{2A_C} + B \quad \text{and} \quad C_2 = X L_C + Q_C \frac{\Delta t}{2A_C} + Z_1 - CM \]

2. Rigid Column Method

Fig. 2 (a) illustrates a typical open surge tank. A pump stopping causes the flow variation, which results in the oscillations of the water level in the tank. A freebody diagram of a horizontal pipeline having constant cross-sectional area is shown in Fig. 2 (b).
The forces acting on the liquid are:

\[ F_{p1} = \gamma A (H_0 + Z) \]  \hspace{1cm} (18)
\[ F_{p2} = \gamma A (H_0 + h_L + h_e) \]  \hspace{1cm} (19)
\[ F_f = \gamma A h_f \]  \hspace{1cm} (20)

where: \( A \) is the cross-sectional area of the pipeline, \( H_0 \) is the static head, \( \gamma \) is the specific weight of liquid, \( h_L \) is the velocity head losses at the intake, \( h_e \) is the intake head losses, \( h_f \) is the friction and form losses in the pipeline between the intake and the closed surge tank, and \( Z \) is the water level difference between the open surge tank and the reservoir level (positive upward). Considering the downstream flow direction as positive, the net force acting on the liquid element in the positive direction is:

\[ \sum F = F_1 - (F_2 + F_3) \]  \hspace{1cm} (21.a)
\[ \sum F = \gamma A (z - h_L - h_e - h_f) \]  \hspace{1cm} (21.b)

According to Newton's second law of motion, the rate of change of momentum is equal to the net applied force. Therefore,

\[ \frac{\gamma L}{g} \frac{dQ}{dt} = \gamma A (z - h_L - h_e - h_f) \]  \hspace{1cm} (22)

in which: \( \gamma L/g \) is the mass of the liquid element, \( L \) is the length of the pipeline, \( g \) is the acceleration due to gravity, \( Q \) is the flow in pipeline and \( t \) is the time.

By defining the total head losses as: \( h = h_L + h_e + h_f = RQ|Q| \) in which \( R \) is the flow resistance due to the singular and linear friction, (22) may be written as:

\[ \frac{dQ}{dt} = \frac{\gamma A}{L} (z - RQ|Q|) \]  \hspace{1cm} (23)

The second differential equation is determined by applying the principle of conservation of mass at the surge tank. Note that if \( z \) increases, this means that the flow in the pipe is negative.

Referring to Fig. 2 (a), the continuity equation for the junction of the pipeline and the open surge tank may be written as:

\[ Q_p = Q_c + Q \]  \hspace{1cm} (24)

in which: \( Q_c \) is the flow into the open surge tank (positive into the tank), \( Q_p \) is the pump flow, and \( Q \) is the flow in pipeline.

Where: \( Q_p = 0 \) in instantaneous pump stopping.

Oscillation in the open surge tank water level is defined by the continuity equation which is written:

\[ A_B dz = Q_B dt \]  \hspace{1cm} (25.a)
\[ \frac{dz}{dt} = \frac{Q_c}{A_c} \]  \hspace{1cm} (25.b)

The flow into the open surge tank being opposite sign of the fluid in the pipe, the continuity equation according to the flow \( Q \) is reduced to:

\[ \frac{dz}{dt} = - \frac{Q}{A_c} \]  \hspace{1cm} (26)

where: \( z \) is the difference in water level in the tank compared to the water level in the reservoir.

The dynamic and continuity equations (23) and (26) constitute the system of differential equations governing the unsteady flow between the surge tank and the reservoir after pumping stops.

**B. Open Surge Tank at Downstream end of the Pipe - "Instantaneous Valve Closure"**

The considered configuration in this case consists of single pipeline with a reservoir at upstream end and open valve at downstream is considered.

Also two methods (rigid column and elastic) have been used to simulate and to analyze the flow transient in pipeline systems.

1. Elastic Method

The unknowns head and flow at node \( N \) at time \( t + \Delta t \) are labeled \( HP_N \) and \( QP_N \) can be calculated by integrating (3), the known head and flow at the previous time step are \( H_N \) and \( Q_N \), before integrating, the equation is multiplied by \( dt/g \), \( V \) changed to \( Q \), and \( dt \) replaced by \( dx \). The \( C^+ \) equation (28) is obtained by integrating from node \( (N-1) \) to node \( (N) \) gives:

\[ \int_{H_{N-1}}^{H_N} dH + B \int_{Q_{N-1}}^{Q_N} dQ + R_1 \int_{Q_{N-1}}^{Q_N} Q|Q|dx = 0 \]  \hspace{1cm} (27)

\[ HP_N + BQP_N = CP \]  \hspace{1cm} (28)

where: \( C^+ \) is the linear friction coefficient.
The equations required to calculate the head and flow at the boundary where the open surge tank is installed at node N (Fig. 3) are illustrated through the following system of equations, in addition to the equation that derived above, the unknown variables in these equations which identified in Fig. 3 are: \( Q_P, Q_D, Q_P_C, H_P_N \) and \( XLP_C \).

\[
XLP_C = XL_C + (Q_P + Q_C) \frac{\Delta t}{2A_C} \tag{29}
\]

\[
H_P_N = XLP_C + Z_N + K_C Q_P_C |Q_P_C| \tag{30}
\]

\[
Q_P = Q_P + Q_D \tag{31}
\]

where: \( Q_D = 0 \) in instantaneous valve closure.

A simplified single non-linear equation in \( Q_P \) that governing the flow at the surge tank location is obtained by combining the above equations.

\[
K Q_P C |Q_P_C| + C_1 Q_P_C + C_2 = 0 \tag{32}
\]

where

\[
C_1 = \frac{\Delta t}{2A_C} + B \quad \text{and} \quad C_2 = XL_C + Q_C \frac{\Delta t}{2A_C} + Z_N - CP
\]

In the case where the chimney is provided with an orifice at its base, the pressure drop that must be added to cause the expression of the force \( F_{P_1} \) the forces acting on the liquid mass contained in the conduit the pressure forces are: \( F_{P_1} \) and \( F_{P_2} \).

\[
F_{P_1} = \gamma A (H_0 - h_i) \tag{33}
\]

\[
F_{P_2} = \gamma A (H_0 + Z - h_0) \tag{34}
\]

Definition of variables and diagrams of forces and the frictional force on the lateral wall of the pipe:

\[
F_f = \gamma A 145 h_f \tag{35}
\]

where: \( A \) is the cross-sectional area of the pipeline, \( H_0 \) is the static head, \( \gamma \) is the specific weight of liquid, \( h_i \) is the velocity head losses at the intake, \( h_f \) is the friction and form losses in the pipeline between the intake and the closed surge tank, \( K_C \) is the coefficient of singular head losses that takes into account the entrance of the surge tank, and \( Z \) is the water level difference between the open surge tank and the reservoir level (positive upward).

\[
\frac{dQ}{dt} = \frac{\Delta A}{L} (z + RQ |Q| + KQ_C |Q_C|) \tag{36}
\]

with: \( h_0 = KQ_C |Q_C| \) is the head loss at the surge tank entrance, \( K \) is the orifice singular loss coefficient, \( Q_C \) is the flow entering the surge tank (positive) or leaving (negative). The equation of motion becomes:

The previously established continuity equation remains valid, namely:

\[
\frac{dz}{dt} = \frac{Q_C}{A_C} = \frac{Q - Q_D}{A_C} \tag{37}
\]
and, if the valve closure is instantaneous, \( Q_B \) is zero, the above equation reduces to

\[
\frac{dz}{dt} = \frac{Q}{A_C}
\]

III. SIMULATION RESULTS

In order to demonstrate the use of the elastic and rigid column method for transient, the configurations considered here are shown in Figs. 5 and 11 consist of a long pipeline in which water is flowing due to effect of pump for the first case and by the effect of a reservoir for the second one. The foregoing case studies illustrate a typical concept to consider when analyzing hydraulic transients.

A. Case 1: Pump Feeds a Reservoir at Downstream End

A pump feeds a reservoir as it shown in Fig. 5, where the water level elevation \( H_R = 30 \text{ m} \), through a conduit having the following characteristics, \( L = 2000 \text{ m} \), \( D = 2 \text{ m} \), \( \lambda = 0.025 \), and \( a = 1100 \text{ m/s} \). At a given moment the pump is stopped instantaneously after a power outage.

An open surge tank installed just immediately downstream of the pumping station. The surge tank has \( 5 \text{ m}^2 \) cross-sectional area and an entrance diameter is \( 0.15 \text{ m} \).

This case study demonstrates the capability of the developed program to simulate the water hammer effect by simulating the sudden pump stopping at the upstream of a long pipe in which water is flowing. The model takes into account the fluid and pipe wall elasticity. For this case study, a simple system is presented in order to best illustrate the water hammer simulation capability of the developed program. The simulation results for the unprotected pipeline are presented in the following figures for each case.

1. Elastic Method: Instantaneous Pump Stopping without Including the Simple Surge Tank

A typical starting point of a transient study is to estimate the worst-case (instantaneous stopping) events in the pipeline systems. If the protection strategy is well designed, the combination of various transient events that creates the pressure force will dissipate.

Fig. 6 shows that the maximum and minimum pressure occurred at the times 7.272 and 3.636 second respectively, and the pressure head amplitude become weaker from one cycle to another till it is vanish due to head losses, the maximum and the minimum pressure envelopes for the unprotected pipeline along the entire pipe length are 205.02m and -148.14m respectively, while in the steady state before generating any transient are 33.228m and 30m respectively.
2. Elastic Method: Instantaneous Pump Stopping with Simple Surge Tank Included

Fig. 7 Transients in a pumping system (a) Head variation versus time at the pomp (b) Hydraulic grade lines (with including the open surge tank) (c) Flow variations versus time at the pomp and (d) Variation of head at each node Vs time (with including the open surge tank)

Fig. 7 shows that the maximum pressure and minimum pressure occurred at the times 87.628 and 30.724 second respectively, and the pressure head amplitude become weaker from one cycle to another till it vanish due to head losses, the maximum and the minimum pressure envelopes for this case are reduced to 43.103m and 14.113m respectively the exchanged discharge between the pipe and the surge tank begins from 5m³/s and it reduced from a cycle to another.

3. Rigid Method: Instantaneous Pump Stopping with Simple Surge Tank Included

The same previous case study has been considered for rigid column method, the simulation results are presented in Fig. 8.

Fig. 8 Transients in a pumping system (a) Head variation versus time at the pomp and (b) Flow variation versus time at the pomp (with including the open surge tank)
4. Comparison between the Two Methods

The compassion between the two methods has been done and represented Fig. 9.

B. Case 2: Simple Reservoir Valve System

A reservoir connected to a horizontal pipeline and a butterfly valve at downstream end. The pipe characteristics are: \( L = 2000 \text{m}, \ D = 2 \text{m}, \ \lambda = 0.025, \ H_{R1} = 30 \text{m} \). Pressure wave speed is assumed to be 1100 m/s. The valve diameter is equal to the pipe diameter and the initial valve opening degree is 50%. An open surge tank installed just immediately upstream of the valve with the same characteristics of the previous one.

The valve flow variation coefficient according to its opening degrees and the case study are presented in Figs. 10 and 11 respectively.

This case study demonstrates the capability of the developed program to simulate the water hammer effect by simulating the valve closure at the pipe downstream in which water is flowing. The model takes into account the fluid and pipe wall elasticity.

1. Elastic Method: Instantaneous Valve Closure without Including Protective Devices

A typical starting point of a transient study is to estimate the worst-case (instantaneous stopping) events in the pipeline systems.
Fig. 12 shows that the maximum and minimum pressure occurred at the times 3.454 and 7.272 second respectively, and the pressure head amplitude become weaker from one cycle to another till it is vanish due to head losses, the maximum and the minimum pressure envelopes for the unprotected pipeline along the entire pipe length are 451.182m and -374.487m respectively, while in the steady state before generating any transient are 30m and 11.867m respectively.

2. Elastic Method: Instantaneous Valve Closure with Open Surge Tank Included

Fig. 13 Transients in simple reservoir valve system (a) Head variation versus time at the valve location (b) Hydraulic grade lines (with including the open surge tank) c) Flow variation versus time at valve location and (d) Variation of head at each node Vs time (with including the open surge tank)
Fig. 13 shows that the maximum pressure and minimum pressure occurred at the times 34.542 and 91.809 second respectively, and the pressure head amplitude become weaker from one cycle to another till it is vanish due to head losses, the maximum and the minimum pressure envelopes for this case are reduced to 61.44m and 7.93m respectively, the exchanged discharge between the pipe and the surge tank begins from 5 m³/s and it reduced from a cycle to another, and the pressure head at the nodes further from the source of transient are less amplitude and become more less whenever it goes further from it.

3. Comparison between the Two Methods
The compassions between the two methods has been done and represented in Fig. 14.

Fig. 14 Transients in a simple reservoir valve system: Head variation versus time at the valve location (Comparison between the two methods)

Approximately identical results have been obtained by using these two methods, due to the fact that the speed of propagation is much greater than the speed at which boundary conditions responds, where the transient is generated. The surge tank acts as a reservoir and prevents most of the transient pressures.

When the analysis had been done for the both cases considered in this work which are a pipes connected to a surge tanks, the changing of the boundary condition (i.e. the changing of the water level inside the tank) is so slow with respect to the propagation of the pressure wave that change the boundary condition (changed level), the wave speed is assumed to be a high value. High value or infinite wave speed means rigid pipe and incompressible fluid, and the pipe which considered in the elastic analysis is rigid enough, which may explaining the obtaining the same results as in the rigid column theory. Moreover, the current case study is a simple pipeline system is presented in order to best illustrate the capability of the developed program to simulate the flow transients.

IV. CONCLUSIONS
Rigid column method effectively avoids the interpolation error occurs in the characteristics method and reduce its calculations complexity. Moreover, this method provides nearly the same simulation results as the full elastic method for some cases, but not always suitable; However, In general, to be in the safe side, the full elastic method should be used for transient analysis; because it takes into consideration all factors that play an important agent on the transient.

Numerical simulation models are helpful tools for the engineers to decide among different technical and economical alternatives regarding to the adverse and dangerous effect occurs in the flow transient state.

Using the simple surge tank for the both cases ensures adequate protection for the pipeline system against overpressure and low pressure.

REFERENCES