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Unsteady Stagnation-Point Flow towards a Shrinking Sheet with Radiation Effect

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***Abstract*—**In this paper, the problem of unsteady stagnation-point flow and heat transfer induced by a shrinking sheet in the presence of radiation effect is studied. The transformed boundary layer equations are solved numerically by the shooting method. The influence of radiation, unsteadiness and shrinking parameters, and the Prandtl number on the reduced skin friction coefficient and the heat transfer coefficient, as well as the velocity and temperature profiles are presented and discussed in detail. It is found that dual solutions exist and the temperature distribution becomes less significant with radiation parameter.

***Keywords*—**Heat transfer, Radiation effect, Shrinking sheet Unsteady flow.

I. Introduction

SHRINKING sheet boundary layer flow problem has attracted many researchers due to its applications such as the shrinking film which is used in packaging of bulk product. Wang [1] was the first to study the shrinking sheet problem by considering the case of stretching deceleration surface. Later, Miklavcic and Wang [2] proved the existence and uniqueness for steady viscous flow due to a shrinking sheet. Wang [3] also studied the stagnation flow towards a shrinking sheet, by considering two-dimensional and axisymmetric stagnation flows. Other related published papers studied the steady boundary layer flow problems induced by shrinking sheets in various aspects can be found in [4] – [7].

To date, many researches have been done on unsteady boundary layer problems due to a stretching sheet. Namely, Surma Devi et al [8] and Lakshmisha et al. [9] studied unsteady three-dimensional boundary layer flow over a stretching surface. Ali et al. [10] investigated the unsteady uniform flow across a stretching surface in an arbitrary direction, where the unsteadiness is caused by the impulsive motion of the stretching surface. While, Abd El-Aziz [11] added radiation effect to the flow over an unsteady stretching sheet and reported that for larger Prandtl number, the effect of radiation parameter becomes more significant. On the other hand, only a few studies have been reported on the problem of unsteady boundary layer flow due to a shrinking sheet. The unsteady boundary layer flow due to a shrinking sheet with mass transfer has been considered by Fang et al. [12] and this idea has been extended by Ali et al. [13] to the case of rotating fluid. Further, Ali et al. [14] included the radiation effect in their study and dual solutions were reported for non zero radiation. Recently, Midya [15] investigated the effect of radiation and heat sink on heat transfer in MHD boundary.

The present paper aims to study the problem of unsteady stagnation-point flow and heat transfer across a shrinking sheet with radiation effect is taken into consideration. The transformed equations are solved numerically using the shooting method. To the best of our knowledge, the present problem has not been considered before, thus the reported results are new.

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II. Basic Equations

Consider the unsteady stagnation-point flow of a viscous and incompressible fluid towards a shrinking sheet which at time $t = 0$ starts impulsively or suddenly from rest. Following Wang [3], it is assumed that the unsteady potential stagnation-point flow at infinity is given by $u_e(\bar{t},x) = (1-\lambda\bar{t})^{-1} ax$ and $w_e(\bar{t},x) = (1-\lambda\bar{t})^{-1} az$, where a is the strength of the stagnation flow, $\bar{t} = at$ is the dimensional time and λ is a parameter associated with the flow unsteadiness. It is also assumed that on the stretching surface, the velocity are $u_w(\bar{t},x) = b(1-\lambda\bar{t})^{-1}(x+c)$ and $w_w(\bar{t},x) = 0$, where b is the stretching rate (shrinking if $b < 0$) and $-c$ is the location of the stretching origin. The governing unsteady boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial \bar{t}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$\frac{\partial v}{\partial \bar{t}} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$\frac{\partial w}{\partial \bar{t}} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\frac{\partial^2 w}{\partial z^2} \tag{4}$$

$$\frac{\partial T}{\partial \bar{t}} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial z} \tag{5}$$

where (u,v,w) are the velocity components along the axes (x,y,z), T is the fluid temperature, ν is the kinematic viscosity, ρ is the fluid density and c_p is the specific heat of the fluid at a constant pressure. The initial and boundary conditions are

$$\begin{aligned}
&\bar{t}<0: u=v=w=0, T=T_\infty \text{ for any } x,y,z \\
&\bar{t}\geq 0: u=u_w(\bar{t},x)=(1-\lambda\bar{t})^{-1}b(x+c) \\
&\qquad w=0=v_w(y,t), T=T_w \text{ at } z=0 \\
&\qquad u=u_e(\bar{t},x)=(1-\lambda\bar{t})^{-1}ax, \\
&\qquad v=0=v_e(\bar{t},y)=0, \\
&\qquad w=w_e(\bar{t},z)=-(1-\lambda\bar{t})^{-1}az, \\
&\qquad T\to T_\infty \text{ as } z\to\infty
\end{aligned} \tag{6}$$

where T_∞ is the ambient temperature and $T_w(x)$ is the surface temperature.

Using the Rosseland approximation for radiation (Raptis et al. [16] or Bataller [17], the radiative heat flux q_r, is simplified as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z} \tag{7}$$

where σ^* and k^* are the Stefan- Boltzmann constant and the mean absorption coefficient, respectively. It is assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature (see Cortell [18]. Hence, expanding T^4 in a Taylor series about T^∞ and neglecting higher order terms, we get

$$T^4\cong 4T_\infty^3T-3T_\infty^4. \tag{8}$$

In view of (7) and (8), (5) reduces to

$$\frac{\partial T}{\partial\bar{t}}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\left(\alpha+\frac{16\sigma^*T_\infty^3}{3\rho c_p k^*}\right)\frac{\partial^2T}{\partial z^2} \tag{9}$$

where $\alpha=k/\rho c_p$ is the thermal diffusivity of the fluid. It is seen from this equations that the effect of radiation is to enhance the thermal diffusivity. If we take $N_R=16\sigma^*T_\infty^3/(3kk^*)$ as the radiation parameter, (9) becomes

$$\frac{\partial T}{\partial\bar{t}}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha(1+N_R)\frac{\partial^2T}{\partial z^2}. \tag{10}$$

III. Two-Dimensional Case

Following Wang [3] and Surma Devi et al. [8], we introduce the similarity transformations

$$\begin{aligned}
&u=(1-\lambda\bar{t})^{-1}\left[axf'(\eta)+bcg(\eta)\right],\ v=0, \\
&w=-(a\nu)^{1/2}(1-\lambda\bar{t})^{-1/2}f(\eta), \\
&\theta(\eta)=(T-T_\infty)/(T_w-T_\infty), \\
&\eta=(a/\nu)^{1/2}(1-\lambda\bar{t})^{-1/2}z.
\end{aligned} \tag{11}$$

Substituting (11) into (2), (3) and (10), we obtain the following coupled ordinary differential equations:

$$f'''+ff''+1-f'^2+A\left(1-f'-\frac{\eta}{2}f''\right)=0 \tag{12}$$

$$g''+fg'-f'g-A\left(g+\frac{\eta}{2}g'\right)=0 \tag{13}$$

$$\left(\frac{1+N_R}{\Pr}\right)\theta''+\left(f-A\frac{\eta}{2}\right)\theta'=0 \tag{14}$$

subject to the boundary conditions

$$\begin{aligned}
&f(0)=0, f'(0)=b/a=\gamma, g(0)=1, \theta(0)=1, \\
&f'(\infty)=1, g(\infty)=0, \theta(\infty)=0
\end{aligned} \tag{15}$$

where $A=\lambda/a$ is the unsteadiness parameter, $\Pr=\nu/\alpha$ is the Prandtl number, γ is the stretching/shrinking parameter where $\gamma>0$ refers to a stretching case and $\gamma<0$ corresponds to the shrinking case and primes denote differentiation with respect to η. The pressure p can be recovered by

$$\frac{p}{\rho}=\frac{p_0}{\rho}-(1-\lambda\bar{t})^{-2}a^2\frac{x^2}{2}-\frac{w^2}{2}+\nu\frac{\partial w}{\partial z}+\frac{\partial}{\partial t}\int_z^\infty w\,dz \tag{16}$$

where p_0 is the stagnation pressure. It is worth mentioning that when $N_R=0$, the thermal radiation's effect is not considered in (14).

For this flow, the normalized streamlines $\bar{\varphi}$ can be defined as

$$\bar{\varphi}=(1-\lambda\bar{t})^{-1/2}\left[xf(\eta)+\gamma c\int_0^\eta g(s)\,ds\right] \tag{17}$$

where $\bar{\varphi}=\varphi/(a\nu)^{1/2}$ with φ is defined in the usual way as $u=\partial\varphi/\partial z$ and $v=-\partial\varphi/\partial x$. The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2},\ Nu = \frac{x q_w}{k\left(T_w - T_\infty\right)} \tag{18}$$

where the skin friction τ_w and the heat flux q_w are defined as

$$\tau_w = \mu\left(\frac{\partial u}{\partial z}\right)_{z=0},\ q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} \tag{19}$$

with μ and k being the dynamic viscosity and thermal conductivity, respectively. Using (11), (18) and (19), we get

$$\mathrm{Re}_x^{1/2}\, C_f = f''(0) + \frac{bc}{u_e} g'(0),\ \mathrm{Re}_x^{-1/2}\, Nu = -\theta'(0) \tag{20}$$

where $\mathrm{Re}_x = u_w x/\nu$ is the local Reynolds number based on the shrinking sheet velocity u_w.

IV. Results and Discussion

Equations (12), (13) and (14) subject to the boundary conditions (15) have been solved numerically using the shooting method as described in a paper by Meade et al [19]. In order to check the accuracy of the result obtained, comparison with those of Wang [3], [20] are made, as shown in Table I (for stretching sheet) and Table II (for shrinking sheet), where Table II also shows the second (dual) solutions. The agreements between the comparison results are very good.

TABLE I
VARIOUS VALUES OF $f''(0)$, $g'(0)$ AND $-\theta'(0)$ WITH γ WHEN $A = 0$, $N_R = 0$ AND PR = 0.7 FOR THE STRETCHING CASE

γ	Wang [20]	Wang [3]		Present		
	$f''(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$-\theta'(0)$
0	1.23259	1.23259	-0.81130	1.23259	-0.81130	0.49587
.1	1.1466	1.14656	-0.86345	1.14656	-0.86345	0.51545
.2	1.0511	1.05113	-0.91330	1.05113	-0.91330	0.53437
.5	0.7133	0.71330	-1.05239	0.71329	-1.05146	0.58767
1	0	0	-1.25331	0	-1.25331	0.66756
2	-1.8873	-1.88731	-1.58957	-1.88731	-1.58957	0.80478
5	-10.2647	-10.2648	-2.3381	-10.2648	-2.3381	1.12213

TABLE II
VARIOUS VALUES OF $f''(0)$, $g'(0)$ AND $-\theta'(0)$ WITH γ WHEN $A = 0$, $N_R = 0$ AND PR = 0.7 FOR THE SHRINKING CASE. RESULTS SHOWN IN ( ) ARE THE SECOND SOLUTIONS

γ	WANG [3]		Present		
	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$-\theta'(0)$
-0.25	1.40224	-0.66857	1.40224	-0.66857	0.44340
-0.5	1.49567	-0.50145	1.49567	-0.50145	0.38439
-0.75	1.48930	-0.29376	1.48930	-0.29376	0.31547
-1	1.32882	0	1.32882	0	0.22833
-1.15	1.08223 (0.11670)	0.297995 (2.76345)	1.08223 (0.11670)	0.297995 (2.76345)	0.15510 (0.00156)
-1.2465	0.55430	0.99904	0.55429	0.99904	0.04966

TABLE III
VARIATION OF $f''(0)$, $g'(0)$ AND $-\theta'(0)$ WITH A WHEN $\gamma = -0.5$, $N_R = 3$ AND PR = 0.7. RESULTS SHOWN IN ( ) ARE THE SECOND SOLUTIONS

A	-0.7	-1.0	-1.5
$f''(0)$	1.07242 (-0.46860)	0.85196 (-0.62115)	0.37021 (-0.62216)
$g'(0)$	-0.06794 (-1.93081)	0.18481 (-5.41045)	0.84587 (8.83243)
$-\theta'(0)$	0.29342 (0.15402)	0.31144 (0.21065)	0.33568 (0.28569)

Further, Table III displays the skin friction coefficient, $f''(0)$, skin friction coefficient, $g'(0)$ and also the heat transfer coefficient $-\theta'(0)$ for various A when $\gamma = -0.5$, $N_R = 3$ and Pr = 0.7. It is found that unsteadiness has increase the surface shear stress, which in turn increases the heat transfer coefficient.

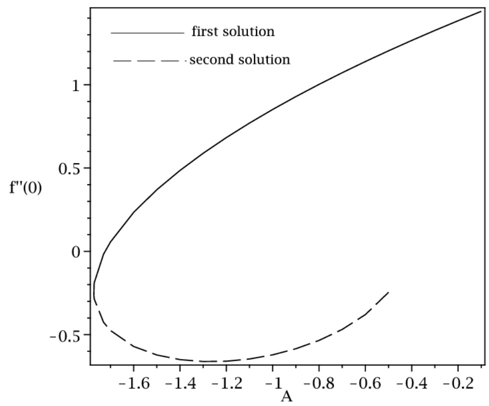

Fig. 1 Variation of the skin friction coefficient, $f''(0)$ with A for different values of N_R when $\gamma = -0.5$ and Pr = 0.7

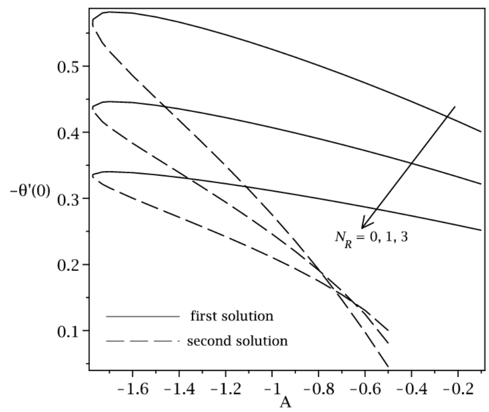

Fig. 2 Variation of the heat transfer coefficient, $-\theta'(0)$ with A for different values of N_R when $\gamma = -0.5$ and Pr = 0.7

In this present study, Figs. 1 and 2 display the skin friction coefficient and the heat transfer rate at the surface for all Nr and A, respectively, when γ and Pr are fixed. As $|A|$ increases, $f''(0)$ decreases, however $-\theta'(0)$ increases until the critical (turning) point, $A_c = -1.7719$. No solutions have been found for $A_c > -1.7719$ due to boundary layer starts to separate from the sheet. Hence, beyond this value, the boundary layer approximation is no longer valid. Therefore, the full Navier-Stokes equation need to be used. Figs. 1 and 2 also show the existence of the dual solutions for the case of non-zero radiation, $N_R = 0$. It is worth mentioning that normally the first (upper branch) solutions are physically stable and this can be verified by performing a stability analysis. Unfortunately, this analysis is beyond the scope of the present paper and such analysis can be found in Merkin [21] and Weidman et al. [22]. The value of $f''(0)$ is not affected by the radiation parameter since the flow is not affected by N_R, as shown in Fig. 1. This phenomenon can be explained by (12). While in Fig. 2, it is observed that the heat transfer rate at the surface found to reduce with Nr.

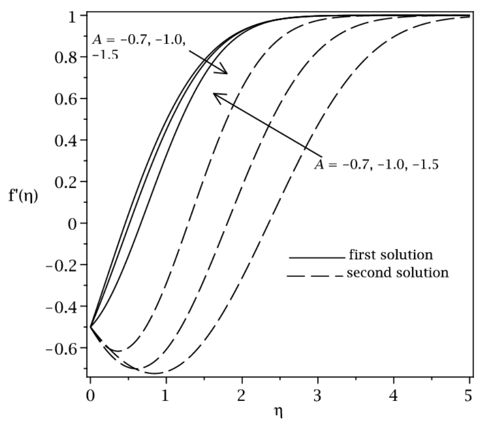

Fig. 3 Velocity profiles for different values of A when $\gamma = -0.5$, $N_R = 3$ and Pr = 0.7

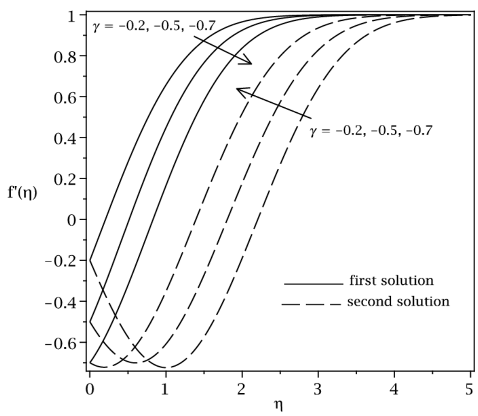

Fig. 4 Velocity profiles for different values of γ when $A = -1$, $N_R = 3$ and Pr = 0.7

Figs. 3 and 4 show the velocity profiles for various A and γ, respectively, when other parameters are fixed. Both figures show that the velocity profiles reduce as A and γ increases, respectively and broaden the boundary layer thickness. Fig. 5 shows the g profiles for various A. It is found that the g profiles increase with A.

Figs. 6 to 8 illustrate the temperature profiles for various A, γ, Pr and N_R, respectively. It is observe that as A, γ, Pr and N_R increases, the temperature profiles decreases, as well as the thermal boundary layer thickness. The surface temperature gradient also increases that causes increases in surface heat transfer rate. Physically, when Pr increases, the fluid has lower thermal conductivity, which in turn reduce conduction, thus reduces the thermal boundary layer thickness. From Fig. 8, it is also observed that the temperature profiles become less significant when N_R increases. Figs. 3 to 8 also show the dual profiles to prove the dual nature of the problem and all the profiles satisfy the far field boundary conditions (15).

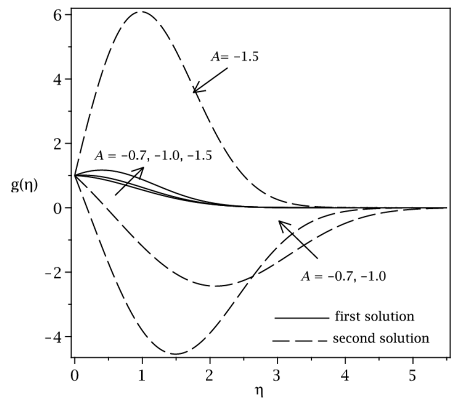

Fig. 5 g profiles for different values of A when $\gamma = -0.5$, $N_R = 3$ and Pr = 0.7

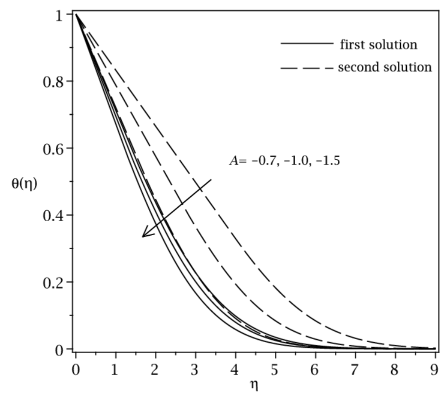

Fig. 6 Temperature profiles for different values of A when $\gamma = -0.5$, $N_R = 3$ and Pr = 0.7

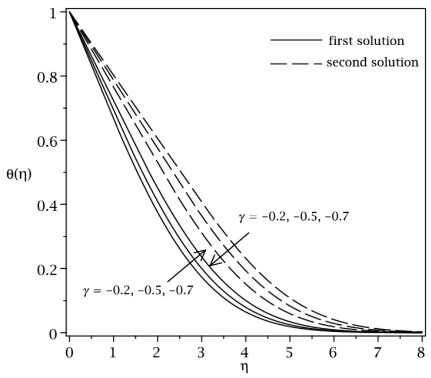

Fig. 7 Temperature profiles for different values of γ when $A=-1$, $N_R=3$ and $\Pr=0.7$

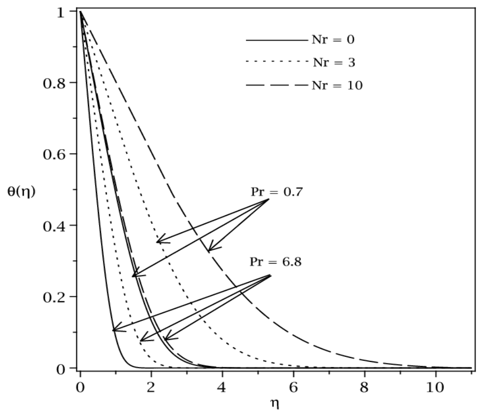

Fig. 8 Temperature profiles for different values of Pr and N_R when $\gamma=-0.5$ and $A=-1$

V. Conclusion

A study is performed for the problem of unsteady stagnation-point flow and heat transfer induced by a shrinking sheet in the presence of radiation effect. It is observed the existence of the dual solutions for the shrinking case. In this study, it is found that the thermal boundary layer thickness reduced with the unsteadiness and shrinking parameters and also with the Prandtl number and radiation parameter. The temperature profile is affected the most by the Prandtl number.

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References

[1] C. Y. Wang, “Liquid film on an unsteady stretching sheet,” *Quarterly of Applied Mathematics*, vol. 48, pp 601–610, 1990.

[2] M. Miklavcic and C. Y. Wang “Viscous flow due to a shrinking sheet,” *Quarterly of Applied Mathematics*, vol. 64, pp. 283–290, 2006.

[3] C. Y. Wang, “Stagnation flow towards a shrinking sheet,” *International Journal of Non-Linear Mechanics*, vol. 43, pp. 377–382, 2008.

[4] T. Fang, “Boundary layer flow over a shrinking sheet with power-law velocity,” *International Journal of Heat and MassTransfer*, vol. 51, pp. 5838–5843, 2008.

[5] T. Fang and J. Zhang, “Thermal boundary layers over a shrinking sheet: an analytical solution,” *Acta Mechanica*, vol. 209, pp. 325–343, 2010.

[6] M. Sajid and T. Hayat, “The application of homotopy analysis method for method for MHD viscous flow due to a shrinking sheet,” *Chaos,Solitons and Fractals*, vol. 39, pp. 1317–1323, 2009.

[7] T. Hayat, Z. Abbas, T. Javed and M. Sajid, “Three-dimensional rotating flow induced by a shrinking sheet for suction,“*Chaos, Solitons and Fractals*, vol. 39, pp. 1615–1626, 2009.

[8] C. D. Surma Devi, H. S. Takhar and G. Nath, “Unsteady, three-dimensional, boundary-layer flow due to a stretching surface,” *International Journal of Heat and Mass Transfer*, vol. 29, pp. 1996–1999, 1986.

[9] K. N. Lakshmisha, S. Venkateswaran and G. Nath, “Three-dimensional unsteady flow with heat and mass transfer over a continuous stretching surface,” *Journal of Heat Transfer*, vol. 110, pp. 590–595, 1988.

[10] F. M. Ali, R. Nazar, N. M. Arifin and I. Pop, “Unsteady flow across a stretching surface,” *International Communications in Heat and Mass Transfer*, vol. 37, pp. 476–479, 2010.

[11] M. Abd El-Aziz, “Radiation effect on the flow and heat transfer over an unsteady stretching sheet,” *International Communications in Heat and Mass Transfer*, vol. 36, pp. 521–524, 2009.

[12] T-G. Fang, J. Zhang and S-S Yao, “Viscous flow over an unsteady shrinking sheet with mass transfer,” *Chinese Physics Letters*, vol. 26, pp. 014703-1–014703-4, 2009.

[13] F. M. Ali, R. Nazar, N. M. Arifin and I. Pop, “Unsteady shrinking sheet with mass transfer in a rotating fluid,” *International Journal for Numerical Methods in Fluids*, vol. 66, pp. 1465–1474, 2011.

[14] F. M. Ali, R. Nazar, N. M. Arifin and I. Pop, “Unsteady flow and heat transfer past an axisymmetric permeable shrinking sheet with radiation effect,” *International Journal for Numerical Methods in Fluids*, vol. 67, pp. 1310–1320, 2011.

[15] C. Midya, “Heat transfer in MHD boundary layer flow over a shrinking sheet with radiation and heat sink,” *Journal of Global Research in Mathematical Archives*, vol. 1(2), pp 63–70, 2013.

[16] A. Raptis, C. Perdikis and H. S. Takhar, “Effect of thermal radiation on MHD flow,” *Journal of Applied Mathematics and Computing*, vol. 153, pp. 645–649, 2004.

[17] R. C. Bataller, “Radiation effects in the Blasius flow,” *Journal of Applied Mathematics and Computing*, vol. 198, pp. 333–338, 2008.

[18] R. Cortell, “Similarity solutions for flow and heat transfer of a quiescent fluid over a nonlinearly stretching surface,” *Journal of Materials Processing Technology*, vol. 203, pp. 176–183, 2008.

[19] D. B. Meade, B. S. Haran and R. E. White, “The shooting technique for the solution of two-point boundary value problems,” *Maple Tech*., vol. 3, pp. 85–93, 1996.

[20] C. Y. Wang, “Impinging stagnation flows,” *Physics of Fluids*, vol. 30(3), pp. 915–917, 1987.

[21] J. H. Merkin, “On dual solutions occuring in mixed convection in a porous medium,” *Journal of Engineering Mathematics*, vol. 20, pp. 171–179, 1987.

[22] P. D. Weidman, D. G. Kubitschek and A. M. J. Davis, “The effect of transpiration on self-similar boundary layer flow over moving surfaces,” *International Journal of Engineering Science*, vol. 44, pp. 730–737, 2006.