Approximate Solution of Some Mixed Boundary Value Problems of the Generalized Theory of Couple-Stress Thermo-Elasticity

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Abstract—We have considered the harmonic oscillations and general dynamic (pseudo oscillations) systems of theory generalized Green-Lindsay of couple-stress thermo-elasticity for isotropic, homogeneous elastic media. Approximate solution of some mixed boundary value problems for finite domain, bounded by the some closed surface are constructed.

Keywords—The couple-stress thermo-elasticity, boundary value problems.

I. INTRODUCTION

PROBLEMS of the connected theory of couple-stress thermo-elasticity are dynamic problems [1]. The general theory of these dynamic problems, involving the proof of the basic existence and uniqueness theorems, is developed on the assumption that the boundaries of the domains under consideration are fixed in the finite part of the space. For the general case when the boundary or some pieces thereof extend to infinity, the boundary and initial-boundary problems of elasticity were studied but little and the progress achieved hitherto in this direction is limited to some particular results. These results are of a more general nature in the case when the boundary of an infinite domain is composed of certain systems of planes or of systems of straight segments (in the plane case); we mean here problems for the half-plane and the half-plane, problems for some other parts of the space and the plane.

Assuming the existence of a solution in some, sufficiently wide, class of functions, many problems of such kind may be solved explicitly and verified by a direct substitution. If the results of the verification are favorable, we may expect the corresponding uniqueness theorems are proved.

Problems of couple-stress thermo-elasticity for which it appears possible to obtain such results are rather great in number. Here will be considered problems for finite domain, bounded by the some closed surface. They are solved explicitly by using the potential method.

II. NOTATIONS AND DEFINITIVE CONCEPTS

Introduce the notations: Let \( E_k \) be three-dimensional Euclidean space, \( x = (x_j); y = (y_j); j = 1,2,3 \) - points of this space, \( D_k \subset E_k \) - finite domain, bounded by the closed surfaces \( S_k \subset \bigcup_{\alpha=0}^{\infty} \), \( \alpha > 0, k = o,...,m \) i.e. by the surface with a continuous curvatures [1]; \( S_k \cap S_j = \emptyset, a > 0, j = o,...,m \) surface \( S_0 \) contains all these surfaces; \( \bar{D}_k = D_k \cup S_k \); \( S = \bigcup_{k=0}^{m} S_k \); \( D^* = D_k \setminus \bigcup_{k=1}^{m} S_k \); i.e. \( D^* \) - finite connected domain with the surface \( S \); \( D^* = E_3 \setminus \bigcup_{k=1}^{m} D_k \) infinite connected domain with the surface \( S \).\( = \bigcup_{k=1}^{m} S_k \).

The model of partial differential equations of the harmonic oscillations and general dynamic (pseudo oscillations) systems of theory generalized Green-Lindsay couple-stress thermo-elasticity for isotropic homogeneous elastic media has the form [2]:

\[
\begin{align*}
\mu \cdot \delta \Delta u + (\lambda + \mu - \alpha) \Delta \div v + 2 \delta \delta \div v - \gamma \tau \div \Delta u - \zeta \tau\div \gamma u &= h^{(1)} \\
(\nu + \beta) \delta \Delta u + (\epsilon + \nu - \beta) \Delta \div u + 2 \delta \delta \div u + (\Omega^2 - 4 \nu) \omega &= h^{(2)}
\end{align*}
\]

where,
\( u = (u_1, u_2, u_3) \) is the displacement vector, \( \omega = (\omega_1, \omega_2, \omega_3) \) is the rotation vector, \( \tau \) is the Temperature variation, \( \gamma, \epsilon, \lambda, \mu, \nu, \beta, \Omega, \sigma \) are elastic and thermal constants of the domain, \( \Delta \) is the three-dimensional Laplacian operator; \( \gamma \) is a general, complex parameter; the case \( \sigma = p > 0 \) corresponds to the harmonic oscillations, while the case \( \sigma = r \) corresponds to the general dynamic problems [1], [2];

\[ H = (h^{(1)}, h^{(2)}, h_3) = (h_1, h_2, h_3, ..., h_\infty) \subset C^0(\bar{D}^*) \quad \alpha > 0 \]

is a given vector of Heldor's class.

III. STATEMENT PROBLEM

Problem \( M^*(\alpha) \). It is required to find in \( D^* \) the regular vector \( U = (u, \omega, u_3) \)- solution of the (1) system with the boundary conditions:

\[
\begin{align*}
(u(y))^* &= F^{(k)}(y), (\omega(y))^* = G^{(k)}(y), (u_3(y))^* = f^{(k)}(y), \\
\end{align*}
\]

\( y \in S_k, k = 0, m_1 \)

\[
\begin{align*}
\left[ \Gamma_1(\partial_n, n)y(y) \right]^* &= F^{(k)}(y), \left[ T^*(\partial_n, n)y(y) \right]^* = G^{(k)}(y), \left[ \frac{\partial y}{\partial n} \right]^* = f^{(k)}(y), y \in S\alpha \quad (3)
\end{align*}
\]

\( k = m_1, 1, m_2 \)
\( (u(y))^* = F^{(k)}(y), (\omega(y))^* = G^{(k)}(y), \left( \frac{\partial u(y)}{\partial n} \right)^* = f^{(k)}(y), \)
\( y \in S_k, k = m_2 + 1, m_3 \)
\( |T_1(\partial_{\nu}, n)U(y)^*|^2 = F^{(k)}(y), |T^4(\partial_{\nu}, n)\omega(y)^*|^2 = G^{(k)}(y), \)
\( (u_1(y))^* = f^{(k)}(y), y \in S_k, k = m_3 + 1, m_4 \)

Here
\[ F^{(k)}(y) = \left( F^{(1)}(y), F^{(2)}(y), F^{(3)}(y) \right), \]
\[ G^{(k)}(y) = \left( G^{(1)}(y), G^{(2)}(y), G^{(3)}(y) \right), f^{(k)}(y) = \left( f_1^{(k)}(y), f_2^{(k)}(y) \right), \]

are corresponding, given vector-functions and scalar-functions of classes:
\[ F^{(k)}(y), G^{(k)}(y), f^{(k)}(y) \in C^{1,0}(S_k), \alpha > 0, y \in S_k, k = 0, m, \]
\[ F^{(k)}(y), G^{(k)}(y), f^{(k)}(y) \in C^{0,0}(S_k), \alpha > 0, y \in S_k, k = m_1 + 1, m_2 \]
\[ F^{(k)}(y), G^{(k)}(y), f^{(k)}(y) \in C^{1,0}(S_k), f^{(k)}(y) \in C^{0,0}(S_k), \alpha > 0, y \in S_k, \]
\[ k = m_2 + 1, m_3 \]
\[ F^{(k)}(y), G^{(k)}(y), f^{(k)}(y) \in C^{0,0}(S_k), \alpha > 0, y \in S_k, \]
\[ k = m_3 + 1, m \]
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