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An Application of Extreme Value Theory as a Risk Measurement Approach in Frontier Markets

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***Abstract*—**In this paper, we consider the application of Extreme Value Theory as a risk measurement tool. The Value at Risk, for a set of indices, from six Stock Exchanges of Frontier markets is calculated using the Peaks over Threshold method and the performance of the model index-wise is evaluated using coverage tests and loss functions. Our results show that "fattailedness" alone of the data is not enough to justify the use of EVT as a VaR approach. The structure of the returns dynamics is also a determining factor. This approach works fine in markets which have had extremes occurring in the past thus making the model capable of coping with extremes coming up (Colombo, Tunisia and Zagreb Stock Exchanges). On the other hand, we find that indices with lower past than present volatility fail to adequately deal with future extremes (Mauritius and Kazakhstan). We also conclude that using EVT alone produces quite static VaR figures not reflecting the actual dynamics of the data.

***Keywords*—**Extreme Value theory, Financial Crisis 2008, Frontier Markets, Value at Risk.

I. Introduction

The impact of the financial crisis on developed and emerging markets has been such that investors may need in a near future to look to other potential investment platforms such as Frontier Stock Markets (FSM).In the light of such developments, a proper framework is required for measuring risk on FSM.

Value at Risk (VaR) was developed as a response to the financial crashes of institutions during the 1990s. Since then, VaR has become a standard for measuring market risks of assets, largely due to its simplicity of understanding although accurate calculation may prove quite tedious. The VaR of an asset represents in a single number, the largest possible loss over a given time horizon with a given confidence interval. This measure is also recommended by the Basle Committee for Banking Supervision for use by banks for calculating the risk exposure of the investment portfolios and subsequently for the calculation of capital requirements. In fact, The Basel II Accord allows for the use of internal models for calculating VaR on a daily basis assuming a 99 percent confidence level.

However, an accurate calculation of VaR entails the use of proper models catering for distributional characteristics of the data being used. Financial returns are often assumed to have a Normal distribution.Pickands[12] discusses various characteristics of financial time series and observes the presence of skewness and excess kurtosis in a set of selected indices and stocks. The assumption of normality may lead to flawed VaR figures. Other distributions such as the Student's t or Skewed t have also been considered. However, a notable comment regarding this type of procedure is that the VaR is found in the tail of the distribution, so that the need to fit a distribution to the whole data is, in a sense, wasteful. Instead, we may concentrate on the left tail of the distribution particularly. This leads to the use of Extreme Value Theory in the calculation of market risk.

Extreme Value Theory is a suitable for modeling and studying the usually fat tails of financial time series. The aim of focusing only on the tail of the data using Extreme Value Theory (EVT) is twofold. First, it avoids having to assume a single distribution for the whole evaluation sample and secondly, it provides a parametric fit to the targeted region of the data hence allows for adequate extrapolation beyond the range of the data. EVT provides more accurate fits to heavy tailed data as discussed in [6] and [7]. Regarding applications in estimation of VaR, the literature is quite extensive. In [9]it is found that EVT outperforms other modeling techniques such as GARCH, historical simulation and variance - covariance in estimating VaR using the daily closing prices of the Istanbul Stock Exchange index. Reference [8] compares the same techniques as in [9] but this time using daily closings of stock market indices for nine different emerging economies. The authors model both the upper and lower tails of the distributions using EVT based on the Generalised Pareto Distribution (GPD). EVT has also been used to calculate VaR in Asian Stock Markets by [5]. Ten Asian Stock Markets, comprising emerging as well as developed economies, are analyzed using the Generalised Extreme Value distribution. Results show that EVT VaR model produces conservative VaR figures as compared to a Normal distribution based VaR model. In [13], the final conclusion is that the FTSE/JSE TOP40 index returns series is clearly non normal and is best fit using EVT. They also point out that although implementation of Extreme Value Theory is more tedious than relying on the normal distribution, "the results obtained are worth the effort" and that "appreciation of the true distribution of returns not only presents us with trading opportunities but also a clearer picture of the risk involved in an investment decision". In another empirical study, [1] examines the relevance of EVT when applied to the S & P 500 in the context of the recent financial crisis. The author concludes that the left tail of the index shows how great an impact, the crisis

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has had. One weakness of using a GPD fit is that it does not cater for the dynamically evolving returns series.

This paper aims at assessing the predictive ability of using EVT for measuring VaR for a set of six indices from FSM. We consider the Mauritius Stock Exchange, Tunisia Stock Exchange, Colombo Stock Exchange, Karachi Stock Exchange, Kazakhstan Stock Exchange and Croatia Stock Exchange. Model evaluation is performed using hypothesis tests and loss functions. The paper is structured as follows: we first provide the necessary theoretical aspects of EVT using the Peaks over Threshold method. We then present in Section III the descriptive statistics of the data being used. The application to the SEMDEX (Mauritius Stock Exchange) is discussed in detail. The same procedure is adopted for each of the other indices. Section IV deals with the backtesting exercise and we conclude in Section V.

II. Extreme Value Theory

For the purpose of our study, all prices series are converted to *returns*. For a series of prices $\{X_t\}$, the *return* r_t is obtained using

$$r_t = \ln \frac{X_t}{X_{t-1}} . \tag{1}$$

Extreme Value Theory pertains to the behavior and occurrence of extreme observations of a random variable. It allows for the modeling of rare events and subsequently for the calculation of tail-related risk measures. There may be two possible ways of using EVT: either by considering maxima or minima over determined observation periods of equal length or by considering the behavior of extreme value above a sufficiently high threshold value. The former is the Block Maxima Method and the latter the Peaks over Threshold (POT) method. Our study makes use of the Peaks over Threshold. The POT method based on the Picklands – Balkema - De Haan theorem (see [3] and[12]), presents a more efficient way for modeling the extreme data available. The observations above a determined threshold value are used to fit a Generalised Pareto Distribution. VaR is extracted from the tail of losses. We thus need to model the lower tail of the returns distribution.

A. The Peaks over Threshold Method

Prior to discussing the POT method, we consider the tail of negative returns. We first make the negative returns positive and denote it$\{R_t\}$. Thus an extreme loss would be the maximum of sequence$\{R_t\}$. The cumulative distribution of the losses is$F(r) = P(R_t \leq r)$. For a sufficiently high threshold$u$, the excesses above$u$ are given by$y = r - u$. The distribution function of y is

$$F_u(y) = P(R_t - u \leq y | R_t > u), \tag{2}$$

which may be rewritten as

$$F_u(y) = \frac{F(r)-F(u)}{1-F(u)} . \tag{3}$$

Further rearrangement of (3) leads to the equation

$$F(r) = F(u) + [1 - F(u)]F_u(y). \tag{4}$$

The estimation of$F(r)$is based on the Picklands - Balkema – De Haan theorem. In [12], it is stated that for a sufficiently high u, the distribution of y approximately belongs to the Generalised Pareto family which is defined as

$$G_{\xi,\mu,\sigma}(y) = \begin{cases} 1 - \left(1 + \xi \frac{y-\mu}{\sigma}\right)^{-\frac{1}{\xi}} if \xi \neq 0 \\ 1 - e^{-\frac{y-\mu}{\sigma}} if \xi = 0 \end{cases} \tag{5}$$

where ξ is the shape parameter, μ, a location parameter and σ, a scale parameter. When $\xi = 0$, the distribution is thin tailed. The case of interest is when $\xi > 0$. Then $G_{\xi,\mu,\sigma}$ is the cumulative distribution function of a heavy tailed Pareto distribution. A key step for proper use of the POT method is the choice of u. In the next section, we discuss the tools to be used in choosing the threshold.

B. Threshold Selection

A threshold needs to achieve balance between bias and variance. When choosing u, one wishes to include a sufficient number of observations in the tail so as to have a reliable approximation to the distribution of excesses. A very high threshold would be problematic in the sense that, we would be left with too few observations for estimating the parameters of the GPD. Similarly, a low threshold value might yield an asymptotic distribution that does not converge to a GPD. In order to help us in choosing a threshold, we make use of three graphical tools: the Mean Excess Plot, the Hill plot and Maximum Likelihood Estimates of the scale parameter with different number of observations in the tails.

For a set of independent and identically distributed observations$X_1, \ldots, X_n$, the mean excess above a given threshold u is defined as

$$E(u) = \frac{\sum_{i=1}^{n}(X_i - u) I_{X_i > u}}{\sum_{i=1}^{n} I_{X_i > u}}, \tag{6}$$

where I is an indicator function. The set of points$\{(X_i, E(X_i))\}$ makes up the Mean Excess plot. A correct threshold is identified by determining a portion of the graph that is linear. Moreover, a positively sloped plot indicates a heavy tailed distribution. We also consider the Hill Plot which is based on the Hill estimator for ξ provided $\xi > 0$. The set of data is first ordered such that$X_1^{(n)} > X_2^{(n)} > \cdots > X_n^{(n)}$ and the Hill estimate is calculated as

$$\hat{\xi} = \frac{1}{n} \sum_{i=j}^{k} \ln \frac{X_j^{(n)}}{X_k^{(n)}}. \tag{7}$$

The Hill plot is constructed using the set of points $\left\{\left(k, \frac{1}{\hat{\xi}}\right)\right\}$. A suitable threshold may be chosen based on the criterion of stability of the estimated shape parameter. Stability would imply a relatively flat part of the graph where the parameter ξ is fairly stable.

Along similar lines as the Hill plot, we consider a plot of Maximum Likelihood Estimates of ξ against the number of upper order statistics included in the estimation. According to the Balkema and de Haan Picklands theorem, if the GPD provides a satisfactory fit at a given threshold, then using the same shape parameter, the excesses of a higher threshold should also follow a GPD. As with the Hill plot, we choose the number of observations to be used in the GPD fit based on the portion of the graph, where the shape parameter remains stable.

C. Value at Risk Using POT

From (4) and (5), we obtain for a chosen threshold$u$,

$$F(r) = F(u) + [1 - F(u)]G_{\xi,\mu,\sigma}(r-u). \qquad (8)$$

By substituting for $G_{\xi,\mu,\sigma}$ and using the estimator $\widehat{F(u)} = \frac{n-N}{n}$, where$n$ is the sample size and N is the number of exceedences above u, we obtain the equation

$$\widehat{F(r)} = 1 - \frac{N}{n}\left(1 + \hat{\xi}\,\frac{r-u}{\hat{\sigma}}\right)^{-1/\hat{\xi}}. \qquad (9)$$

A simple rearrangement of (9) yields

$$r^{(\alpha)} = u + \frac{\hat{\sigma}}{\hat{\xi}}\left(\left(\frac{n}{N}\,(1-\alpha)\right)^{-\hat{\xi}} - 1\right), \qquad (10)$$

from which the 100α percent quantile may be obtained, hence the VaR.

III. Data and Methodology

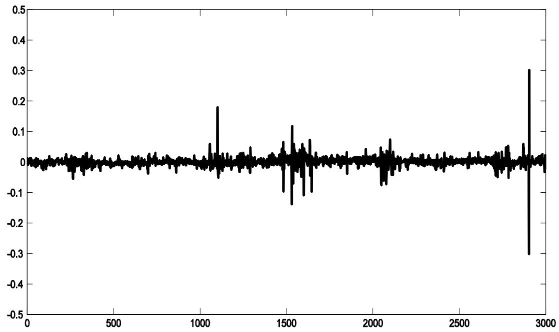
We use as data, the daily closing prices of the indices from the Stock Exchanges mentioned. The prices series start at various dates but end in 2009. We thus use data pre and post 2008 financial crisis. The time series are also of different lengths due to each Stock Exchange having different closing dates. Plots of the returns series are shown in Fig. 1 where we may observe the relative dynamics of the indices. Indices from same continents have comparative dynamics. Mauritian and Tunisian indices have generally lower volatility as compared to their European (Croatian and Kazakh) counterparts. Table I shows the descriptive statistics of the indices. We see that all the return series are close to zero. The unconditional standard deviations of the different series are relatively comparable. The Tunisia and Mauritius indices resemble each other in terms of mean and standard deviation. The Kazakhstan index has highest standard deviation. The six returns series considered all exhibit kurtosis above 3. Another feature of the returns series is the presence of skewness. In fact, four of the indices (Mauritius, Sri Lanka, Tunisia and Kazakhstan) are positively skewed. These indicate that the returns series are non-normal. We confirm this non normality via the JarqueBera test performed at 5% level. The test statistics obtained lead to the rejection of the null hypothesis of normality. We further illustrate the fattailedness of the data by observing the QQ plots in Fig. 2. The data is plotted against quartiles of the thin tailed standard exponential distribution. We observe that for most of the returns series, there is a concave departure from the straight line thus confirming the fattailedness of the data. The Karachi index is the only of the indices considered that does not clearly show concave departure. However, a slightly concave shape may be observed at higher quantiles.

For the purpose of applying a GPD fit to the tails of the different returns series, we determine the threshold values using the methods described in Section II (C). The plots discussed are used in order to determine the number of upper order statistics to be used in the GPD fit. To illustrate the threshold selection procedure, we present the results obtained for the SEMDEX returns. The mean excess plot in Fig. 3 shows a positively sloped graph further establishing the fat tailed nature of the data. A clear cut choice for the shape parameter is difficult to identify clearly from this graph. We next consider the Hill plot and identify here a relatively flat portion. The Hill estimates are seen to be rather stable when the number of observations used in the tail index is between 300 and 370 (Fig. 4). Finally, we attempt to justify the number of losses to be used in the tail index estimate using a plot of Maximum likelihood estimates of the latter against the number of observations included for estimation (Fig. 5). The region where the data is fairly stable is seen to be within the range 360 to 440. It is worth pointing out that the choice of threshold based on the graphical tools described is a subjective exercise as the threshold choice is up to the user.

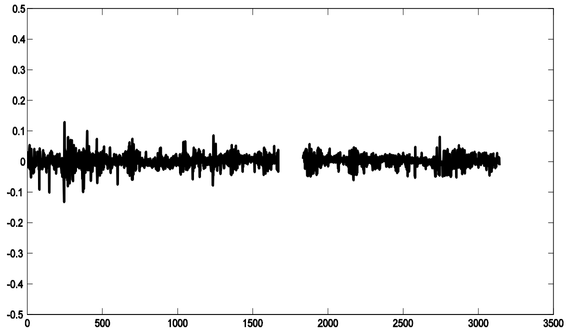
The same exercise is performed on the other indices. The losses are extracted and satisfactory thresholds are decided upon based on the plots mentioned. Table II shows the ranges we identify in each case.

TABLE I
DESCRIPTIVE STATISTICS OF THE RETURNS SERIES

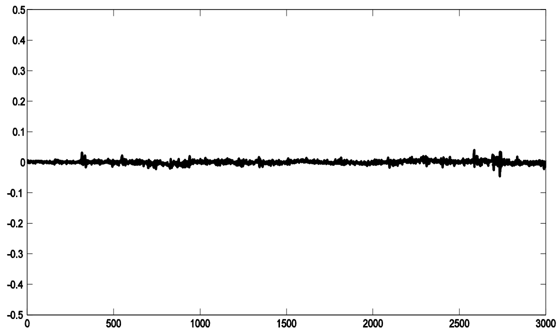
	Colombo	Karachi	Tunis	Kazakhstan	Zagreb	Mauritius
Mean	0.00048	0.00058	0.00049	0.00121	0.00031	0.00055
Std. Dev	0.01506	0.01797	0.00494	0.03268	0.01844	0.00790
Minimum	-0.29677	-0.13213	-0.05004	-0.48644	-0.19423	-0.06383
Maximum	0.30535	0.12762	0.03613	0.48759	0.17575	0.07655
Kurtosis	123.29601	7.68352	11.42786	51.99664	18.38701	20.64550
Skewness	0.30971	-0.35397	0.02877	0.59299	-0.03498	0.23177
JarqueBera	1806527.192	2832.617	8855.338	237305.206	31854.771	31573.302



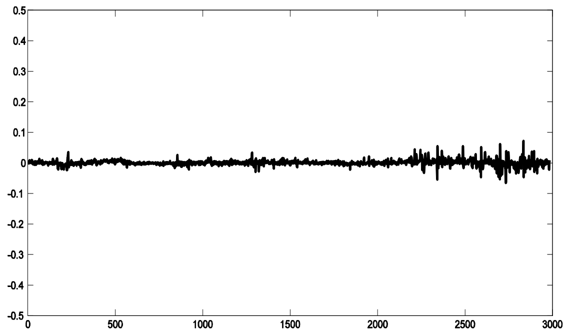
(a) Colombo



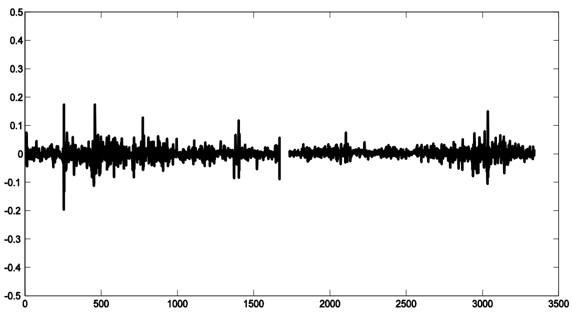
(b) Pakistan



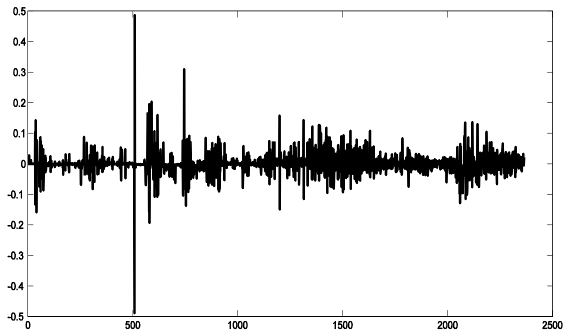
(c)Tunisia



(d) Mauritius



(e)Zagreb



(f) Kazakhstan

Fig. 1 Plots of returns for the different indices considered

TABLE II
RANGE OF OBSERVATIONS OVER WHICH TAIL INDEX ESTIMATES ARE STABLE

	Hill Plot	MLE estimates plot
Colombo	250 – 300	260 – 290
Karachi	60 -100	60 - 80
Tunisia	400 – 450	400 – 440
Mauritius	300 – 370	330 – 440
Zagreb	420 – 480	350 – 420
Kazakhstan	400 – 450	440 – 500

The number of observations to be included in the tail for satisfactory tail estimation, represent about 10 % of the whole set of observations except for the Karachi Stock Exchange. These figures confirm that the returns from the Karachi Stock Exchange index may not be appropriately modeled by a GPD. We nevertheless consider an EVT VaR model for this index for the sake of consistency and for comparison purposes with the other indices.

The next step consists in calculating the VaR with the estimated tail index, using (10). In our case, the 99 % VaR is found. The returns series is first split into two parts. We consider an estimation and an evaluation sample of length 500. The estimation sample is used to find the tail index using the predetermined threshold. One VaR figure is forecasted. The estimation sample is next updated with a new return and the procedure of recalculating the tail index and subsequently the VaR is performed. This process is carried out 500 times to produce 500 VaR forecasts for the purpose of backtesting. By considering an updated returns series at each step, we ensure that the tail index estimated, reacts to potential new extreme events and we expect that this is reflected in the VaR.

IV. Backtesting

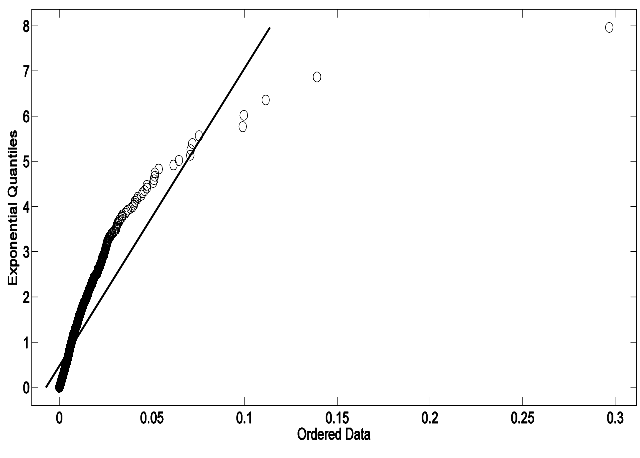
In order to evaluate the predictive ability of the model, we count the number of exceptions occurring. The actual returns are used as a substitute for the actual profit or loss and we consider a negative return less than a VaR forecast, a violation. The number of violations N occurring over an evaluation period of length T may be defined as

$$N = \sum_{i=1}^{T} \mathbb{E}_i,$$

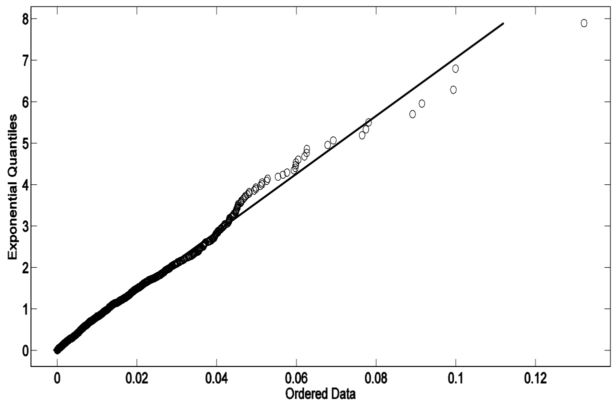
where,

$$\mathbb{E}_i = \begin{cases} 1 \; if r_i < VaR_{t,} \\ 0 \;\; otherwise. \end{cases}$$

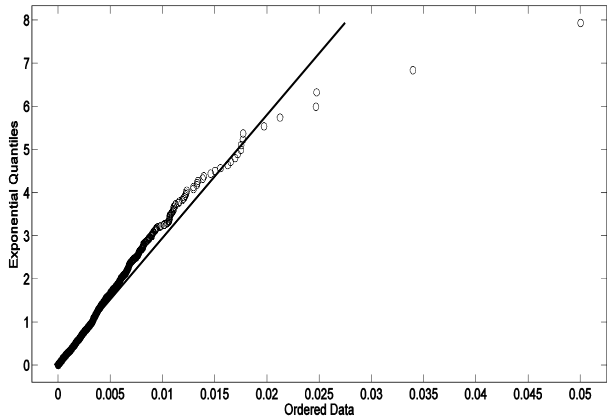
The fittingness of the model considered in forecasting VaR is determined using a two step approach. We first use statistical tests in order to assess the adequacy of the model considered and secondly use loss functions to determine which models produce VaR figures closer to actual losses. The statistical tests used are the test of Kupiec (see [10]) and the Christofferson test (see [4]).



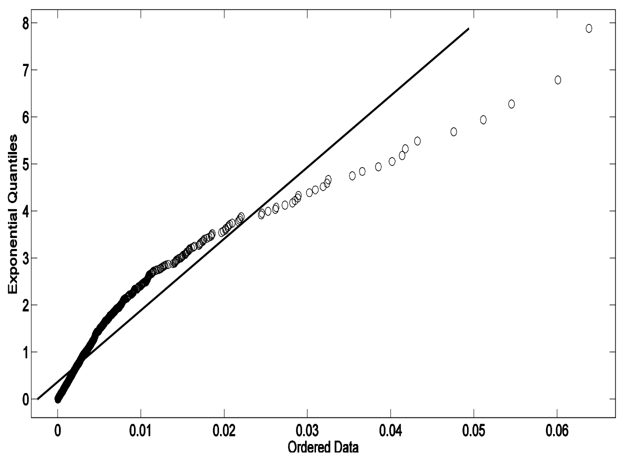
(a) Colombo



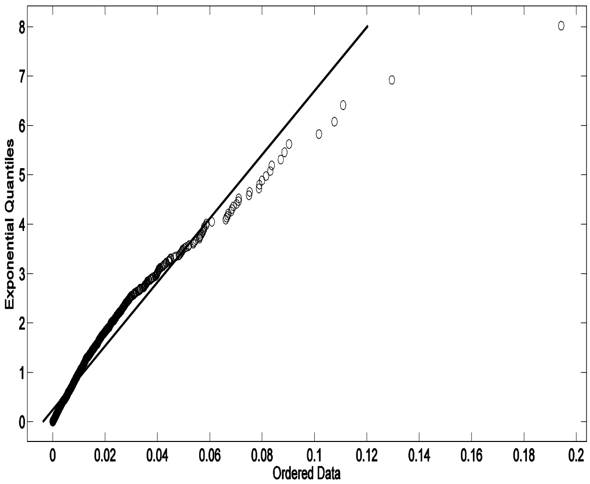
(b) Pakistan

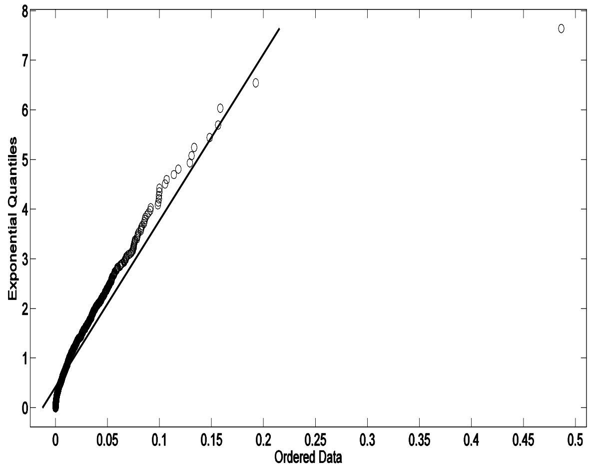


(c) Tunisia



(d) Mauritius





(f) Kazakhstan

(e) Croatia

Fig. 2 QQ plots of returns data for each series against standard exponential quantiles

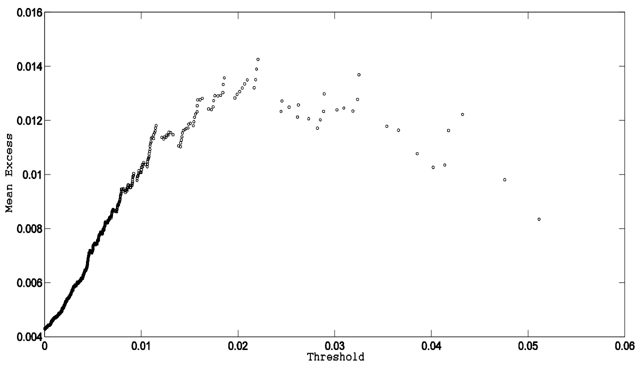


Fig. 3 Mean Excess Plot for SEMDEX losses

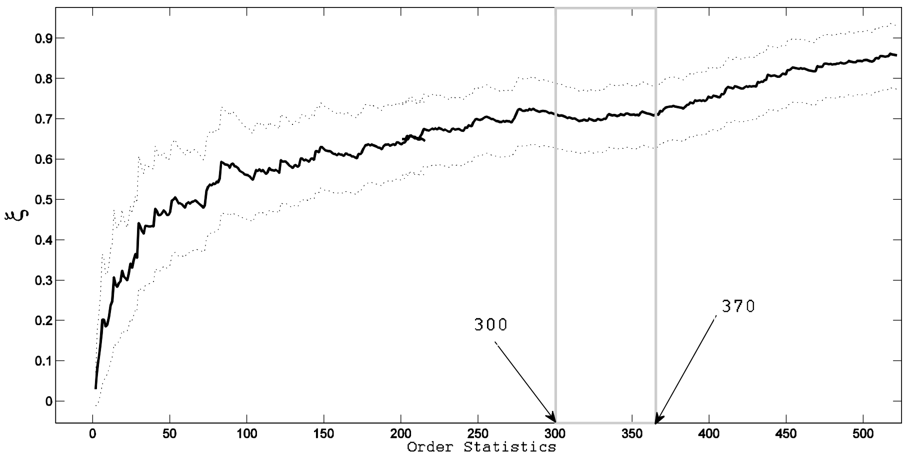


Fig. 4 Hill estimates for tail index of the SEMDEX returns series

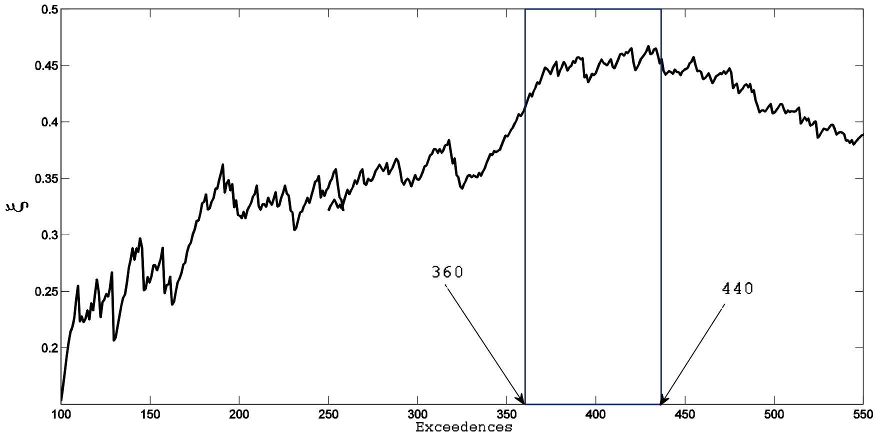


Fig. 5 Maximum Likelihood Estimates of shape parameter using SEMDEX losses data

The Kupiec test is a test of unconditional coverage while the Christofferson test also confirms for independence in the exceptions. The proportion of exceptions is $p = \frac{N}{T}$ and the unconditional coverage test statistic is given by

$$LR_{unc} = -2 \ln \frac{p^N (1-p)^{T-N}}{\alpha^N (1-\alpha)^{T-N}},$$

which has a $\chi^2(1)$ distribution. The test is performed under the null hypothesis that$p = \alpha$. Given a confidence level of 99 % and an evaluation sample of length 500, the target number of violations is 5. The Christofferson test extends the test for unconditional coverage by considering the clustering of exceptions. The variable n_{ij} denotes the number of days on which transitions from state i to j occur. The associated probabilities are

$$\pi_{01} = \frac{n_{01}}{n_{00} + n_{01}} and \pi_{11} = \frac{n_{11}}{n_{10} + n_{11}} \ .$$

The likelihood ratio test statistic for independence is calculated as follows:

$$LR_{ind} = -2 \ln \frac{\pi^{n_{01}+n_{11}} (1-\pi)^{n_{00}+n_{11}}}{\pi_{01}{}^{n_{01}} (1-\pi_{01})^{n_{00}} \pi_{11}{}^{n_{11}} (1-\pi_{11})^{n_{10}}},$$

where

$$\pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}.$$

LR_{ind}also has a chi squared distribution with 1 degree of freedom.The conditional coverage test statistic is $LR_{cc} = LR_{ind} + LR_{unc}$ and has a $\chi^2(2)$ distribution.

Both tests are performed at 5 % significance level. The critical values for the test are 3.841 for a $\chi^2(1)$ distribution and 5.991 for a $\chi^2(2)$ distribution.If the model, when applied to an index, does not produce any violations, the test statistics cannot be calculated and we consequently reject the model for the particular index.

The second stage of our backtesting procedure consists in using loss functions. A loss function assigns a score to the model based on the difference between an actual loss and the VaR forecast when an exception occurs. We consider four loss functions in this study namely the quadratic (QL), absolute (AL), asymmetric linear (ASL) and quantile loss (QuL) functions. The loss functions have the following forms:

$$Quadratic: \quad L_t = \begin{cases} (r_t - VaR_t)^2 \; if r_t < VaR_t, \\ 0 \quad otherwise. \end{cases}$$

$$Absolute: \quad L_t = \begin{cases} |r_t - VaR_t| \; if r_t < VaR_t, \\ 0 \quad otherwise. \end{cases}$$

$$AsymmetricLinear: \quad L_t = \begin{cases} (\alpha - 1)(r_t - VaR_t) \; if r_t < VaR_t, \\ \alpha(r_t - VaR_t) \quad otherwise. \end{cases}$$

$$Quantile: \quad L_t = \begin{cases} (r_t - VaR_t)^2 \; if r_t < VaR_t, \\ (R - VaR_t)^2 \quad otherwise. \end{cases}$$

With reference to the QuL function, R corresponds to the 100α percentile of the returns data available at time $t-1$. The QL and AL functions do not penalize a model when exceptions do not occur while the ASL and QuL assign a score to the model whenever this is the case. A smaller loss function score indicates that the model is performing well.

A. Backtesting Results and Discussions

We present in Table III the test statistics of the unconditional and conditional coverage tests as well as the scores that the EVT model produces index-wise. Two clear cut observations are that the EVT model does not work out for the

Mauritian and Karachi Indices: In the case of the former, the model produces 18 violations which are well above the target of 5. Moreover, the model gets rejected for unconditional and conditional coverage tests. The LR_{IND} of 9.2546 is a further indication that the violations produced are clustered. Regarding the Karachi Stock Exchange index, the results confirm the diagnostics established in Section III regarding the unsuitability of the EVT model. No violations occur and so we reject the model for this particular index. The EVT model also proves inappropriate for the Kazakhstan index. Only one violation happens and the model is rejected for unconditional coverage.

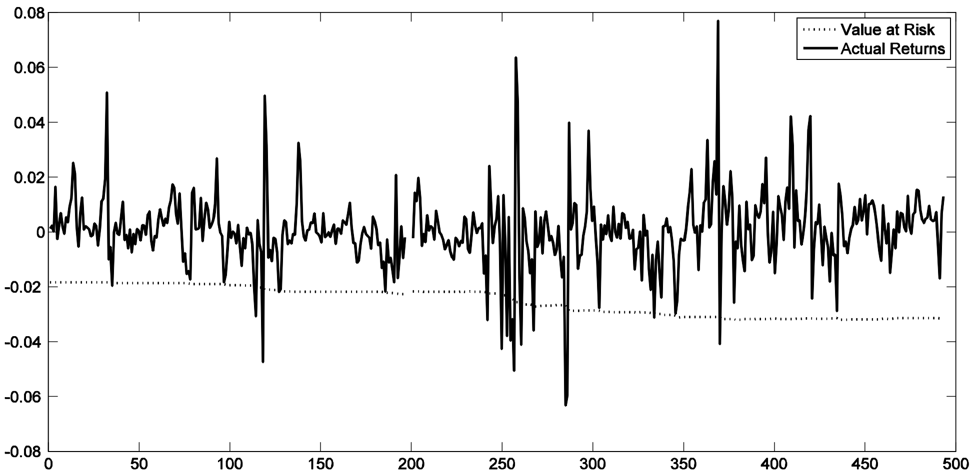
On the other hand, the model provides satisfactory results in the case of the Tunisia, Colombo and Zagreb Stock Exchange indices with 8, 3 and 4 violations respectively. The model is not rejected in any of the coverage tests while producing independent violations. We note that the violation rate for the Colombo and Zagreb indices are very close to the target rate of 1 %. Given that the model is not rejected statistically for these indices, we further analyze the accuracy of the VaR forecasts by looking at the loss function scores. We first consider the QL and AL scores and find that the EVT model works best for the Zagreb index with lowest scores. However, the asymmetric loss function scores are not the best. This may imply relatively high VaR forecasts when losses are not occurring. Regarding the Sri Lankan index, we may observe the loss scores are comparatively high. Finally, the EVT model for VaR of the Tunisian index scores well in terms of all loss functions. The scores obtained are better than for the Zagreb index with respect to the ASL and QUL functions.

We illustrate the relative performance of the EVT model for each index by producing plots of the predicted VaR figures alongside actual returns (Figs. 6 and 7). The plots may be used to understand the loss function scores produced. Fig. 6 shows the plots for the indices where the model is rejected. The violations are seen to occur quite frequently in the case of Mauritius (Fig. 6 (a)) while the VaR forecasts are seen to be well below for the Karachi index (Fig. 6 (b)).

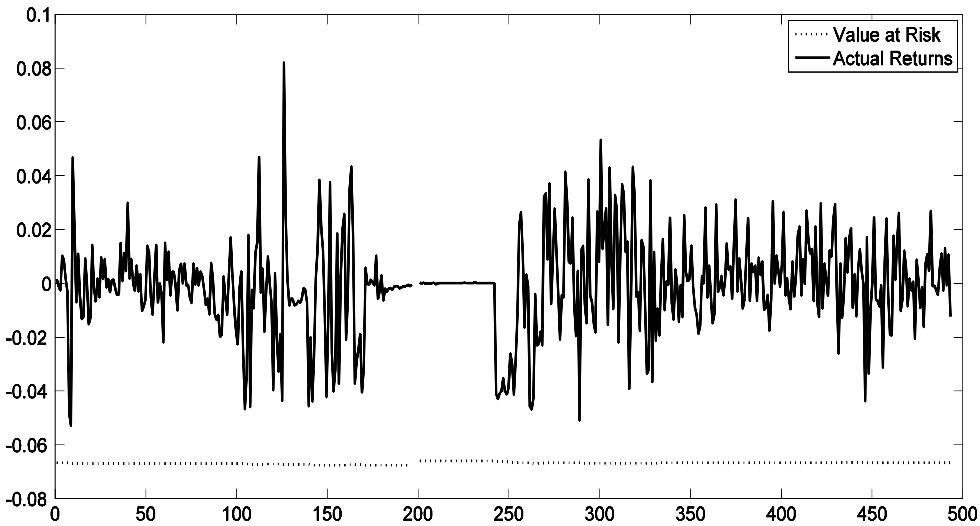
Referring to Fig.7 is most relevant to understand the loss function scores produced. In the case of the Colombo Stock Exchange index, we see that the model fails to capture the peak occurring midway between the 350th and 400th forecasts. The magnitude of the difference is visible and this accounts for rather high loss function scores. The low loss function scores for the EVT model when applied to the Tunisian index may be explained by the fact that the VaR forecasts remain close to the actual returns. Finally, the same may be said for the Zagreb index where the VaR forecasts remain rather close to the returns data and the magnitude of the difference when a violation occurs is not as consequent as for the Colombo index. A general observation for all the indices considered is that despite updating the estimation sample and reevaluating the tail index constantly, the VaR forecasts remain fairly static.

TABLE III
BACKTESTING RESULTS – COVERAGE TEST STATISTICS AND LOSS FUNCTION SCORES

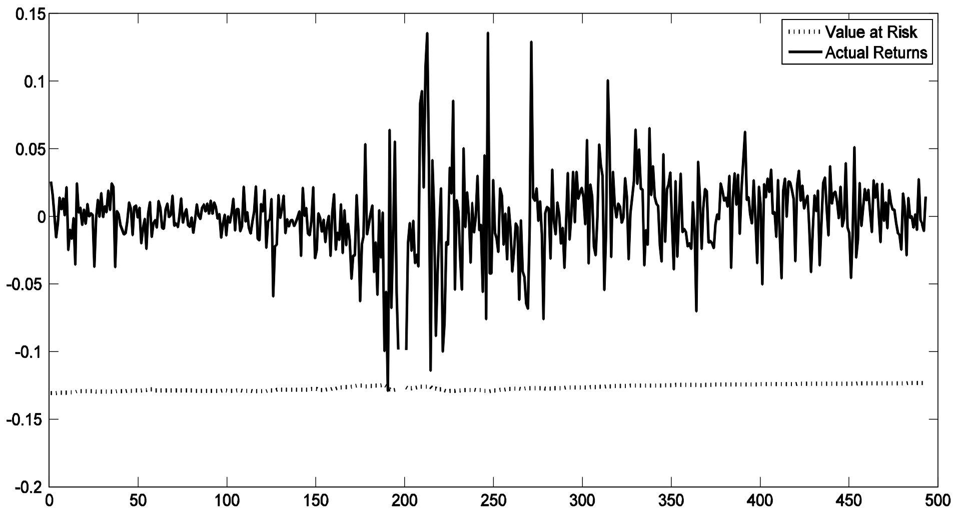
	Vio.	**LR_{UNC}**	**LR_{IND}**	**LR_{CC}**	**QL**	**AL**	**ASL**	**QUL**
Mauritius	18	**20.4581**	**9.2546**	**29.7126**	0.005272	0.2387	0.3709	0.0502
Tunisia	8	1.5383	2.5662	4.1044	0.001958	0.0886	0.1670	0.0092
Colombo	3	0.9431	0.0000	0.9431	0.061248	0.2506	0.5035	0.2197
Karachi	0	**0.0000**	**0.0000**	**0.0000**	0.000000	0.0000	0.3259	0.1825
Zagreb	4	0.2169	0.0000	0.2169	0.001885	0.0712	0.4196	0.1690
Kazakhstan	1	**4.8134**	0.0007	4.8141	0.000023	0.0048	0.6291	0.9629



(a) Mauritius

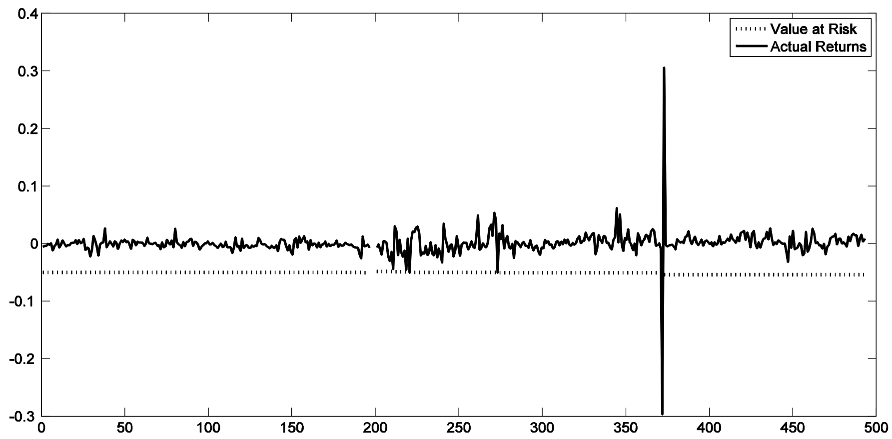


(b) Karachi

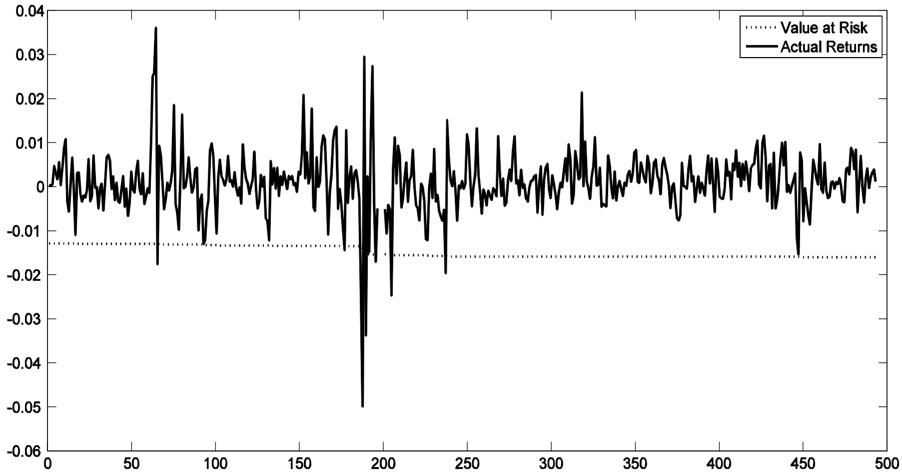


(c) Kazakhstan

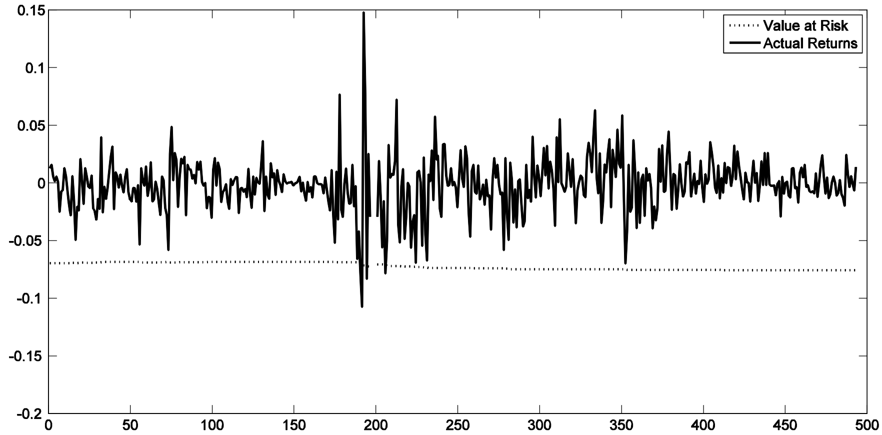
Fig. 6 Plots of VaR forecasts and actual returns for indices where EVT model is rejected



Colombo



Tunisia



Zagreb

Fig. 7 Plots of VaR forecasts and actual returns for indices where EVT model is not rejected

V. Conclusions

We have considered in this paper Extreme Value Theory as a tool for calculating VaR for a set of Frontier markets. With reference to the indices used, we saw that all were fat tailed, except for the Karachi Stock Exchange index and according to our diagnostics, would be well modeled by a GPD. As expected, the EVT VaR model did not properly estimate VaR for the Karachi index. However, we also found that the model was not particularly useful when applied to the Mauritius (too many violations) and Kazakhstan (too few violations) indices. Concerning the Mauritius index, the inability of the EVT VaR model to work correctly may be explained by the very behavior of the returns data. The index has relatively low volatility which however increases during the financial crisis period. The data used in the estimation of the tail index does not contain enough extreme values that would allow the model to cope with future extremes. This explains the large number of violations. The Kazakhstan index on the other hand has quite a few extremes occurring within the estimation sample. The returns within the evaluation sample, on the other hand, are relatively less volatile. This may account for the frequent overestimation of the VaR.

Out of the six indices, the model gave satisfactory results for the Colombo, Tunisia and Zagreb indices. Again, we may explain these by the behavior of the data. Low and high volatility periods occur quite regularly for the Colombo index. Based on past data, the model is able to capture other extremes satisfactorily.. However, a notable extreme occurs which the model fails to capture resulting in high loss function scores. The Tunisia index also has quite consistent volatility as compared to the Mauritius index. This consistency may account for the acceptable results obtained. We last consider the Zagreb index where VaR was very well modeled by EVT. Here our results agree with those obtained by [2] and [11].

In general, we may conclude that the application of Extreme Value Theory to these indices should be done with caution. We observe that the past behavior of the data impacts on the VaR forecasting ability of the model. An existing history of extreme returns helps a model cope well with future extremes. This is particularly important for Frontier markets which are characterized by increasing volatility as the market develops. Our study provides an insight as to how well Extreme Value Theory might suit Frontier Market indices. Such studies are generally not very common especially for African markets where the most literature pertains to South Africa whose market is more developed than Mauritius or Tunisia. One such study by [14] on the FTSE/JSE TOP 40 index showed that unconditional EVT works best in this case. Finally, one major drawback of relying solely on EVT to estimate VaR is that the forecasts produced are quite static and by no means react to volatility. A worthy consideration in the context of VaR modeling would be the use of a combo of Extreme Value Theory and volatility models such as Generalised Auto Regressive Heteroscedasticity models.

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