Node Pair Selection Scheme in Relay-Aided Communication Based On Stable Marriage Problem

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Abstract—This paper describes a node pair selection scheme in relay-aided multiple source multiple destination communication system based on stable marriage problem. A general case is assumed in which all of source, relay and destination nodes are equipped with multiantenna and carry out multistream transmission. Based on several metrics introduced from inter-node channel condition, the preference order is determined about all source-relay and relay-destination relations, and then the node pairs are determined using Gale-Shapley algorithm. The computer simulations show that the effectiveness of node pair selection is larger in multihop communication. Some additional aspects which are different from relay-less case are also investigated.

Keywords—Relay, multiple input multiple output (MIMO), multiuser, amplify and forward, stable marriage problem, Gale-Shapley algorithm.

I. INTRODUCTION

RECENTLY, importance of multiple point to multiple point communication [1] is increasing, as represented by multi-cellular [2], multi-agent system [3], and ad-hoc networks [4]. In those systems, together with the data transmission and signal processing algorithms, the system scheduling is an important topic which is studied hard and many works has been provided [5]. Scheduling scheme in multiuser communication contains allocation of transmission power, time, and frequency resources. The best way is that all the parameters are optimized simultaneously, but it is a so much complicated problem, and generally intractable as it is. Particularly in case of multiple data stream transmission using multiple input multiple output (MIMO) structure [6], the complexity of the problems increases because of existence of plural streams for one user. A popular way to mitigate the scheduling complexity is separate optimization of each resource, and in some cases, resource allocation is carried out under the condition that the node pairs are already determined, which means the derived suboptimal system throughput is limited by node selection even if other parameters are adequately chosen. In this sense, the total system efficiency highly depends on the choice of node pairs among with data is actually transmitted/received.

A typical way of node selection is to choose the one with the nearest location or the largest channel gain, but in case of multiple point to multiple point communication, the situation is more complicated because of the existence of interuser interferences among plural nodes. As an approach to determine the node pair efficiently based on rational rule, we have presented node pair selection scheme based on stable marriage problem [7], which is a kind of matching theory. The improvement of performance by this method is not drastic, but we should remark that the capacity increment is achieved only by exchanging the pair of nodes without adding any communication resource.

In this paper, we consider to extend the method of [7] to amplify and forward (AF) relay-aided case where all of source, relay, and destination nodes are equipped with multiantenna and multistream transmission is carried out inside of each node pair. In this initial-step study, the source-destination direct channel is assumed to be negligible, and half-duplex transmission mode under frequency nonselectivity is considered. One main objective of this paper is to verify how much effect of the node selection scheme is derived under relay-aided transmission where the additional noise is transmitted from the AF relay. In addition, the possibility of signal to interference metrics for the preference determination is studied while in previous work, only the preference functions based on direct channel strength are adopted.

The organization of the rest of this paper is given as follows; in Section II, a relay-aided multiuser MIMO system considered in this study is briefly described. Then actual node pair selection scheme based on stable marriage problem using some metrics which are in this paper derived from the channel condition is provided in Section III. In Section IV, computer simulations are carried out to verify the effectiveness of the proposed approach and to investigate its performance under different conditions. Finally in Section V, concluding remarks are shown together with some future works.

II. SYSTEM DESCRIPTION

In this section, the total construction of relay-aided multiple source multiple destination system is described. In the following, a vector is expressed by bold slant font in lowercase letter, and a matrix is represented by uppercase letter with normal slant font.

Consider a multiple point to multiple point (multiple source multiple destination) relay-aided communication system as shown in Fig. 1. The system consists of $M_s$ source nodes ($m_s = 0, \ldots, M_s - 1$), $M_r$ relay nodes ($m_r = 0, \ldots, M_r - 1$), and $M_d$ destination nodes ($m_d = 0, \ldots, M_d - 1$). The number of user $M$ is same as $M_s$ (M = ...}
Fig. 1. Model of multiple point to multiple point relay-aided communication system considered in this study. Each of $M_s = M$ sources transmits multiple data streams to a target destination chosen from $M_d$ nodes through one of $M_r$ relays.

$M_s$, and the relation $M_d \geq M_r \geq M_s$ is assumed (each node is used by only one user, and some of relay and destination nodes may not be used). Source node $S_{m,s}$, relay node $R_{m,s}$, and destination node $D_{m,d}$ have $N_{s,m,s}$, $N_{r,m,s}$, and $N_{d,m,d}$ antennas, respectively, hence the system is expressed as a MIMO multiuser model. The MIMO channel between source $S_{m,s}$ and relay $R_{m,r}$ is defined by $N_{r,m,s}$ by-$N_{s,m,s}$ matrix $H_{r,m,s,m,s}$ whose $(k_r,k_s)$-th element denotes the response between the $k_s$-th antenna of the source $S_{m,s}$ and $k_r$-th antenna of the relay $R_{m,r}$. The MIMO channel $H_{d,m,r,m,r}$ between relay $R_{m,r}$ and destination $D_{m,d}$ is similarly defined.

The direct channel between the sources and destinations are ignored assuming they are weak and cannot be used for the communication (if they have certain strength and not negligible, it may become another future work). The relays work in half-duplex mode which is widely used to avoid self-interference between transmit and receive antennas in relay which normally becomes very strong without some kind of self-interference shield.

In the first transmission phase, source $m$ transmits $L_m$ data streams $\{s_{m,0}(t), \ldots, s_{m,L_m-1}(t)\}$ to relay $R_{m,s}$ which is chosen as the pair node from $M_r$ relays using $N_{r,m}-by-L_m$ complex weight matrix $W_{s,m}$ through channel $H_{r,m,s,m,s}$ and the AF relay derives the estimate $\hat{s}_{m,(t)}$ of $\ell$-th stream signal using $N_{r,m}$-dimensional complex weight vector $w_{r,m,\ell}$. Here we should remark that signal $\hat{s}_{m,(t)}$ contains noise component generated at the relay. Then in the second transmission phase, estimated signal $\hat{s}_{m,(t)}$ is again transmitted from the $r_{m}$-th relay using $N_{r,m}$-dimensional complex weight vector $w_{r,m,\ell}$. The signal arrives at the $d_{m}$-th destination which is similarly chosen as the pair with source $m$ through MIMO channel $H_{d,m,\ell}r_m,m,r_m$, and the destination produces the estimate $\hat{s}_{m,\ell}(t)$ of $\ell$-th stream signal using $N_{d,m}$-dimensional complex weight vector $w_{d,m,\ell}$.

Here our problem is to determine total $M$ combinations of source, relay, and destination nodes. For this aim, we utilize stable marriage problem which is known in the area of social science like economics.

### III. NODE PAIR SELECTION SCHEME

In this section, an actual node pair selection method based on stable marriage problem is given for the system described in the previous section. The former half of this section describes the determination of the preference function, and the latter half shows node pair selection by Gale-Shapley algorithm. In the third subsection, the weight design used in this study is also briefly shown.

Here, for the convenience of the later discussion, we define the problem of this section as follows:

#### Problem Setting

Find the set of node pairs $\{(S_m,R_{m},D_{d_m}); m = 0, \ldots, M - 1\}$ ($r_m \in \{0, \ldots, M_r - 1\}$ and $d_m \in \{0, \ldots, M_d - 1\}$) which have high performance as possible under the condition the transmission procedure and other resource allocation is kept same.

#### A. Preference Function

This subsection describes the definition of four types of functions which is used for obtaining the preference order in each of source, relay, and destination. In this paper, all of
them are based on the channel condition between transmitter and receiver (source-relay or relay-destination).

The preference function is a real number function defined between a source and a relay or a relay and a destination at the both sides. For example, in case of source-relay relation, it is defined as \( f_{s,m_r,r_m} \) at the source as the preference of \( S_{m_r} \) against \( R_{r_m} \), which means how much \( S_{m_r} \) prefers \( R_{r_m} \). Similarly, at the side of \( R_{r_m} \), the preference to \( S_{m_r} \) can be defined as \( f_{r_m,s_m,r_m} \). The same definitions are derived for the relay-destination link, namely, as \( f_{r_m,d,m_2} \) and \( f_{d,m_2,r_m} \). In the rest of this paper, this function is determined from the inter-node channels, hence in the strict sense, preference function should be expressed like \( f_{s,m_r,m_r}(H_{r_m,s_m,m_1}) \), but we usually adopt the simple notation \( f_{s,m_r,m_r} \) since the variable is clear from the subscript.

**Function 1: Fixed**

The pairs are all fixed even when the situation of channels (and/or other parameters) changes, which means preference functions are constant not depending on the instantaneous change of the channels. Namely, \( f_{s,m_r,y,m_0}(H_{y,m_0,x_m}) = \text{const.} \) for any \( (x,y) \in \{(s,r),(r,d)\} \). This choice requires no computation, and a certainly reasonable choice if a source is connected to the nearest relay, and then the relay communicates with a destination within the shortest distance, and furthermore, channel variation is negligible. This is the conventional approach in which a dynamic node pair selection is not adopted. If the statistics of all source-relay and relay-destination channels are same, this strategy is equivalent to the case where the preference function is random.

**Function 2: The Largest Eigenvalue of Channel Matrix**

The functions are defined as \( f_{x,m_r,y,m_0} = f_{y,m_0,x_m} = \lambda_{\ell}(H_{y,m_0,x_m}) \), where \( \lambda_{\ell}(A) \) is the largest eigenvalue of matrix \( A \). This metric simply considers the largest eigenvalue, and a reasonable choice when single stream transmission is adopted, but it becomes a rough metric in multistream cases.

**Function 3: Sum Capacity**

The function is given by

\[
    f_{x,m_r,y,m_0} = f_{y,m_0,x_m} = \sum_{\ell=0}^{L_m-1} \log_2 \{1 + \lambda_{\ell}(H_{y,m_0,x_m})\}.
\]

This equation represents the approximation of sum capacity (it is not the exact one since the energy allocation has not been introduced), and reflects the situation of multistream different from Function 2 (but the computational cost increases).

**Function 4: Channel Matrix Norm**

This metric adopts norm (here Frobenius norm is used) as a preference function.

\[
    f_{x,m_r,y,m_0} = f_{y,m_0,x_m} = \|H_{y,m_0,x_m}\|
\]

If \( L_m = \text{rank} H_{y,m_0,x_m} \), this metric becomes the approximation of sum capacity, which reflects the condition of all the data streams.

**Function 5: Absolute Sum of Channel Matrix Elements**

The preference function is defined as follows:

\[
    f_{x,m_r,y,m_0} = f_{y,m_0,x_m} = \sum_{m,n} |H_{y,m_0,x_m,m,n}|,
\]

where \( H_{y,m_0,x_m,m,n} \) is the \((m,n)\)-th element of matrix \( H_{y,m_0,x_m} \). This function is the only approximation of Function 3, but it requires no multiplication operation 1, hence has very low computational burden.

**Function 12-15: Signal to Interference Ratio Versions**

Above metrics consider only the strength between \( S_{m_r} \) and \( R_{r_m} \), or \( R_{r_m} \) and \( D_{m_2} \). But in multiuser system, generally used index is the ratio of target signal and interferences. Hence the possibility of such metrics is investigated in this study. Namely, signal to interference ratio version of preference function is defined by

\[
    f_{x,m_r,y,m_0} = \sum_{m_0 \neq m} f_{x,m_r,y,m_0} = f_{y,m_0,x_m},
\]

where \( f_{x,m_r,y,m_0} \) is one of the Function 1~5 (e.g., if Function 2 is used for \( f_{x,m_r,y,m_0} \), \( f_{x,m_r,y,m_0} \) is denoted as Function 12).

**B. Node Pair Selection**

This subsection describes the actual procedure of the determination of preference order, and the node pair selection using Gale-Shapley algorithm.

First, based on the preference function given in the previous section, preference order list is determined. Preference order is defined as \( P_{s,m_0} = \{P_{s,m_0},0, \cdots, P_{s,m_0},M_0\} \) for \( S_{m_0} \), where \( P_{s,m_0},k \in \{R_0, \cdots, R_{M_r-1},b\} \) and \( b \) denotes the ‘bachelor,’ namely, the situation the corresponding node does not need its pair any more \( (P_{s,m_0},k \neq P_{s,m_0},\ell) \) and the order of elements in \( P_{s,m_0} \) is important). List \( P_{s,m_0} \) shows that source \( S_{m_0} \) prefers relays in order of \( P_{s,m_0},0, \cdots, P_{s,m_0},M_0 \), which means \( P_{s,m_0},0 \) is the most preferable. The list elements appearing after ‘\( b \)’ are not chosen, but in communication system, normally \( b \) is located in the end of the list. The preference list for \( D_{m_2} \) similarly defined as \( P_{d,m_2} = \{P_{d,m_2},0, \cdots, P_{d,m_2},M_2\} \) \( P_{d,m_2,k} \in \{R_0, \cdots, R_{M_d-1},b\} \). Only relays have two preference lists, one is the list \( P_{r,m_0} = \{P_{r,m_0},0, \cdots, P_{r,m_0},M_0\} \) of source nodes \( P_{r,m_0,k} \in \{S_0, \cdots, S_{M_r-1},b\} \), and the other is the list \( P_{r,m_2} = \{P_{r,m_2},0, \cdots, P_{r,m_2},M_2\} \) of destination nodes \( P_{r,m_2,k} \in \{D_0, \cdots, D_{M_d-1},b\} \).

The way to make the preference list is quite simple; the elements are located in the list in the descent order of preference function. For example, if \( f_{s,m_r,r_m} > \cdots > f_{s,m_r,r_m,t_m-1} \), the list becomes \( P_{s,m_r} = \{R_0, \cdots, R_{M_r-1},b\} \). Similarly, \( P_{r,m_0}, P_{r,m_2}, \) and \( P_{d,m_2} \) are determined from \( f_{r,m_2,d,m_2}, f_{r,m_2,d,m_2}, \) and \( f_{r,m_2,r_m}, \) respectively.

Once the preference order list of all the nodes is determined, node pair is actually made based on them. Here, each of pairing problems in source-relay and relay-destination links can be considered as two sided matching, which can be solved
by using well-known Gale-Shapley algorithm. This algorithm is used twice for both links, so here we exemplify the case of source-relay link. The relay-destination link can be transacted in the same manner by changing the role of source to relay, and relay to destination.

**Gale-Shapley Algorithm** [8] (proposal from relay type)

**Input** Preference Order of Nodes
- Source $(P_{s,0}, \ldots, P_{s,M-1})$
- Relay $(P_{r,0}, \ldots, P_{r,M-1})$

**Output** Node Pair Set
- $(S_0, R_{r_0}), \ldots, (S_{M-1}, R_{r_{M-1}})\)

**Procedure**

**Step 1** proposal of relay ($M_r \geq M_s$)
- All relays suggest connection (like a proposal of marriage) to a source which is in the top of their preference list. For example, relay $R_{r_k}$ requests the connection to $P_{s, r_m, 0}$. 

**Step 2** acceptance/rejection of source
- Sources $S_{s_m}$ makes the list $C_{s_m}$ of relays who proposed the connection to it. Then $S_{s_m}$ chooses a relay from $C_{s_m}$ which is most preferable in its preference list. Other relays are notified the rejection of the proposal, and add $S_{s_m}$ into rejected relay list $\mathcal{J}_{s_m}$. This operation is carried out for all source nodes.

**Step 3** stop or continue
- If the criterion shown below is satisfied, the algorithm is terminated, and the pairs become $(S_0, R_{r_0}), \ldots, (S_{M-1}, R_{r_{M-1}})$. If not, go to Step 4, and repeat the procedure until the stopping criterion is satisfied.

**Stopping Criterion:**
- All relay node have a pair source node or reached to $\mathcal{J}_{s_m}$.

**Step 4** repeat proposal/acceptance/rejection
- The rejected relays suggest connection to a source which is preferable next to the previously rejected one in the preference list (and not in $\mathcal{J}_{s_m}$). Then repeat the operation of Step 2-4. Remark that even if a source node $S_{s_m}$ already has a pair node, if more preferable relay $R_{r_k}$ provides a proposal, it betrays the current partner and changes the pair node to $R_{r_k}$.

This procedure is assured to converge into a certain solution. In addition, except the calculation of the preference function, this algorithm does not need multiplication operation, and can be achieved with very few computations (hence a simple preference function is important). To keep the fairness among users (destination nodes), one way is to put the destination nodes who has finished the communication into lower preference order, but such variations are out of scope of this paper.

**C. Multiuser MIMO Design**

This section briefly describes the design method of relay-aided communication system used in the simulations of the next section.

After the selection of node pairs in the previous section, any resource allocation and design method can be used. Here, our aim is not the system design but to measure the effect of node pair selection, hence the most fundamental channel inversion method is used, but to reduce the degrees of freedom of minimum requirement, it is applied after the receiver weight is first designed [7] (this method can be easily extend to the investigation with interferences, namely, $M > N_{s,m}$ and/or $M > N_{r,m}$ different from another popular method, block diagonalization [9]). Namely, in the first transmission phase, receiver (relay) weight vector $w_{r,t,m,d}$ for the $\ell$-th stream of $m$-th user is derived as the normalized singular value vector corresponding to the $\ell$-th largest singular value of channel matrix $H_{r,m,s,m}$. Then the transmitter (source) weight matrix for $0 \sim L_{m-1}$-th streams is designed as $W_{s,m} = H_{r,m,s,m}$, where $A^T$ is the generalized inverse of matrix $A$. Matrix $H_{r,m,s,m}$ is defined as $H_{r,m,s,m} = \left[h_{0,0}^T, h_{1,1}^T, \ldots, h_{M-1,L_{m-1}}^T\right]$ and $h_{m,\ell} = w_{r,t,m,d}^H H_{r,m,s,m}$. Also in the second phase, the transmit (relay) weight matrix $W_{r,m,d}$ is defined as $W_{r,t,m,d} = [w_{r,t,m,d,0}, \ldots, w_{r,t,m,d,L_{m-1}}]$ and receiver (destination) weight vectors $\{w_{d,m}\}$ are similarly derived. By those operations, only one degree of freedom is demanded for elimination of one stream.

In the above design, only the allocation of other resources like transmission power is not taken into account, but the proposed method is considered to have a similar effect also in case of other adequate design approach including optimization of those parameters.

**IV. SIMULATION**

In this section, to verify the effectiveness and features of the proposed node pair selection algorithm, computer simulations are carried out. First, a simple result is shown for the relay less case as a reference, and then the graphs and discussions are given for relay-aided system.

The simulation conditions are as follows; the default number of the source, relay destinations are $M_s = 3$ (hence user number is also $M = 3$), $M_r = 5$, and $M_d = 8$, respectively. The antenna numbers of them are $N_{s,m} = 6$, $N_{r,m} = 6$, and $N_{d,m} = 2$, where the source is assumed to have relatively large chassis like base station or access point, and the destination is small equipment like portable phone or laptop. The channels between source-relay and relay-destination are considered to be under independent and identically distributed (i.i.d.)
Rayleigh fading with unit variance. (Relays are sometimes located in the line-of-sight (LOS) position from the source nodes, and then the fading becomes Ricean. But in this case, the connection can be fixed taking into account the LOS component which is normally dominant in the wave propagation. On the other hand, some applications put the relays in non LOS positions, and it well reflects the effect of the node selection, which is the reason Rayleigh channel is dealt with in this paper.) As described in section II, the direct channels $H_{d,m,s,m}$ are all ignored. The default simulation conditions are summarized in Table I.

As a measure of the effect of the proposed node pair selection approach in multistream transmission, the approximated sum capacity defined in the below is used:

$$C = \sum_{m=0}^{M-1} \sum_{\ell=0}^{L_{m}-1} \log_2(1 + \text{SINR}_{m,\ell}),$$

where $\text{SINR}_{m,\ell}$ is the signal to interference plus noise ratio (SINR) of the output signal of the $\ell$-th stream of the $m$-th user, and given by

$$\text{SINR}_{m,\ell} = \frac{|\rho_{m,\ell}|^2}{(1 - |\rho_{m,\ell}|^2)E[|s_{m,\ell}(t)|^2]E[|s_{m,\ell}(t)|^2]}.$$

It should be discriminated from signal to noise ratio (SNR) in Table I which is defined by SNR$_{x,m} = P_{x,m}/P_{n,x,m}$ for the $m$-th user ($x = r, d$). Here, $P_{x,m}$ is the total transmit powers of $S_m$, and $P_{n,x,m}$ and $P_{d,x,m}$ are the powers of noise generated at $R_m$ and $D_m$, respectively. The simulations are carried out over 2,000 samples of fading channels, and then mean value is calculated depending on the necessity.

Fig. 2 shows the distribution functions of capacity of user 0 (the curves are almost same also for other users because of the symmetry of channel statistics) in relay-less systems for the reference aim. The curve with broken line is fixed, and others are using the node pair selection, which include schemes adopting signal to interference ratio version of preference functions. From this figure, it is clear that the proposed selection scheme can improve the total performance without adding any additional resources.

The distribution functions of capacity in relay-aided case is given in Fig. 3 for user 0. Subplot (a) shows the case with same number of users as Fig. 2. Compared with relay-less case (Here comparison is not in the sense the replacement of the system of Fig. 2 by relay-aided one. The use of relays is under the assumption of negligible direct channels, and here we investigate is the effect of the additional noise and fading), the performance is degraded because of the existence of relay noise and double fading which do not exist in single hop case (Fig. 2). But the proposed method still achieves better characteristics than fixed case (broken line), which means the effectiveness of node selection is maintained also in relay-aided systems. The capacity of signal to interference version (Function 12~15) is very close to or even slightly worse than that of Function 2~5 (it is difficult to read from the
Fig. 4. Number of destination nodes versus sum capacity.

In subplot (a) of Fig. 3, the influence of interuser interference does not appear since the zero forcing by channel inversion is applied, but in subplot (b), the total stream number exceeds the degrees of freedom, hence the interuser interference affects the performance. In this case, signal to interference version has higher effect than the case of subplot (a), but it does not provide significant advantage compared with Function 1∼5, so in the rest, we consider only those simple preference functions (Function 1∼5) from the viewpoint of computational cost (so we conclude that the Function 1∼5 is normally sufficient choice).

Fig. 4 plots the number of destination versus capacity curves, namely, $M_d - C$ characteristics. The effect of node pair selection becomes slightly larger as the increment of destination number while that of fixed strategy is unchanged. This is a natural result, and similar tendency can be observed also by changing the number of relays.

Fig. 5 shows how the capacity changes as relay SNR increases. The right end of the horizontal axis labelled with 'infinity' denotes the case of noiseless relay which is equivalent to error-free decode-and-forward (DF) relay-aided system. It can be also interpreted as the pure effect of cascade connection of multiple source multiple destination system. Compared to this reference case, the actual system with additional relay noise has lower capacity as the relay SNR decreases. In high SNR region, the capacity improvement is saturated because the total performance is limited by the destination noise. It can be observed that, by using low noise relay, the effect of relay choice becomes dominant, and the effectiveness of the proposed node pair selection becomes larger.

The relation between destination SNR and capacity is drawn in Fig. 6. Similar tendency as Fig. 5 is observed, and the capacity increases as SNR increases, but because of the influence of the relay noise, the performance cannot be improved in proportion to SNR. It can be also seen that, different from Fig. 6, the effect of selection scheme is not dependent on destination SNR.

V. CONCLUSION

This paper has proposed a novel node pair selection scheme for relay-aided multiple source multiple destination communication system. Assuming multistream transmission under the condition all the nodes are equipped with multiantenna, the node pair selection is carried out based on stable marriage problem. For this aim, several types of preference functions are considered, and based on them, preference orders between source-relay and relay-destination are determined. The computer simulations have shown that the performance of the proposed approach is degraded from the simple connection of two hops because of the existence of the relay noise, but the degradation is quite limited by using low noise relay.

The future works are the improvement of the proposed method and performance investigation in more various cases. The examinations in case of...
nonnegligible source-destination channel, full-duplex relay transmission (where self-interference and inter-relay interferences should be taken into account), and/or frequency selectivity are also important topics. Simultaneous determination of source-relay-destination pairs utilizing three sided matching problem [10] is another attractive theme of the study.

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REFERENCES