Estimating 3D-Position of A Stationary Random Acoustic Source Using Bispectral Analysis of 4-Point Detected Signals

Katsumi Hirata

Abstract—To develop the useful acoustic environmental recognition system, the method of estimating 3D-position of a stationary random acoustic source using bispectral analysis of 4-point detected signals is proposed. The method uses information about amplitude attenuation and propagation delay extracted from amplitude ratios and angles of auto- and cross-bispectra of the detected signals. It is expected that using bispectral analysis affects less influence of Gaussian noises than using conventional power spectral one. In this paper, the basic principle of the method is mentioned first, and its validity and features are considered from results of the fundamental experiments assumed ideal circumstances.

Keywords—4-point detection, a stationary random acoustic source, auto- and cross-bispectra, estimation of 3D-position.

I. INTRODUCTION

To develop the useful acoustic environmental recognition system, it is the problem to be solved to localize the acoustic sources. We, human beings have the ability to localize acoustic sources using information about propagation delay and/or amplitude attenuation. In this study, we will realize the function of acoustic localization with simpler engineering way.

For such kind of problem, various methods are being developed [1]–[3], but most of those target the deterministic signal such as the voice with little consideration to influences of the observation noise. On the other hand, we propose localization methods which are applied to the random signal. Our conventional methods are based on the power (second order) spectral analysis and assumed that mutually uncorrelated noises are added to the detected signals. However, If the noise sources locate spatially, same or mutually correlated noises are added at detectors, and there is a possibility that the power spectral methods are difficult to be applied [4].

Then, we propose a method to estimate the 3D-position of a stationary random acoustic source using bispectral (third order spectral) analysis. We think that the bispectral method makes the localization under the circumstance with correlated Gaussian noises possible because the bispectrum of Gaussian signal vanishes [5].

In this paper, the basic principle of the proposed method is explained, then results of fundamental experiments under the ideal circumstance without acoustic reflections are presented,

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II. PRINCIPLE OF THE PROPOSED METHOD

A. Arrangement of Detectors and Detected Signals

This method is to estimate the 3D-position of a stationary random acoustic source using auto- and cross-bispectral analysis of signals detected at 4 spatially fixed microphones, where the signals are radiated from the source. The geometrical arrangement of a source and 4 microphones is illustrated in Fig.1. In the figure, P is a source, M1, M2, M3 and M4 are microphones, and r1, r2, r3 and r4 denote distances between the source and each microphone respectively.

\[ x_i(t) = \frac{1}{r_i} s(t - \frac{r_i}{c}) + n_i(t), \quad i = 1, 2, 3, 4 \]  

(1)

and the validity and features of the localization method are made clear.

\[ \Phi_{x_i,x_i}(f_1, f_2) \]  

(2)
where \( \Phi_{aa}(f_1, f_2) \) is the auto-bispectrum of the radiated signal \( s(t) \). The bispectrum is a third order spectrum. On the other hand, the cross-bispectra \( \Phi_{xz}(f_1, f_2) \) between \( x_i(t) \) and \( x_k(t) \) are

\[
\Phi_{xz}(f_1, f_2) = \frac{1}{r_i} r_4 \Phi_{aa}(f_1, f_2) e^{-j2\pi(f_1+f_2)(r_4-r_i)/c}, \quad i = 1, 2, 3.
\]

There is no term including noise in the auto- and cross-bispectra because the third order statistics of Gaussian vanish.

**B. Estimation of 3D Position of A Source**

Equations (2) and (3) denote that the auto- and cross-bispectra of detected signals are inversely proportional to the cubic (or third-order) of the distances \( r_i \), and the cross-bispectra have the phase term proportional to the differences of the distances \( r_4 - r_i \). The following information is extracted from the ratio of amplitude and phase of the bispectra.

\[
A_i(f_1, f_2) := \frac{\Phi_{xz}(f_1, f_2)}{\Phi_{zz}(f_1, f_2)} = \frac{r_i}{r_4}, \quad i = 1, 2, 3, \quad (4)
\]

\[
d_i(f_1, f_2) := \frac{c}{2\pi(f_1 + f_2)} \arg \left[ \Phi_{xz}(f_1, f_2) / \Phi_{zz}(f_1, f_2) \right] \quad (5)
\]

= \( r_4 - r_i \), \quad i = 1, 2, 3.

Equations (4) and (5) denote that \( A_i(f_1, f_2) \) and \( d_i(f_1, f_2) \) are the ratios and the differences of the distances \( r_i \), respectively. The distances \( r_i \) are derived by solving these simultaneous equations, and then the 3D-position of the source is calculated with the microphones coordinates. This is the principle to estimating the source position by the method.

If the auto-bispectra of radiating signal is spread over the analyzing band, \( A_i(f_1, f_2) \) and \( d_i(f_1, f_2) \) must be constant irrespective of the frequencies \( f_1 \) and \( f_2 \). It is thought that the information is extracted accurately by averaging them over the analyzing band \( B_H \),

\[
\hat{A}_i = \frac{1}{B_H} \int_{B_H} A_i(f_1, f_2) df_1 df_2, \quad i = 1, 2, 3, \quad (6)
\]

\[
\hat{d}_i = \frac{1}{B_H} \int_{B_H} d_i(f_1, f_2) df_1 df_2, \quad i = 1, 2, 3. \quad (7)
\]

### III. FUNDAMENTAL EXPERIMENTS

**A. Conditions**

Fundamental experiments were conducted to make clear the validity and features of the proposed method. In the experiments, estimations of the 3D-position were carried out many times for four kinds of positions under noiseless or noisy environments, and the statistics of estimates are evaluated. The detected signals were made from abnormal sound radiated from a small fan of 50 mm in a diameter with amplitude attenuation and time delay which depend on each position of the source. The main experimental conditions are shown in Table I.

**B. Results**

The distributions of the estimated positions from 50 independent estimations are shown in Fig. 2, 3 and 4, where red circles, black dots and green triangles indicate true positions, estimated positions of sources and microphones respectively. Fig. 2 show the estimates under the noiseless environment \( (S/N = \infty) \), and (a) - (d) are for the position A - D, respectively. For any positions, it is found that the estimates are very good with small errors because 50 dots are placed at the circle. Therefore, the source positions are able to be estimated by the method using bispectral analyses of detected signals.

Next, Fig. 3 show the estimates under the noisy environment for the position A, where the Gaussian noises are added to the detected signals, and the S/N’s are \( (a) 20 \text{dB}, (b) 15 \text{dB}, (c) 10 \text{dB} \) and \( d) 5 \text{dB} \). It is found that the variances of estimates are larger as the S/N is lower. But the estimates distribute around the true positions (the red circles). Though the 5dB-S/N is the very ill condition with loud noise, it is indicated the possibility of localization to a certain degree of accuracy with our method. Table II shows (a) bias and (b) RMS errors of estimates of positions in meters under each S/N, though the figures of the distributions of estimates for the position B, C and D are omitted for lack of space. The bias error is the root of squared sum of the bias and the variance, and the error means the averaged magnitude of the error of the estimated positions. For noiseless estimation \( (S/N = \infty) \), the bias and RMS errors are very small, and it is found that

![Table I](image)

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<th>Position</th>
<th>( S/N )</th>
<th>Bias Error [m]</th>
<th>RMS Error [m]</th>
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![Table II](image)

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Fig. 2  The distributions of estimated positions from 50 independent estimations under the noiseless environment, where red circles, black dots and green triangles indicate true positions, estimated positions of sources and microphones respectively.

Fig. 3  The distributions of estimated positions from 50 independent estimations under the noisy environment for the position A, where red circles, black dots and green triangles indicate true positions, estimated positions and microphones respectively.
the estimations are implemented in errors of about 3 cm at maximum. Especially, the errors of estimates for the position A and C are nearly zero because the positions are closer from the microphones than others. The lower S/N cause increasing the RMS errors. The increases of bias errors are not as much as the RMS. Therefore, for examples, it is thought that the error can be suppressed by meaning multiple estimates for a estimation.

IV. SIMPLIFIED BISPECTRAL ANALYSIS
The bispectrum is the 2-dimensional information with 2 parameters of frequencies $f_1$ and $f_2$. Therefore bispectral analysis takes time for calculation, and a memory is consumed. Then we introduce the simplified bispectral analysis to estimate a source position. In this section, the result of estimating using auto- and cross-bispectra only at $f_1 = f_2$ is reported as an example. The simplified auto- and cross-bispectra $B_{x_i},x_i,\ldots, (f)$, $B_{x_i},x_i,\ldots(x)$ are described following equations.

$$B_{x_i},x_i,\ldots, (f) = \frac{1}{r_i^2} B_{sss}(f), \quad i = 1, 2, 3, 4,$$

(8)

where $B_{sss}(f)$ is the simplified auto-bispectrum of radiated signal $s(t)$, and

$$B_{x_i},x_i,\ldots(x) = \frac{1}{r_i^2} B_{sss}(f)e^{-j2\pi(f)(x_4-x_1)/c}.$$  

(9)

Fig. 4 shows examples of (a) the conventional auto-bispectra and (b) the simplified one of the signal detected at the microphone $M_i$, where a white broken line indicates $f_1 = f_2$. It is found that most of the components of the signal are on and near the line.

Table III shows (a) bias and (b) RMS errors of estimates of positions under each S/N. With a very few exceptions, the errors of estimates using simplified bispectra are as small as using conventional ones. It is thought that the information about the ratios and the differences of the distances from the microphones to the source is extracted efficiently because the most of the information is concentrated on the $f_1 = f_2$. Therefore, for the source signal having such a bispectral density, the method used simplified bispectra brings the efficient and effective estimation.

V. CONCLUSION
To develop the useful method of 3D-localization of an acoustic source, the method applicable to a stationary random acoustic source using the bispectral analysis of 4-point detected signals is proposed, and the validity and the features were made clear through the evaluations of the fundamental experimental results. As a result, first, the validity of the method was confirmed under the ideal and noiseless condition. Next, it was found that it is a task to heighten accuracy of the bispectral estimation and the information extraction. Moreover, we found that the estimates by the simplified bispectral analysis is as accurate as the conventional bispectral one but depending on the spectral density of the source signal.

In addition, it is thought that it is necessary to consider applying to multiple sources or echoic environment.

REFERENCES

Katsumi Hirata was born in Osaka, Japan, in 1975. He received the Ph.D. degree in engineering from University of Tsukuba in 2002. He is currently an assistant professor at Oyama National College of Technology, Japan. Dr. Hirata is a member of the IEEE, the Institute of Electronics, Information and Communication Engineers of Japan and the Society of Instrument and Control Engineers of Japan.