MHD Unsteady Free Convection of Heat and Mass Transfer Flow through Porous Medium with Time Dependent Suction and Constant Heat Source/Sink

Praveen Saraswat, Rudraman Singh

Abstract—In this paper, we have investigated the free convection MHD flow due to heat and mass transfer through porous medium bounded by an infinite vertical non-conducting porous plate with time dependent suction under the influence of uniform transverse magnetic field of strength $H_0$. When Temperature $(T)$ and Concentration $(C)$ at the plate is oscillatory with time about a constant non-zero mean. The velocity distribution, the temperature distribution, co-efficient of skin friction and role of heat transfer is investigated. Here the partial differential equations are involved. Exact solution is not possible so approximate solution is obtained and various graphs are plotted.

Keywords—Time Dependent Suction, Convection, MHD, Porous.

I. INTRODUCTION

FLOW of fluid past an oscillating plate is of great importance owing to its application in aerodynamics and automobile industries, etc. The problem of heat transfer from an oscillating plate has been discussed by several authors as Stuart [6], Rao et al. [4], etc. Free convection flow especially in porous medium is one of the most interesting subject matter's because of importance in petroleum, chemical and nuclear industries.

The magnetic effect can also be used in power generator. Lin and Wu [3] analyzed the problem of simultaneous heat and mass transfer with entire range of buoyancy ratio for most practical chemical especially in dilute solution and aqueous solution. Yan et al. [7] have investigated numerically the laminar mixed convective flow in a channel and simultaneous influence of the combined buoyancy effects and the thermal and mass diffusion for a water system. Chitti Babu and Rao [1] analyzed the free convective flow of heat and mass transfer past a vertical porous plate taking Viscous and Darcy resistance terms into account constant permeability. Sharma and Sharma [5] studied unsteady flow of incompressible viscous fluid and heat transfer along hot infinite, porous vertical surface bounded by porous medium in presence of oscillating free stream and cross flow velocity. Here we are investigating the case dealt by Gupta et al. [2]. Here we are dealing with unsteady free convective MHD flow due to heat and mass transfer through porous medium bounded by an infinite vertical oscillatory porous plate in presence of transverse magnetic filed $H_0$ along with time dependent suction.

II. MATHEMATICAL ANALYSIS

Consider the unsteady free convection flow of an electrically conducting incompressible viscous fluid in presence of transverse magnetic field of strength $H_0$ through a porous medium (assume highly porous of permeability $K$) bounded by an infinite vertical porous plate with heat source/sink. Let x-axis be taken in vertically upward direction along the infinite vertical plate and y-axis normal to it. Neglecting the induced magnetic field and applying Boussinesq Equations:

$$\frac{\partial v}{\partial y} = 0$$
$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g \beta (T - T_0) + \nu \frac{\partial^2 u}{\partial y^2} + \frac{g \beta}{K} (C - C_\infty) - \frac{\mu}{k} \frac{\partial H_0 u}{\partial y}$$
$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q(T - T_0)}{k}$$
$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2}$$

where $u$ and $v$ are velocities along x- and y-axes respectively, $T$ and $T_0$ are temperatures of fluid near and away from the plate respectively, $C$ and $C_\infty$ are the boundary layer species concentration in the boundary layer near and away from the plate respectively, $\rho$ is the density, $\sigma$ is the co-efficient of electric conductivity. $\mu_0$ is the magnetic permeability, $g$ is the acceleration due to gravity, $t$ is the time and $\beta$ and $\beta'$ are the co-efficient of volume expression and volume expression with concentration respectively, $\nu$ is the kinematic viscosity of fluid, $\alpha$ is thermal conductivity, $Q$ is the heat source, $D$ is chemical molecular deficiency. Here, effect of induced magnetic field is neglected Reynolds theory number is assumed for suction velocity is here taken as such:

$$v = -v_0 (1 + e^{\int_{0}^{t} \sigma dt})$$

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The negative sign indicates that the suction is towards the plate. The boundary conditions are:

\[
u = 0, \quad T = T_\infty + \varepsilon (T_\infty - T_e) e^{i\omega t},
\]

\[
C = \frac{C_\infty + \varepsilon (C_\infty - C_e) e^{i\omega t}}{F},
\]

\[
u \to 0, \quad T \to T_e, \quad C \to C_\infty
\]

where the mean temperature of plate is \(T_e\), \(C_\infty\) is the species concentration near the plate and \(\omega\) is the frequency.

Introducing the following non dimensional quantities we have

\[
u^* = \frac{u}{v_0 F G}, \quad T^* = \frac{T - T_e}{(T_\infty - T_e) F}, \quad C^* = \frac{C - C_\infty}{(C_\infty - C_e) F},
\]

\[
y^* = \frac{v y}{v_0}, \quad t^* = \frac{v t}{v_0^2}, \quad w^* = \frac{w}{v_0}, \quad k^* = \frac{k v_0^2}{v}.
\]

Substituting the above values in (1) and (2) we obtain,

\[
\frac{v_0^3 F G}{v} \left( \frac{\partial u^*}{\partial t^*} \right) - v_0 \left( 1 + \varepsilon e^{i\omega \tau} \right) \frac{v_0^3 F G}{v} \frac{\partial u^*}{\partial y^*} = g \beta T^* F (T_\infty - T_e) + \frac{v_0^3 F G}{v} \frac{\partial u^*}{\partial y^*} + g \beta (C_\infty - C_e) F G
\]

\[
\frac{v_0^3 F G}{v} \frac{\partial u^*}{\partial y^*} - \frac{v_0^3 u^* F G}{v k^*} - \frac{\mu}{\sigma} H_n^2 u^* F G V_e
\]

Dropping * and using the following parameters, we have

\[
F = 1 + \varepsilon e^{i\omega t},
\]

\[
N = \frac{G e}{G}, \quad G = \frac{v_0 g \beta (T_\infty - T_e)}{v_0} \text{ is Grashof Number},
\]

\[
P = \frac{\nu}{\alpha} \text{ Prandtl Number},
\]

\[
M = \frac{\mu}{\sigma} H_n \sqrt{\frac{\sigma v}{v_0}} \text{ is magnetic field},
\]

\[
S = \frac{v_0 Q}{v_0} \text{ is heat source parameter and } S_e \text{ is Schmidt Number}.
\]

We obtain

\[
\frac{\partial \tilde{u}^*}{\partial y^*} + \frac{\tilde{u}^*}{(1 + \varepsilon e^{i\omega t})} - \frac{u^*}{k} \tilde{u} + \frac{\partial u^*}{\partial t^*} + T + N C = 0
\]

(7)

Similarly,

\[
\frac{1}{\rho} \frac{\partial^2 T^*}{\partial y^*} + \frac{\partial T^*}{\partial y^*} (1 + \varepsilon e^{i\omega t}) - \frac{\partial T^*}{\partial t^*} + TS = 0
\]

(8)

and boundary conditions are reduced as

\[
u = 0, \quad C = 1 + \varepsilon e^{i\omega t} F \quad \text{at } y = 0,
\]

\[
T \to 0, \quad C \to 0, \quad u \to 0 \quad \text{as } y \to \infty
\]

III. SOLUTION OF THE PROBLEM

In solving (7), (8) and (9) and using of boundary conditions, we here assume:

\[
u_{(y,j)} = u_{0(y)} + \varepsilon e^{j\omega t} u_{1(y)}
\]

(10)

\[
T_{(y,j)} = T_{0(y)} + \varepsilon e^{j\omega t} T_{1(y)}
\]

(11)

\[
C_{(y,j)} = C_{0(y)} + \varepsilon e^{j\omega t} C_{1(y)}
\]

(12)

Here (7) becomes:

\[
\frac{d^2 u}{dy^2} + \varepsilon e^{i\omega t} \frac{d^2 u}{dy^2} + \left( \frac{du}{dy} + \varepsilon e^{i\omega t} \frac{du}{dy} \right)(1 + \varepsilon e^{i\omega t})
\]

\[
- \left( \frac{u_{0} + \varepsilon e^{i\omega t} u_{1}}{K} \right) - M^2 \left( u_{0} + \varepsilon e^{i\omega t} u_{1} \right) - i \omega e^{i\omega t} u_{1}
\]

\[
+ T_{0} F e^{i\omega t} T_{1} + N \left[ C_{0} + \varepsilon e^{i\omega t} C_{1} \right] = 0
\]

(13)

Using Harmonic and non–Harmonic term and neglecting higher power terms, the above equation gives:

\[
u_{0} + u_{0} - \frac{u_{0}}{K} = M^2 u_{0} + T_{0} + N C_{0} = 0
\]

(14)

\[
u_{1} + u_{1} + \frac{u_{1}}{K} = M^2 u_{1} - C_{0} u_{0} + T_{1} + N C_{1} = 0
\]

(15)

Similarly (8) and (9) give:

\[
\frac{1}{\rho} \frac{\partial T_{0}}{\partial y} + \frac{\partial T_{0}}{\partial y} (1 + \varepsilon e^{i\omega t}) - \frac{\partial T_{0}}{\partial t} + TS_{0} = 0
\]

(16)

\[
\frac{1}{\rho} \frac{\partial T_{1}}{\partial y} + \frac{\partial T_{1}}{\partial y} - i \omega t_{1} + ST_{1} = 0
\]

(17)

\[
\frac{C_{0}}{S C} + C_{1} = 0
\]

(18)

\[
C_{1} i \omega C_{1} + C_{0} + C_{1} = 0
\]

(19)

On account of (10), (11), (12), the boundary conditions are reduced to:
\[ u = 0, u_i = 0, C_0 = 1, C_i = 1, T_0 = 1, T_i = 1, \text{ at } y = 0 \]
\[ T_0 = 0, T_i = 0, u_0 = 0, u_i = 0, C_0 = 0, C_i = 0, \text{ as } y \to \infty \]

The various parameters used in the problem are calculated as follows:

\[ C_0 = e^{-S_{1,y}}, \]
\[ C_i = \left(1 + \frac{S}{iw}\right)e^{-S_{1,y} - \frac{e^{-S_{1,y}}}{iw}}. \]
\[ T_0 = e^{-\beta}, \]
\[ T_i = (1 + \delta)e^{-\beta y} - \delta e^{-\beta y}, \]
\[ u_0 = \left(\xi + \phi\right)e^{-\beta y} - \xi e^{-\beta y} - \phi e^{-S_{1,y}}, \]
\[ u_i = \left(\xi + \phi\right)e^{-\beta y} + \left(\delta - \xi - \beta\right)e^{-\beta y} + S_i \left(\frac{N}{io} - \phi\right). \]

\[ e^{-S_{1,y}} \frac{S_i - S - \theta}{\gamma^2 - \gamma - \theta} = -\frac{Ne^{-\theta}}{\alpha^2 - \alpha - M^2 - \frac{1}{k}} + i\omega. \]
\[ + e^{\gamma t} \left[1 + \frac{1 + 4\delta}{\gamma^2 - \gamma - \theta} - \frac{(\xi + \phi) - \left(\delta - \xi - \beta\right)}{\alpha^2 - \alpha - M^2 - \frac{1}{k}} \right] + \frac{1}{\gamma^2 - \gamma - \theta} \frac{N}{io} \left(\frac{S_i}{io} - \phi\right). \]

Where we have used

\[ \alpha = \left[\frac{S + 4ioS_i}{2}\right], \beta = \left[\frac{\rho + \sqrt{\rho^2 - 4ps}}{2}\right], \gamma = \left[\frac{\rho + \sqrt{\rho^2 - 4ps + 4i\rho}}{2}\right], \delta = \left[\frac{\beta}{i\rho}\right], \phi = \left[\frac{N}{S_i - S - M^2 - \frac{1}{k}} \right], \theta = M^2 + \frac{1}{k} + i\omega. \]
\[ \xi = \frac{1}{\beta^2 - \beta - M^2 - \frac{1}{k}}. \]

IV. Numerical Computation

Let us take the value of the following k, m, Sc, S, t, \( \omega, \Omega \) respectively k = 5; m = 0, 1, 2, 3; Sc = 1, 16, 32; S = 1.2, .5; N = 1, p = .5, \( \epsilon = 2, \omega = .2. \)

From the above values we have:

\[ \alpha = .319 + .979, .219 + .144, .439 + .229, \text{ when } S_c = 1, .16, .32, \beta = .3618, (.25 + .19)i, (.25 + .433i), \text{ when } S = 1.2, .5. \]
\[ \gamma = .673 + .372, .372 + .53, (.22 + .909), \text{ when } S = .1, .2, .5 \]
\[ \phi = .468, .319, .16, .089 \text{ at } m = 0, 1, 2, 3 \]
\[ \psi = 2, 3, 2.302, 3, 3.85 \text{ at } m = 0, 1, 2, 3, \]
\[ \delta = 3.618i, (.19 + .25), (4.33 - .25), \text{ when } S = 1, .2, .5 \]
\[ \theta = 2 + .2i, 3 + 2i, 5 + 2i, 11 + 2i \text{ at } m = 0, 1, 2, 3, \]
\[ \xi = .44, -.301, -.16, -.089 \text{ at } m = 0, 1, 2, 3, \]
\[ C_0 = e^{-1.1y}, e^{-1.6y}, \text{ when } S_c = 1, 16, 32, \]
\[ C_i = (1 - .5i)e^{-(319 + .979)y} + .5i e^{1.1y} \text{ at } S_c = 1, \]
\[ C_i = (1 - .8i)e^{-(219 + 1.44)y} + .8i e^{1.16y} \text{ at } S_c = 16, \]
\[ C_i = (1 - 1.61i)e^{-(439 + .229)y} + 1.6i e^{32y} \text{ at } S_c = 32, \]
\[ T_0 = e^{-3.618y}, e^{-2.5 + 1.9y}, e^{-(2.5 + 3.43)y}, \text{ when } S = 1, 2, 5. \]
\[ T_i = (1 - 3.618i)e^{1.979y} + 3.618i e^{-3.618y} \text{ at } S = 1, \]
\[ T_0 = (2.9 - 2.5i)e^{3.725y} - .19 + .25i e^{(2.5 + 3.43)y} \text{ at } S = 2, \]
\[ T_i = (5.33 - 2.5i)e^{3.725y} - (4.33 - 2.5i) e^{(2.5 + 3.43)y} \text{ at } S = 5, \]
\[ u_0 \text{ at } m = 0 = - .908e^{-2.2y} - .44e^{3.618y} - .468e^{1.16y}, \]
\[ u_0 \text{ at } m = 1 = .62 e^{3.2y} - .301e^{3.618y} - .319e^{1.16y}, \]
\[ u_0 \text{ at } m = 2 = .323 e^{3.2y} - .16e^{3.618y} - .16e^{1.16y}, \]
\[ u_0 \text{ at } m = 3 = .783 e^{-3.85y} - .898e^{-3.618y} - .898e^{1.16y}, \]
\[ u_0 \text{ at } m = 0 = .908e^{-1.2y} - (.168 + .18) e^{-3.618y} - (.57 + 1.08) e^{(2.37 + .979)y} - (.42 + 2.67) e^{(2.29 + 1.44)y} \]
\[ + e^{y2(3.002 + .387)} (1.85 + 2.107i) \]
\[ u_0 \text{ at } m = 1 = (.0967 - .735i) e^{1.32y} + (1.082 - .6669) e^{3.618y} \]
\[ + .001 - .255i e^{1.16y} + (.2030 - 1.142i) e^{(2.37 + .979)y} -(3.119 - 2.53i) e^{(2.29 + 1.44)y} + e^{y2(3.002 + .387)} (1.85 - .6669 + .222) e^{(2.29 + 1.44)y} \]
\[ + e^{y2(3.002 + .387)} (1.85 + 2.107i) \]

Putting the values of \( u_0 \) and \( u_i \) at \( m = 0 \) in (10) we obtain,

\[ u_0 = .908e^{-2.2y} - .44e^{3.618y} - .468e^{1.16y} + e^{2.2y} (-908) i e^{1.6y} \]
\[ - (1.2 + .04) e^{3.618y} + (1.68 + .18) e^{1.16y} - (57 + 1.08) e^{(2.37 + .979)y} \]
\[ + e^{y2(3.002 + .387)} (1.85 + 2.107i) \]

Taking \( \omega t = \pi / 2 \), the real part equation becomes of (19)
Similarly, we have found the values of \( u(y, z) \) at \( m = 1, 2 \) and 3. Also putting the values of \( T_0 \) and \( T_1 \) in (11) at \( S = 1 \),

\[
T(y, z) = e^{-360y} + e^{360y} [(1-3.618) e^{(237-197)y} + 3.618 e^{360y}] \quad (21)
\]

Putting \( \omega t = \pi / 2 \) and taking real part, we have:

\[
T(y, z) = 2764 e^{360y} + (.7226 G_0 \cdot 197y + 2 \cdot 197y) e^{237y} \quad (22)
\]

Similarly we have found the values of \( T(y, z) \) at \( m = 2 \) and \( 5 \).

Putting the different values of \( C_0 \) and \( C_1 \) in (12), we have the real part of \( C(y, z) \) at \( S_c = 1 \) is

\[
C(y, z) = .99 e^{3y} + (.1 \cdot 197y + 2 \cdot 197y) e^{319y} \quad (23)
\]

Similarly we have found the values of \( C(y, z) \) at \( S_c = .16 \) and \( .32 \).

V. SKIN FRICTION

Co-efficient of Skin Friction is given by

\[
\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad (24)
\]

where \( u \) is the real part of (25) for \( m = 0 \)

\[
u_{(y, z)} = -908 e^{-2y} - 448 e^{-360y} - .468 e^{-360y} + e^{3y} \left[ (-907e^{-16y} - (.12 + .04i) e^{-361y} + (.168 + 1.8) e^{-16y} - (.57 + 1.08i) e^{-237y} \right. \\
(C_0 \cdot 197y - i \cdot 197y) - (42 + 267i) (C_0 \cdot 144y - i \cdot 144y) e^{219y} + e^{-160y} (1.85 + 2.107i) (C_0 \cdot 1935y - i \cdot 1935y)] \quad (25)
\]

Taking real part and then partial differentiation w.r.t. \( y \) and taking \( y = 0 \), at \( m = 0 \), from (21), we obtain:

\[
\left( \frac{\partial u}{\partial y} \right)_{y=0} = .05535 - .6264 \cdot \sin \omega t + .24816 \cdot \cos \omega t \quad (26)
\]

In the same manner we obtain: at \( m = 1 \),

\[
\left( \frac{\partial u}{\partial y} \right)_{y=0} = 1.5859 - 1.0514 \cdot \sin \omega t + 1.72616 \cdot \cos \omega t \quad (27)
\]

Similarly, at \( m = 2 \), we obtain:

\[
\left( \frac{\partial u}{\partial y} \right)_{y=0} = .885512 - 1.3822 \cdot \sin \omega t - .768 \cdot \cos \omega t \quad (28)
\]

Similarly, at \( m = 3 \) we obtain:

\[
\left( \frac{\partial u}{\partial y} \right)_{y=0} = .0218 - 7.30 \cdot \cos \omega t - 17.54 \cdot \sin \omega t \quad (29)
\]

VI. RESULTS AND DISCUSSION

The effect of material parameter on velocity distribution has been shown in Fig. 1. We observe that increase in magnetic field causes a rapid decrease in velocity but after a certain distance it is reversed; in the case of no magnetic field, it increases first and then decrease.

Fig. 2 exhibits the fact that increases in \( S_c \) number, causes a decrease in concentration of fluid mass. It is also seen that concentration is more near the plate; as we move further, it decreases rapidly but gradual decrease is less.

Fig. 3 shows that the presence of heat source causes a decrease in temperature but far from plate it becomes stationary, if heat source is of high intensity then it first decreases and then gradually increases.

In Fig. 4, skin friction is plotted for a fix value of \( \omega t = .2 \). It is seen that increase in magnetic field shows a gradual decrease in skin friction but after a certain time, this skin friction shows a very-very low change.

Fig. 5 shows that if time value is constant, \( \omega t = 1 \), then skin friction decrease, increases in magnetic field and increase in frequency values also create a decrease in skin friction.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>VELOCITY (U) PROFILES WITH RESPECT TO MAGNETIC FIELD (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( u_{\omega M = 0} )</td>
</tr>
<tr>
<td>0</td>
<td>-0.9918</td>
</tr>
<tr>
<td>1</td>
<td>0.255042</td>
</tr>
<tr>
<td>2</td>
<td>0.117913</td>
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<tr>
<td>3</td>
<td>0.01851</td>
</tr>
<tr>
<td>4</td>
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</tr>
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<td>5</td>
<td>-0.06931</td>
</tr>
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<td>6</td>
<td>-0.08597</td>
</tr>
<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
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</tr>
<tr>
<td>9</td>
<td>-0.08679</td>
</tr>
<tr>
<td>10</td>
<td>-0.07932</td>
</tr>
</tbody>
</table>
Fig. 1 Velocity ($u$) profiles at different values of $M=0, 1, 2, 3$ and other parameters are fixed at $k=.5, N=1, P=.5, \varepsilon=.2, \omega=.2$

![Velocity (u) profiles at different values of M=0, 1, 2, 3 and other parameters are fixed at k=.5, N=1, P=.5, \varepsilon=.2, \omega=.2](image)

**TABLE II**

<table>
<thead>
<tr>
<th>CONCENTRATION (C) WITH RESPECT TO SCHMIDT NO ($Sc$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONCENTRATION (C)</td>
</tr>
<tr>
<td>$y$</td>
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<td>8</td>
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</table>

**TABLE III**

<table>
<thead>
<tr>
<th>TEMPERATURE (T) WITH RESPECT TO HEAT SOURCE PARAMETER ($S$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEMPERATURE (T)</td>
</tr>
<tr>
<td>$y$</td>
</tr>
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</tr>
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</tr>
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<tr>
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</tbody>
</table>

Fig. 2 Concentration (C) profiles at different values of $Sc=.1, .16, .32$ and other parameters are fixed at $k=.5, N=1, P=.5, \varepsilon=.2, \omega=.2$

![Concentration (C) profiles at different values of Sc=.1, .16, .32 and other parameters are fixed at k=.5, N=1, P=.5, \varepsilon=.2, \omega=.2](image)

Fig. 3 Temperature (T) profiles at different values of $S=.1, .2, .5$ and other parameters are fixed at $k=.5, N=1, P=.5, \varepsilon=.2$

![Temperature (T) profiles at different values of S=.1, .2, .5 and other parameters are fixed at k=.5, N=1, P=.5, \varepsilon=.2](image)
TABLE IV
Skin Friction (τ) profiles with respect to time (t)

<table>
<thead>
<tr>
<th>t</th>
<th>S.F. at M=0</th>
<th>S.F. at M=1</th>
<th>S.F. at M=2</th>
<th>S.F. at M=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.30351</td>
<td>3.31206</td>
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<td>-1.9128</td>
<td>-10.6173</td>
</tr>
<tr>
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<td>0.0399892</td>
<td>3.134855</td>
<td>-2.13114</td>
<td>-13.5523</td>
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<td>2.951195</td>
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Fig. 4 Skin Friction (τ) profiles at k=.5, N=1, P=.5, ε=.2, with respect to time (t)

TABLE V
Skin friction (τ) profiles with respect to frequency (ω)

<table>
<thead>
<tr>
<th>τ</th>
<th>S.F. at M=0</th>
<th>S.F. at M=1</th>
<th>S.F. at M=2</th>
<th>S.F. at M=3</th>
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Fig. 5 Skin Friction (τ) profiles at k=.5, N=1, P=.5, ε=.2, τ=1 with respect to Frequency (ω)

REFERENCES