Abstract—Multiprocessor task scheduling is a NP-hard problem and Genetic Algorithm (GA) has been revealed as an excellent technique for finding an optimal solution. In the past, several methods have been considered for the solution of this problem based on GAs. But, all these methods consider single criteria and in the present work, minimization of the bi-criteria multiprocessor task scheduling problem has been considered which includes weighted sum of makespan & total completion time. Efficiency and effectiveness of genetic algorithm can be achieved by optimization of its different parameters such as crossover, mutation, crossover probability, selection function etc. The effects of GA parameters on minimization of bi-criteria fitness function and subsequent setting of parameters have been accomplished by central composite design (CCD) approach of response surface methodology (RSM) of Design of Experiments. The experiments have been performed with different levels of GA parameters and analysis of variance has been performed for significant parameters for minimisation of makespan and total completion time simultaneously.

Keywords—Multiprocessor task scheduling, Design of experiments, Genetic Algorithm, Makespan, Total completion time.

I. INTRODUCTION

The multiprocessor task scheduling is considered to be NP hard problem in which the tasks or jobs are to be processed on more than one processor at a time such that optimal objectives can be achieved. There are several applications of multiprocessor task scheduling such as information processing, fluid flow, weather modeling, database systems, real-time high-speed simulation of dynamical systems, and image processing [1]. Scheduling of tasks is very important as inappropriate scheduling of tasks can fail to exploit the true potential of a parallel system and can offset the gains from parallelization due to excessive communication overhead or under-utilization of resources. Thus it falls to one’s scheduling strategy to produce schedules that efficiently utilize the resources of the parallel systems and minimize the total execution time [2].

Most of research in field of multiprocessor task scheduling is concerned with the minimization of the single criteria i.e.

makespan. However, in practice, many fields have tradeoffs in their scheduling problems where multiple objectives need to be considered in order to optimize the overall performance of the system. Obviously, the multi-objective scheduling problems are more complex than the scheduling problems with one criterion, and it is hard to find a compromise solution as the objectives are often inconsistent, conflicting or even contradictory.

The importance of the multiprocessor task scheduling problem led to several comparative studies. Several heuristics & metaheuristics have been developed for the solution of the multiprocessor task scheduling. In the case of Parallel machine scheduling, there are many literatures surrounding the multi-objective problem. The use of Holland’s genetic algorithms [3] (GAs) in scheduling, which apply evolutionary strategies to allow for the fast exploration of the search space of schedules, allows good solutions to be found quickly and for the scheduler to be applied to more general problems [4]. E. Kim et al. [5] considered a deterministic scheduling problem where multiple jobs with s-precedence relations are processed on multiple identical parallel machines. The objective is to minimize the total completion time. The precedence relation between two jobs i and j represents the situation where job j is constrained from processing until job i starts processing, which is different from the standard definition of a precedence relation where j cannot start until i completes. Hwang et al. [6], addresses the challenge of multiprocessor task scheduling parallel programs, represented as directed acyclic task graph (DAG), for execution on multiprocessors with communication costs. Genetic algorithm was used for solving this problem and design the new encoding mechanism with a multi-functional chromosome that uses the priority representation—the so-called priority-based multi-chromosome (PMC).

Hou et al. [7] developed efficient method based on genetic algorithm for multiprocessor scheduling. They developed crossover operator which are based on task graphs with dependencies but without communication delays. They showed that the results of GA were within 10% of the optimal schedules when compared with others.

Wu et al. [8] proposed a novel GA which allows both valid and invalid individuals in the population. This GA uses an incremental fitness function and gradually increases the difficulty of fitness values until a satisfactory solution is found. This approach is not scalable to large problems since
much time is spent evaluating invalid individuals that may never become valid ones.

Ceyda Oguj et al. [9] proposed a genetic algorithm for hybrid flow shop scheduling problem with multiprocessor tasks. They developed a new crossover operator (NXO) and compare it with PMX crossover. Some preliminary tests were performed for tuning of different parameters of GA such as population size, crossover rate and mutation rate.

Correa et al. [10] proposed a modified GA by the use of list heuristics in the crossover and mutation in a pure genetic algorithm. This method is said to dramatically improve the quality of the solutions that can be obtained with both a pure genetic algorithm and a pure list approach. Unfortunately, the disadvantage is that the running time is much larger than when running the pure genetic algorithm.

M. R. Bonyadi and M.E. Moghaddam [11] proposed Bipartite Genetic Algorithm (BGA) for minimizing the maximum completion time for a multiprocessor task scheduling problem. They performed a preliminary test to set the parameters of GA for better performance and compared the results with GA-based & heuristic based algorithms from literature in terms of STD, average makespan, best obtained makespan and iterations. Goh et al.[12] presented a hybrid evolutionary algorithm for task scheduling heterogeneous multiprocessor environment and proved that the proposed genetic operators, when coupled with the local search operators performed better than in the case where any one of the operators were omitted.

P. Chitra et al. [13] considered the multi-objective task scheduling problem in heterogeneous distributed computing systems (HDCS) with two objectives of makespan & reliability index. They developed two Multi-Objective Evolutionary Algorithms and experiments were performed on various random task graphs and a real-time numerical application graph. They showed that, MOEA algorithms are well-suited for obtaining good Pareto optimal solutions in a single run for task scheduling problem.

M. R. Mohamed and M. H. A. Awadalla [14] developed a modified list scheduling heuristic (MLSH) & a hybrid approach composed of GA and MLSH for task scheduling in multiprocessor system. They proposed three different representations for the chromosomes of genetic algorithm: task list (TL), processor list (PL) and combination of both (TLPLC) and found that proposed approach outperforms the others in terms of best makespan, average makespan & processor efficiency.

Thus, several methods have been considered till now to solve this problem based on GAs. But, most of these methods considers single criteria multiprocessor task scheduling problem. In the present work, minimization of the bi-criteria multiprocessor task scheduling problem has been considered which includes weighted sum of makespan (time at which last task in the schedule finishes) & total completion time (overall time in which all the tasks of a schedule get finished) using Genetic algorithm. Genetic Algorithm belongs to the approximate techniques and an optimal solution depends on the different parameters like type of crossover & mutation, crossover & mutation rate, selection function etc. Optimal combination of genetic algorithm parameters is necessary for the solution of multiprocessor task scheduling problem. The present work is an attempt for optimization of genetic algorithm parameters for the considered bi-criteria multiprocessor task scheduling problem.

II. MULTIPROCESSOR TASK SCHEDULING PROBLEM

In the present work, a multiprocessor task scheduling problem with ‘n’ tasks & ‘m’ processors has been considered. There are some tasks which are dependent on other tasks & cannot be started until their predecessors have been processed. After a task is processed, its successor task may be processed only after a predefined time (communication cost) [11]. To show this dependency, precedence & communication cost, the input is considered in terms of Directed acyclic Graph (DAG).

In a DAG G= (V, E), V the set of vertices represent the tasks & E, set of directed edges show the dependency between tasks. The computation weight of each vertex show the number of CPU cycles required by a task & the computation weight on each directed edge shows the communication cost. The sample problem considered for parameter setting & experimental results is a standard Gauss Elimination of 18 tasks & 4 processors along with variable communication cost for each task as shown in Fig. 1.

Fig. 1 Standard Gauss Elimination of 18 tasks & 4 processors [11], [13]
A. Assumptions

The different assumptions considered in the formulation of bi-criteria multiprocessor task scheduling are:

- Problem is based on deterministic model i.e. task dependencies, data communication time & execution time are known in advance.
- The dependencies along with execution time & communication cost are represented by a DAG.
- Two tasks scheduled on same processor have no communication cost & any two tasks scheduled on different processor have the communication cost specified by the edge weight in DAG.
- Architecture is based on network of homogeneous processors i.e. all processors have same execution time to run a task individually.
- Pre-emption of tasks is not allowed.
- All processors & tasks are available at time \( t = 0 \).

B. Objective Function

The proposed work considers the parameters optimization of genetic algorithm for the bi-criteria multiprocessor task scheduling problem. The parent & offspring in the genetic algorithm are evaluated by bi-criteria objective function i.e weighted sum of makespan & total completion time.

Makespan \( (F_1) \) of a schedule \( (S) \) is calculated as

\[
F_1 = C_{\text{max}}(S),
\]

where \( C_{\text{max}}(S) \) is the time at which the last task completes for a particular schedule \( S \) [13].

Total completion time \( (F_2) \) of a schedule is calculated as

\[
F_2 = \sum_{i=1}^{n} C_i
\]

where \( C_i \) is the completion time of \( i_{\text{th}} \) task of a schedule.

The Objective function is

\[
f = \text{Min} \left( \alpha F_1 + (1 - \alpha) F_2 \right)
\]

where \( F_1 \) is the makespan function, \( F_2 \) is the total completion time function and \( \alpha \) is the weighting coefficient in the range 0 and 1. When \( \alpha = 1 \), only the makespan objective is considered and when \( \alpha = 0 \), only the total completion time objective is considered. By varying the values of \( \alpha \), the trade-off between the makespan and total completion time can be determined over the range of values of \( \alpha \) [13].

III. GENETIC ALGORITHM

As genetic algorithm is considered to be an effective population based approach for solving NP hard problems like multiprocessor task scheduling. A genetic algorithm tries to mimic the natural evolution process and generally start with an initial population of chromosomes which can be either generated randomly or based on some other rules, heuristics and algorithms. Then in each generation the population goes through the processes of encoding, fitness evaluation, selection, crossover & mutation [14]. The basic detail of the algorithm is given below:

**Step 1 Encoding**

a) Encoding give the representation of a chromosome. In the present work, chromosome is represented as \((T, P)\) pair where \( T \) is task sequence \( t_1, t_2, \ldots, t_n \) & \( P \) is allocated processor sequence \( p_1, p_2, \ldots, p_n \).

b) Each task sequence is a permutation of task numbers & each processor sequence is a permutation of processor numbers \((1, 2, \ldots, m)\) with length equal to number of tasks.

**Step 2 Initialization**

Each task sequence is a permutation of task numbers, so each task will be processed according to its appearance. As dependency exists between tasks, each task in the task sequence should appear before all of its children and after all of its parents. Therefore, some permutations of the tasks may not be valid and some mechanism would be needed to validate the invalid sequences. The initial population in the present work is generated randomly by the following procedure:

Generate the valid task sequences \( (TS) \) of population size using the algorithm as stated by M. R. Bonyadi and M.E. Moghaddam [11]. Generate the processor sequences \( (PS) \) of population size randomly. Map each task sequence \( (T) \) from \( TS \) to randomly selected processor sequence \( (P) \) from \( PS \) giving each chromosome in the form \((T, P)\) i.e. task sequence followed by mapped processor sequence.

**Step 3 Reproduction**

Reproduction operators (crossover & mutation) are used for producing new offsprings. In the present work, different crossover & mutation operators are used for task sequence & processor sequence. There are three crossover (PMX, Order & Position based) & three mutation (swap, inversion & scramble) operators used for task sequences. Similarly for Processor sequence one point crossover & uniform mutation has been used. The task sequence operators are considered for analyzing the optimal crossover, mutation operator using design of experiments.

Different reproduction operators are used for task and processor sequences due to different nature. The sequences are firstly separated from a chromosome & then used individually. As generated task sequences after reproduction may not be valid in terms of dependency, so a mechanism is used for validating the task sequences as stated by M. R. Bonyadi and M.E. Moghaddam [11]. Then valid task sequences after reproduction \((TS')\) are mapped to processor sequences after reproduction \((PS')\) based on minimum fitness value.

To generate the new off springs the algorithm uses the following steps:

a) Scores each member of the current population by computing fitness (i.e. weighted sum of makespan and total completion time).
b) Selects parents based on the fitness function (i.e. Tournament and roulette wheel selection).

c) Some of the individuals in the current population that have best fitness are chosen as elite and these elite individuals are used in the next population.

d) Production of offsprings from the parents by crossover of the pair of parents or by making random changes to a single parent (mutation).

e) Replaces the current population with the children to form the next generation.

**Step 4: Stopping Criteria**

The algorithm stops when the maximum number of iterations reaches 100 with the stall generation limit 20.

The overall work is divided into two parts: the first part finds a best sequence of tasks along with best suited processor sequence using GA & in second part an attempt has been made to optimize the parameters of GA using Design of Experiments. Approximate 270 experiments have been done to optimize the parameters of GA using RSM method of design of experiments.

**IV. RESULTS & DISCUSSIONS**

The present work considers the optimization of different parameters of genetic algorithm for bi-criteria multiprocessor task scheduling problem. The different parameters of genetic algorithm like crossover, mutation etc. greatly determine the degree of solution accuracy and the convergence speed. A number of methods have been developed for improving the performance of GAs. One of the methods for improving the performance of Genetic Algorithms is the optimal parameters selection for the solution of the particular problem.

A Central Composite Design (CCD) of Response surface methodology (RSM) using Design Expert 6.0 for optimizing the different genetic algorithm parameters have been considered for the minimization of the bi-criteria multiprocessor task scheduling i.e. weighted sum of makespan and total completion time. The GA parameters with its range & levels are shown in Table 1.

The design matrix has been obtained by the combination of different variables and total of 270 experiments are required to be performed for half factorial which shows 72 factorial points, 108 axial points and 90 centre points. The genetic algorithm belongs to the category of approximate algorithms and run five times for taking final average. Hence, total of \(270 \times 5 = 1350\) results were obtained for the optimization of GA parameters for which the design summary is shown in Table II in which three factors (A, B & C) are numeric and the other three factors (D, E & F) are categorical.

**Table I**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameters (Factors)</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Population size (A)</td>
<td>20 – 80 (5 levels)</td>
</tr>
<tr>
<td>2.</td>
<td>Crossover Probability (B)</td>
<td>0.50 – 0.90 (5 levels)</td>
</tr>
<tr>
<td>3.</td>
<td>Weight Coefficient (C)</td>
<td>0.2 – 0.8 (5 levels)</td>
</tr>
<tr>
<td>4.</td>
<td>Crossover (D) Level 1 of D Level 3 of D Levels: 3</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Mutation (E) Level 1 of E Level 3 of E Levels: 3</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Selection (F) Level 1 (Tournament), Level 2 (Roulette Wheel)</td>
<td></td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Study Type: Response Surface</th>
<th>Initial Design: Central Composite</th>
<th>Design Model: Quadratic</th>
<th>Response Name</th>
<th>Observation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Model</th>
<th>Experiments: 270</th>
<th>Blocks: No Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1 Makespan</td>
<td>270</td>
<td>520.00</td>
<td>726.67</td>
<td>Quadratic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y2 Completion Time</td>
<td>270</td>
<td>5550.00</td>
<td>7260.00</td>
<td>Quadratic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**A. ANOVA for the Response Surface Quadratic Model**

The model for the makespan and completion time is quadratic in nature and hence the predicted model is good predictor of the optimum conditions. To check the significance of the models, F-test and probability test have been performed. F-ratio is defined as the ratio between groups mean square to within group mean square values. P-values are used to investigate the significance of each coefficient which also shows the interaction strength of each variable. A smaller value of p shows a higher significance of the corresponding coefficient. If P-value for proposed model is less than 0.05 then it is significant at 5% level of significance.

**Table III**

<table>
<thead>
<tr>
<th>Factors</th>
<th>Name</th>
<th>Type</th>
<th>Low Actual</th>
<th>High Actual</th>
<th>Low Coded</th>
<th>High Coded</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Population size</td>
<td>Numeric</td>
<td>20.00</td>
<td>80.00</td>
<td>-1.00</td>
<td>1.000</td>
</tr>
<tr>
<td>B</td>
<td>Crossover Probability</td>
<td>Numeric</td>
<td>50%</td>
<td>90%</td>
<td>-1.00</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>Weight Coefficient</td>
<td>Numeric</td>
<td>0.20</td>
<td>0.80</td>
<td>-1.00</td>
<td>1.000</td>
</tr>
<tr>
<td>D</td>
<td>Crossover</td>
<td>Categorical</td>
<td>Level 1 of D Level 3 of D</td>
<td>Levels: 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Mutation</td>
<td>Categorical</td>
<td>Level 1 of E Level 3 of E</td>
<td>Levels: 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Selection</td>
<td>Categorical</td>
<td>Level 1 of F Level 2 of F</td>
<td>Levels: 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The model F-value of 6.68 as shown in Table III implies the model terms are significant. There is only 0.01% chance that a "Model F-Value" & this large could occur due to noise. The term "Prob > F" in the table less than 0.05 indicates that the model terms are not significant.

B. Optimum GA Parameters Predicted by RSM

There are different techniques to find the optimum Genetic Algorithm parameters by RSM (i.e. numerical, graphical etc.) Optimum parameters are predicted by applying numerical optimization of Design expert 6.0 version using RSM as shown in Table VI. From the different experiments conducted by Genetic Algorithm in the MATLAB environment, the optimum values for minimizing the makespan and completion time simultaneously for multiprocessor task scheduling are:

- Population size: 75
- Crossover probability: 50%
- Weight coefficient: 0.2
The present work considers the parameter optimization of Genetic Algorithm for Bi-criteria Multiprocessor task scheduling problem with minimizing the weighted sum of makespan and total completion time. Genetic algorithm belongs to the category of approximate algorithms and useful for the solution of NP hard problems. The solution of genetic algorithm mainly depends on its different parameters like type of crossover, mutation, selection function; crossover probability etc. and every problem have specific GA parameters. The standard Gauss Elimination problem of 18 tasks & 4 processors along with variable communication cost for each task have been used for parameters optimization using Design Expert 6.0 software. A Central composite design of response surface model (RSM) which considers 5 levels of numeric factors is used. Total of 270 experiments have been performed in genetic algorithm by varying its different parameters. All the parameters except Selection and crossover have contributed significant effect on the quality of solution at 5% level of significance. From the different experiments conducted by proposed Genetic Algorithm in the MATLAB environment, the optimum values for minimizing the makespan and completion time simultaneously for the bi-criteria multiprocessor task scheduling are- population size: 75, Crossover probability: 50%, weight coefficient: 0.2, crossover: position based, mutation: swap and selection: tournament.

### References


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