Abstract—Historically, actuators’ redundancy was used to deal with faults occurring suddenly in flight systems. This technique was generally expensive, time consuming and involves increased weight and space in the system. Therefore, nowadays, the on-line fault diagnosis of actuators and accommodation plays a major role in the design of avionic systems. These approaches, known as Fault Tolerant Flight Control systems (FTFCs) are able to adapt to such sudden faults while keeping avionics systems lighter and less expensive. In this paper, a (FTFC) system based on the Geometric Approach and a Reconfigurable Flight Control (RFC) are presented. The Geometric approach is used for cosmic ray fault reconstruction, while Sliding Mode Control (SMC) based on Lyapunov stability theory is designed for the reconfiguration of the controller in order to compensate the fault effect. Matlab®/Simulink® simulations are performed to illustrate the effectiveness and robustness of the proposed flight control system against actuators’ faulty signal caused by cosmic rays. The results demonstrate the successful real-time implementation of the proposed FTFC system on a non-linear 6 DOF aircraft model.

Keywords—Actuators’ faults, Fault detection and diagnosis, Fault tolerant flight control, Sliding mode control, Geometric approach for fault reconstruction, Lyapunov stability.

I. INTRODUCTION

The safety of flight control systems is a key issue for the aerospace industry. The challenge of maintaining acceptable performances and preserving the aircraft stability when unexpected scenarios occur, requires different strategies rather than just having simple conventional controllers, designed only on the basis of the sensors’ actual measurements. Indeed, Fault Tolerant Flight Control Systems (FTFCs) are very important to increase the reliability of an aircraft when actuators’ fault occur and which may lead to a loss of control during a flight. These strategies allow a safe landing of the aircraft and help avoid serious accidents and disasters.

Generally, FTFC systems react instantly to the occurrence of actuator faults by using the faults’ parameters provided by a Fault Detection and Diagnosis (FDD) process. Then, the reconfiguration of the remaining healthy actuators is performed to compensate the faulty actuator effect on the aircraft behavior. The reconfiguration of the controller is usually necessary in the event of severe faults such as total actuator loss, considered as the critical components of the aircraft. A wide survey on FTFC and FDD systems can be found in [1]–[6].

A reliable FDD process is assumed to provide accurate information about the aircraft’s health status to avoid false alarms. This ensures robustness against external disturbances, model uncertainties and sensor noise measurements. Model based FDD processes can be classified into two major categories; residual generation based FDD and fault reconstruction based FDD [1]. In residual generation based FDD, a residual signal is formed by comparing the mathematical model outputs and the sensor measurements. In this way, in normal conditions, the residual signal is supposed to be close to zero. It will be nonzero when faults occur. In fault reconstruction based FDD, the process estimates and reconstructs the actuator deflection. This reconstruction can be used directly to correct the faulty actuator before it is used by the controller. Among the methods used for the design of such FDD processes is the geometric approach has been selected and will be considered later in this paper [7]–[9]. The fundamental characteristic of this approach is that it handles simultaneous faults more accurately. It consists of a decomposition of the aircraft state space into two planes: one is tangent to the faulty signal and the other is transverse. The input signals are then constructed using the minimum invertibility concept of systems.

Once the geometric based FDD process detects, locates and identifies the source of the fault, the fault parameter information is then used by a reconfiguration mechanism. This mechanism tries to adapt and to compensate the fault’s effect by using the remaining healthy actuator signals, therefore preserving the entire stability and maintaining acceptable performances. Just like FDD process, the reconfigurable controller needs to be robust against external uncertainties and disturbances. Among recent research on reconfigurable flight control used specifically for FTFC systems was found the Sliding Mode Control (SMC) [10]–[15]. The SMC controller design depends primarily on the design of a so called ‘sliding surface’. The trajectory of the states will be driven towards this surface. Once they have reached their destination, the states are forced to remain on it ensuring robustness in regards to uncertainties and to the stability of the system. This makes it a strong candidate for the design of the FTFC systems to handle actuator faults.

This paper is organized as follows. In Section II, the actuator fault models are defined and described. Then, the
origins of radiation faults and their emulation using Xilinx® and mathematical modelling are explained. Section III presents the geometric fault reconstruction based FDD formulation. Section IV presents the reconfigurable sliding mode control design. Section V briefly presents the integration between FDD and SMC. To demonstrate the performances of the proposed system, Matlab®/Simulink® numerical simulations are performed on the nonlinear 6 DOF aircraft model in Section VI. Section VII concludes the paper.

II. ACTUATOR FAULT MODELLING

According to Isermann’s definition of fault [1], an actuator fault corresponds to any abnormal system behavior. They may be small or hidden, so they can be hard to diagnose. In literature, several types of actuator faults are listed [1]–[3]. The actuator may be stuck and motionless, it may move freely without providing any moment to the aircraft or it may lose some effectiveness or totally hard over. When a fault occurs in the actuator, the first thing that should be done is to diagnose the kind of fault, and then decide how to deal with it. It must be detected, isolated and identified. The fault detection consists of the system health monitoring and the determination of the time of fault occurrence. The fault isolation determines the kind and location of such fault. The fault identification determines the form and the time varying of the fault.

In last decade, new types of faults affecting aircrafts were a topic of interest. The neutrons generated by cosmic rays could cause Single-Event Upsets (SEUs) in avionic systems at high flight altitudes [16]. Indeed, because of the high technology of used to fabricate integrated circuits, semiconductor-based components are being increasingly sensitive to cosmic rays events and become the target of many such faults. These types of faults can be emulated on a Field-Programmable Gate Array (FPGA) device using the soft error mitigation (SEM) IP core provided by Xilinx® [17]. In the remainder of the paper, one type of cosmic ray fault models previously published in [17], is used. It is illustrated in Fig. 1.

![Fig. 1 Cosmic rays fault: Noisy oscillations around zero between 4s and 6s](image)

Fig. 1 Cosmic rays fault: Noisy oscillations around zero between 4s and 6s

Equation (1) defines the faulty control signal \( u_f(t) \) of the \( k^{th} \) actuator affected by a faulty input signal \( v(t) \).

\[
u_f(t) = F u(t) + (I_{m \times m} - F)v(t)
\]

(1)

Where \( v_{j}, \ldots, v_{i}, \ldots, v_{m} \) are the faulty actuators signals, and \( u_{1}, \ldots, u_{i}, \ldots, u_{m} \) are the controller outputs.

III. GEOMETRIC FAULT RECONSTRUCTION BASED FDD

In this section, a non-linear dynamic system for a 6 DOF aircraft model is considered. Equation (4) presents the state space of the non-linear dynamic system.

\[
\begin{align*}
\dot{x}(t) &= f(x, t) + b(x)u(t) \\
y(t) &= h(x)
\end{align*}
\]

(4)

where \( f, b \) and \( h \) are respectively the system, input and output functions. \( x(t), y(t) \) and \( u(t) \) represent respectively the state vector variables, the output vector variables and the control input variables. The main objective of this approach is to design a geometric projector \( \Pi(x) \) as below [9]:

\[
\Pi(x) = I_n - L A_{proj}^{-1} \nabla \sigma^T
\]

(5)

where details on \( L, A_{proj} \) and \( \nabla \sigma^T \) matrices can be found in Appendix A. \( I_n \) is the identity matrix and \( n \) is the state vector size. Using (5), the dynamic vector \( f(x) \) can be decomposed into tangent and transverse parts along a so called sub-manifold \( S \) as below [9]:

\[
f(x) = \Pi(x)f(x) + (I_n - \Pi(x))f(x), \quad \forall x \in S
\]

(6)

The terms \( \Pi(x)f(x) \) and \( (I_n - \Pi(x))f(x) \) represent respectively the tangent part and the transverse part. Then, the projector \( \Pi(x) \) is used to reconstruct the faulty inputs by using the minimum invertibility system concept illustrated by the followed [9]:

\[
\begin{align*}
\dot{\hat{x}}(t) &= \Pi(x)f(x) + L A_{proj}^{-1} \nabla \sigma^T f(x) \\
\dot{\nu}(t) &= A_{proj}^{-1}(\dot{\hat{x}}(t) - \nabla \sigma^T f(x))
\end{align*}
\]

(7)

Fig. 3 illustrates the general concept of the geometric fault reconstruction based FDD. Substituting (6) and (7) in (4), the overall system takes the form below [9]:

\[
\begin{align*}
\dot{\hat{x}}(t) &= \Pi(x)f(x) + L A_{proj}^{-1} \nabla \sigma^T f(x) + L \nu(t)
\end{align*}
\]

(8)
IV. RECONFIGURABLE SLIDING MODE CONTROL DESIGN

The SMC design process starts defining a so called sliding surface $S(t)$. Then, a first control law is designed to drive the trajectory of the states towards this surface. Once the surface is reached, a second control law is then designed to force the trajectory of the states towards this surface. Once the surface is designed, the stability of the sliding control law, assuming that $\dot{S}(t) = 0$, is ensured by using Lyapunov approach. Once the sliding surface is designed, the stability of the sliding control law, assuming that $\dot{S}(t) = 0$, is ensured by using Lyapunov approach. Using (1), (4) and (10) and by choosing $\mu(t)$ as below [10]:

$$s(x,t) = \left( \frac{d}{dt} + \lambda \right) \tilde{x}(t) = 0, \lambda > 0 \quad (9)$$

where $\tilde{x}(t) = x(t) - x_d(t)$ is the state error and $x_d(t)$ is the desired state. Using the sliding surface design, the stability of the sliding control law is ensured by using Lyapunov approach. Using (1), (4) and (10) and by choosing $\mu(t) = \mu_d(t)$ to be the reconfigurable flight control law, $U(t)$ will take the form:

$$U(t) = b(x)^{-1}[(\dot{x}_d(t) - \lambda \tilde{x}(t) - f(x)) - b(x)(I - F)v(t)] - b(x)^{-1}(k \text{sign}(s)) \quad (11)$$

Finally, the overall scheme of the controller designed using the geometric fault reconstruction based FDD process and the reconfigurable SMC is shown in Fig. 4.

VI. CASE STUDY

In this section, the approach presented below is applied on a general 6 DOF nonlinear aircraft model, using Matlab®/Simulink® simulations. First, the fault reconstruction process is designed to compensate the effect of the occurred fault on the aircraft behavior. The flight dynamic equations of a general nonlinear aircraft model can be rewritten as in (4). Where $x(t)$, $u(t)$ and $y(t)$ are defined in Table I illustrated below. $f(x)$, $g(x)$ and $h(x)$ are deduced using the aerodynamic equations of forces and moments [9]. Numerical aerodynamics parameters used in this paper can be found in [9].

TABLE I

<table>
<thead>
<tr>
<th>State vector $x(t)$</th>
<th>Control surface $u(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ Longitudinal velocity, m/s</td>
<td>$\delta_\alpha$ Aileron, rad</td>
</tr>
<tr>
<td>$\alpha$ Angle of attack, rad</td>
<td>$\delta_e$ Elevator, rad</td>
</tr>
<tr>
<td>$\beta$ Side slip angle, rad</td>
<td>$\delta_r$ Rudder, rad</td>
</tr>
<tr>
<td>$p$ Roll rate, rad/s</td>
<td>$q$ Pitch rate, rad/s</td>
</tr>
<tr>
<td>$q$ Pitch rate, rad/s</td>
<td>$r$ Yaw rate, rad/s</td>
</tr>
<tr>
<td>$\phi$ Roll angle, rad</td>
<td>$\theta$ Pitch angle, rad</td>
</tr>
<tr>
<td>$\theta$ Pitch angle, rad</td>
<td>$\psi$ Yaw angle, rad</td>
</tr>
<tr>
<td>$y(t) = [V \ a \ \beta \ p \ q \ r]^T$</td>
<td></td>
</tr>
</tbody>
</table>

The fault model used in the simulation is one of the cosmic rays faults models cited in [17]. It is modelled as oscillations around zero. For realistic situations, the fault model is corrupted by a zero-mean white Gaussian noise. The simulation runs for over 10s, and the rudder surface fails between $t=4s$ and $t=6s$, then it becomes healthy until the end of the simulation. The sensors’ measurements are corrupted with zero-mean white Gaussian noise with an error covariance matrix $0.012 \times I$ [rad$^2$]. This specification corresponds to low-cost sensors.

Fig. 5 illustrates the geometric fault reconstruction based FDD processes detection and isolation results. A value of 1, means that the actuator is in the healthy state. While a value of zero means that the actuator fails. Notice that the FDD react instantly to the occurrence of the fault. Indeed, the rudder fails at $t=4s$, after less than 0.1s the health status monitor changes from 1 to 0, indicating that a fault occurs at this time on the rudder. In the other hand, the FDD process takes also 0.1s to detect that the fault disappeared.
Fig. 5 FDD Detection and Isolation for the rudder fault

Fig. 6 illustrates the SMC outputs in both healthy and faulty states, and those reconstructed using the geometric fault reconstruction based FDD process. The reconstructed control output of the rudder is close to the actuator deflection in both states: faulty and healthy. Fig. 7 illustrates the geometric fault reconstruction error, and shows better the fault reconstruction performance. One can note that even the remaining two actuators are still healthy, their geometric reconstruction is perfect. Notice that the SMC outputs don’t suffer from the chattering problem and they do not exceed actuator mechanical and rates’ limits.

Fig. 8 illustrates the aircraft’s attitude angles $\phi$, $\theta$, and $\psi$. The roll angle $\phi$ has not been affected by the fault. That is because the rudder does not act on the roll angle directly. The pitch and yaw angles $\theta$ and $\psi$ show a minimal degradation when the fault occurs, but the whole stability is still preserved.

Fig. 9 illustrates the aircraft’s omega rates. Unlike the roll and yaw rates $p$ and $r$, which not present any trace of the effect of the fault, the pitch $q$ is affected by the fault and suffers from some degradation. The reason is that the reconfigurable control requests the elevator actuator, which
acts directly on $q$, to compensate the rudder fault and to minimize its effect. Notice that due to the designed FTFC system, the fault impact still remains so minimal, that the aircraft’s stability is preserved and the performances are maintained close to those desired by the pilot.

Fig. 10 illustrates the aircraft’s wind parameters: the forward velocity $V(t)$, the angle of attack $\alpha$ and the sideslip angle $\beta$. One can easily see that the parameters have not been affected by the rudder fault. This can be explained by the facts that requesting the remaining healthy actuators has compensated the rudder fault effect without affecting the acceptable performances. Here again, the FTFC system designed, was able to compensate the effect of the fault and has preserved aircraft’s stability.

Fig. 11 illustrates the state of the aircraft northeast path trajectory, in both cases: when the controller used is based on a conventional technique, and when it is used with the reconfiguration technique proposed in this paper. The rudder fault degrades more the tracking error than in the second case. Here again, actuators’ redundancy plays a major role in the fault compensation, and helps to minimize more the tracking error and to get a better performances. The figure shows also that compensation gives better results than in the case without reconfiguration where the aircraft stability and performance are totally lost. One can see a minimal degradation in the tracking error. This can be avoided or at least attenuated by adding redundant actuators. This will provide aerodynamic redundancy to the existing three actuators and the compensation effect will be more accurate at this time.

The fault detection and reconstruction based FDD system designed shows good performance also. It can reconstruct accurately the controller outputs. Even if the actuator faults are time varying and corrupted, the FDD process error is maintained closed to zero.

The FDD results allow the SMC controllers to have an accurate idea on the faults’ parameters. Then, it reconfigures the remaining actuators to compensate the effect of the fault on the aircraft behavior, safe navigation and landing.

The two processes have a complementary role in the success of the whole FTFC system. Each one is important for the other. By exchanging accurate data at specific times, they ensure a reliable solution for actuators’ faults in avionics, even in the presence types of faults including those caused by radiation in high flight altitudes.

APPENDIX A: THE GEOMETRIC APPROACH FORMULATION

Consider anon-linear dynamic system described by the following state space equations:

$$
\begin{align*}
\dot{x} &= f(x) + b(x)u, & b(x) &= \sum_{i=1}^{p} b_i u_i \\
y(t) &= h_i(x), & 1 \leq i \leq p
\end{align*}
$$

(A1)

In this case $L = [b_1 \ldots b_m]$, details on $A_{proj}$ and $\nabla \sigma^T$ matrices are defined as below [9]:

$$
A_{proj} = \begin{bmatrix}
l_{x_1}^{-1} h_1(x) & \ldots & b_n l_{x_1}^{-1} h_1(x) \\
n_{x_2}^{-1} h_2(x) & \ldots & b_m n_{x_2}^{-1} h_2(x) \\
\vdots & \ddots & \vdots \\
l_{x_p}^{-1} h_p(x) & \ldots & b_m l_{x_p}^{-1} h_p(x)
\end{bmatrix}
$$

(A2)

$$
\nabla \sigma^T = \begin{bmatrix}
\frac{\partial}{\partial x_1} l_{x_1}^{-1} h_1(x) & \ldots & \frac{\partial}{\partial x_1} l_{x_1}^{-1} h_1(x) \\
\frac{\partial}{\partial x_2} n_{x_2}^{-1} h_2(x) & \ldots & \frac{\partial}{\partial x_2} n_{x_2}^{-1} h_2(x) \\
\vdots & \ddots & \vdots \\
\frac{\partial}{\partial x_p} l_{x_p}^{-1} h_p(x) & \ldots & \frac{\partial}{\partial x_p} l_{x_p}^{-1} h_p(x)
\end{bmatrix}
$$

(A3)

The term $dl_{x_i}^{-1} h_i(b_j)$ is called the $(r_i - 1)^{th}$ Lie Derivative of $h_i(x)$ in the direction of the vector field $\dot{x}$. $[r_i]$ is a set of numbers called relative degrees, such that for $1 \leq i \leq
p (p number of outputs):

\[
\begin{align*}
\{dL_\xi^{-1} h_i(b_j)\} &\neq 0 \\
\{dL_\xi^{-1} h_i(b_k)\} &= 0, \forall s \neq j \text{ and } 1 \leq k \leq r_i
\end{align*}
\]

(A4)

\[\Pi(x)\] must satisfy three characteristics:
- \[^2 \Pi(x)\]
- \[^3 \Pi(x)\]
- \[^7 \Pi(x)\]

For more details see [7]–[9].

APPENDIX B: THE KINEMATIC EQUATIONS

\[
\begin{align*}
\psi &= p + (q \sin \phi + r \cos \phi) \tan \theta \\
\theta &= q \cos \phi - r \sin \phi \\
\psi &= \frac{q \sin \phi + r \cos \phi}{\cos \theta}
\end{align*}
\]

(B1)

APPENDIX C: THE NAVIGATION EQUATIONS

\[
\begin{align*}
\dot{p}_n &= u_x \cos \psi \cos \theta + u_y (\cos \theta \sin \psi + \sin \theta \cos \psi) + u_z (\sin \theta \sin \phi) \\
\dot{p}_x &= u_x \cos \psi \cos \theta + u_y (\cos \theta \sin \phi + \sin \theta \cos \phi) + u_z (\sin \theta \sin \phi) \\
\dot{h} &= -u_x \sin \theta + u_y \sin \theta \cos \phi + u_z \cos \phi \cos \theta
\end{align*}
\]

\[(C1)\]

where:

\[
\begin{align*}
u_x &= u \cos \phi \\
u_y &= u \sin \theta \\
u_z &= u \sin \phi \cos \theta
\end{align*}
\]

(C2)

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