The Long Run Relationship between Exports and Imports in South Africa: Evidence from Cointegration Analysis

Sagaren Pillay

Abstract—This study empirically examines the long run equilibrium relationship between South Africa’s exports and imports using quarterly data from 1985 to 2012. The theoretical framework used for the study is based on Johansen’s Maximum Likelihood cointegration technique which tests for both the existence and number of cointegration vectors that exists. The study finds that both the series are integrated of order one and are cointegrated. A statistically significant cointegrating relationship is found to exist between exports and imports. The study models this unique linear and lagged relationship using a Vector Error Correction Model (VECM). The findings of the study confirm the existence of a long run equilibrium relationship between exports and imports.

Keywords—Cointegration lagged, linear, maximum likelihood, vector error correction model.

I. INTRODUCTION

The cointegration relationship between imports and exports has been researched extensively over the past decade. The existence of a cointegration relationship between imports and exports may imply that the trade deficits of a country are short-term and sustainable in the long run. This long run equilibrium may further imply effective macroeconomic policy.

Exports and imports play an important role in every country. Monitoring the current account is very important especially when monitoring the performance of the economy. Several studies were conducted to determine the relationship between imports and exports. Researchers [7] analysed the relationship between exports and imports in Pakistan using cointegration and vector error correction modelling techniques. Their findings were that real imports positively influence real exports, that imports and exports are cointegrated, confirming the existence of a long run equilibrium relationship between exports and imports. In another study [8] researched the long-run relationship between exports and imports in transition European countries using the cointegration method, their findings detected one cointegration vector in ten out of the sixteen transition countries namely Bulgaria, Armenia, Russia, Czech Republic, Slovakia, Lithuania, Croatia, Slovenia, Poland and Romania.

Evidence of cointegration between exports and imports was also shown to exist in Chile for the period 1973 to 1998 [1]. The objective of this paper is to explore the link between import expenditure and export earnings in South Africa by using quarterly data from 1985 to 2012. The theoretical framework for the study is based on Johansen’s cointegration approach and vector error correction modelling.

II. THEORETICAL BACKGROUND

A simple framework for a long-run relationship between exports and imports is given below [6]. In this relationship the individual current-period budget constraint is given by

\[ C_0 = Y_0 + B_0 - I_0 - (1 + r) B_{-1}, \] (1)

where \( C_0 \) is current consumption; \( Y_0 \) is output, \( I_0 \) is investment, \( r \) is the one-period world interest rate, \( B_0 \) is the international borrowing, and \( (1 + r_0)B_{-1} \) is the historically given initial debt. An empirically testable model (based on several assumptions) was then developed from (1):

\[ X_t = \alpha + \beta M_t + \epsilon_t \] (2)

Reference [1] tested (2) as:

\[ M_t = a + bX_t + \epsilon_t \] (3)

where \( M_t \) is imports of goods and services and \( X_t \) is exports of goods and services. The intertemporal budget constraint is stable when cointegration exists between imports and exports.

III. DATA

For this study, import expenditure and export income data was obtained from the South African Reserve Bank. The imports \( M_t \) and exports \( X_t \) are evaluated in local currency (Rand) at current prices and expressed in natural logarithms.

IV. METHODOLOGY AND RESULTS

A. Unit Root Test

The first step in the time series analysis was to determine whether the two series are stationary or non-stationary in nature. If the time series are I (1), they have to be characterized by the presence of a unit root and their first difference by the absence of unit roots [5].

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The Augmented Dickey-Fuller (ADF) [2] unit root test was used to determine whether the series is stationary or non-stationary. The ADF test constructs a model with higher order lag terms and tests the significance of the parameter estimates using a non-standard $t$-test. The model used for this test is

$$
\Delta x_t = \alpha_1 x_{t-1} + \beta_1 \Delta x_{t-1} + \ldots + \beta_{p-1} \Delta x_{t-p+1} + \epsilon_t
$$

where the $t$-test checks significance of the $\alpha_1$ term. If $\alpha_1 = 0$ the series has a unit root. The ADF test is more efficient than the Dickey-Fuller test since it takes into account the correlation between the error terms by adjusting the differenced terms of the dependent variables.

The Augmented Dickey-Fuller tests for non-stationarity of each of the series is shown below (Table I). The null hypothesis is to test a unit root. In the Dickey-Fuller tests, the second column specifies three types of models, which are zero mean, single mean, or trend. The third column (Rho) and the fifth column (Tau) are the test statistics for unit root testing. Other columns are the $p$-values. Consequently, both series have a unit root and their first differences do not have any. Thus, the variables $M_t$ and $X_t$ are first order difference stationary and are integrated, I (1). Thus first differencing of the series yields a stationary series with finite variance.

Having established that both series are integrated of order one, the next section employs Johansen’s Maximum Likelihood cointegration technique which tests for both the existence and number of cointegration vectors that exists between the series for imports and exports.

### TABLE I

<table>
<thead>
<tr>
<th>Variable Type</th>
<th>Rho</th>
<th>Pr&lt; Rho</th>
<th>Tau</th>
<th>Pr&lt; Tau</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports($\chi$) Zero Mean</td>
<td>0.29</td>
<td>0.7506</td>
<td>4.55</td>
<td>0.9999</td>
</tr>
<tr>
<td>Single Mean</td>
<td>-0.52</td>
<td>0.9243</td>
<td>-0.78</td>
<td>0.8204</td>
</tr>
<tr>
<td>Trend</td>
<td>-20.76</td>
<td>0.0481</td>
<td>-3.06</td>
<td>0.1211</td>
</tr>
<tr>
<td>Imports ($M$) Zero Mean</td>
<td>0.33</td>
<td>0.7608</td>
<td>3.82</td>
<td>0.9999</td>
</tr>
<tr>
<td>Single Mean</td>
<td>-0.59</td>
<td>0.9188</td>
<td>-0.75</td>
<td>0.8282</td>
</tr>
<tr>
<td>Trend</td>
<td>-29.52</td>
<td>0.0059</td>
<td>-3.70</td>
<td>0.0266</td>
</tr>
</tbody>
</table>

**B. Cointegration Test**

Reference [4] developed the theory that there exists the special case where linear combinations of nonstationary processes are stationary. They defined this linear combination of nonstationary processes as cointegration and used the notation CI $(d, b)$, where $d$ represents the order of integration of the nonstationary processes and $b$ represents the number of stationary linear combinations between the nonstationary processes. Consider the two I (1) processes, $M_t$ and $X_t$, if there exists a linear combination of the two processes such that the linear combination is I (0), the two I (1) processes are considered to be CI (1,1).

Based on the graph (Fig. 1), it would seem that a linear trend term should be included in the model, thus the cointegration rank test without restriction on the intercept would be appropriate. The time series does not run approximately parallel and has a drift; hence cointegrating restrictions on the intercept parameters are not appropriate. The SAS procedure PROC VARMAX with the NOINT option was used to test for cointegration and model fitting. The
NOINT option specifies that there is no constant in the error correction mechanism but there is a constant included in the long–term relationship. The Minimum Information Criterion was used to inform the selection of an autoregressive order of \( p = 10 \). The results of the cointegration tests are shown below (Table II).

**TABLE II**

<table>
<thead>
<tr>
<th>Rank=r</th>
<th>Rank&gt;r</th>
<th>Eigenvalue</th>
<th>Trace</th>
<th>5% Critical Value</th>
<th>Drift in ECM</th>
<th>Drift in Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.09777</td>
<td>13.2750</td>
<td>12.21</td>
<td>NOINT</td>
<td>Constant</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.0270</td>
<td>2.7930</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the cointegration rank test, the last two columns explain the drift in the model. Since the NOINT option was specified, the model is given by the VECM \((p)\) form; \( p = 10 \):

\[
\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{9} \Phi_i \Delta x_{t-i} + \epsilon_t
\]

where:
- \( x_t \) is a \( k \times 1 \) random vector
- the sequence \( x_t \) is a \( \text{Var}(p) \) process
- \( x_t \sim \text{CI}(1) \)
- \( \Pi = \alpha \beta^t \) where \( \alpha \) is the adjustment coefficient and \( \beta \) the cointegrating vector
- \( \Phi_i \) are fixed coefficient matrices
- \( \epsilon_t \) is a \( k \times 1 \) white noise process

The Johansen and Julius \( \lambda_{\text{trace}} \) cointegration statistic test for testing the null hypothesis that there are at most \( r \) cointegrated vectors is used versus the alternative Hypothesis of more than \( r \) cointegrated vectors. Where: \( \lambda_{\text{trace}} \) is given by:

\[
\lambda_{\text{trace}} = -T \sum_{\lambda_i > 1}^k \log(1 - \lambda_i)
\]

and \( T \) is the available number of observations and \( \lambda_i \) the eigenvalues. The critical values at 5% significance level are used for testing.

The column Drift in ECM means there is no separate drift in the error correction model, and the column Drift in Process means the process has a constant drift before differencing.

There is one cointegrating process (Table II) since the Trace statistic for testing \( r = 0 \) against \( r > 0 \) is greater than the critical value (13.27> 12.21), but the Trace statistic for testing \( r = 1 \) against \( r > 1 \) is smaller than the critical value (2.79< 4.14). Thus, Johansen’s test indicates a single \((r=1)\) cointegrating vector.

**TABLE III**

<table>
<thead>
<tr>
<th>Long-Run Parameter Beta</th>
<th>Adjustment Coefficient Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates When RANK=1</td>
<td>Estimates When RANK=1</td>
</tr>
<tr>
<td>Variable</td>
<td>1</td>
</tr>
<tr>
<td>Exports ((X))</td>
<td>1.04177</td>
</tr>
<tr>
<td>Imports ((M))</td>
<td>-0.94142</td>
</tr>
</tbody>
</table>

The estimates of the long–run parameter \( \beta \), and the adjustment coefficient, \( \alpha \), are given in the table above. Since the cointegration rank is 1 in the bivariate system, \( \alpha \) and \( \beta \) are two dimensional vectors. The estimated cointegrating vector is \( \beta = [1 -0.94142] \). The first element of \( \beta^t \) is 1 since exports \((X)\) is specified as the normalised variable. The impact matrix is:

\[
\Pi = \alpha \beta^t
\]

becomes

\[
\begin{bmatrix}
0.0412 & 0.04784 \\
0.0479 & 0.0451
\end{bmatrix}
\]

The long run relationship of the series is

\[
\beta^t X = [1- 0.94142] X_t - 0.94142 M_t
\]

\[
X_t = 0.94142 M_t
\]

The VECM \((10)\) model can be written in the following 10th order vector autoregressive model:

\[
Y_t=
\begin{bmatrix}
0.715 & 0.463 \\
0.350 & 0.954
\end{bmatrix}
Y_{t-1} + 
\begin{bmatrix}
0.027 & -0.342 \\
-0.361 & -0.168
\end{bmatrix}
Y_{t-2} + 
\begin{bmatrix}
0.281 & -0.355 \\
0.581 & 0.082
\end{bmatrix}
Y_{t-6} + 
\begin{bmatrix}
0.160 & -0.085 \\
-0.085 & 0.171
\end{bmatrix}
Y_{t-4} + 
\begin{bmatrix}
0.133 & -0.149 \\
0.199 & 0.581
\end{bmatrix}
Y_{t-8} + 
\begin{bmatrix}
0.138 & 0.022 \\
-0.156 & 0.551
\end{bmatrix}
Y_{t-5} + 
\begin{bmatrix}
0.172 & 0.423 \\
0.082 & 0.423
\end{bmatrix}
Y_{t-6} + 
\begin{bmatrix}
0.001 & 0.122 \\
0.051 & 0.161
\end{bmatrix}
Y_{t-9} + 
\begin{bmatrix}
0.078 & 0.095 \\
0.551 & 0.423
\end{bmatrix}
Y_{t-10} + 
\begin{bmatrix}
0.001 & -0.452 \\
0.122 & 0.095
\end{bmatrix}
Y_{t-10} + 
\begin{bmatrix}
0.026 & 0.078 \\
-0.156 & 0.551
\end{bmatrix}
Y_{t-10}
\]

V. MODEL DIAGNOSTICS

Checking the assumptions of the model, (i.e., checking the white-noise requirement of the residuals, and so on), is not only crucial for correct statistical inference, but also for the economic interpretation of the model as a description of the behaviour of rational agents [5]. Various tests such as tests for autocorrelation in the squares are able to detect model failures [3].

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of \( \Delta X \) resulted in a model F test 3.15 and R-square of 0.4221. Similarly the regression of \( \Delta M \) resulted in a model F test of 4.30 and R-square of 0.4991 (Table IV).
The residuals are checked for normality and autoregressive conditional heteroskedasticity or ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are homoscedastic. The results also show that there are no ARCH effects on the residuals since the “no ARCH” hypothesis cannot be rejected given the F values (Table V).

There are no AR effects on the residuals - for both residual series the autoregressive model fit to the residuals up to 4 lags show no significance indicating that the residuals are uncorrelated (Table VI).

### VI. CONCLUSION

The main objective of this study was to investigate the long run relationship between exports and imports in South Africa. To this end cointegration techniques and vector error correction modelling was employed using quarterly data form 1985 to 2012. After establishing the non stationarity and order of integration of each series, Johansen’s cointegration techniques were applied to investigate the long run relationship between exports and imports. The results indicate the existence of one cointegrating vector amongst exports and imports. The value of the coefficient of current imports is 0.94, which is close to unity. This may indicate that the trade deficit is sustainable in the long run. These findings could be explored to determine if any policy interventions are necessary within the context of imports and exports.

### REFERENCES


