Abstract—This research investigates the distribution of food demand for animal food and the optimum amount of that food production at minimum cost. The data consist of customer purchase orders for the food of laying hens, price of food for laying hens, cost per unit for the food inventory, cost related to food of laying hens in which the food is out of stock, such as fine, overtime, urgent purchase for material. They were collected from January, 1990 to December, 2013 from a factory in Nakhonratchasima province. The collected data are analyzed in order to explore the distribution of the monthly food demand for the laying hens and to see the rate of inventory per unit. The results are used in a stochastic linear programming model for aggregate planning in which the optimum production or minimum cost could be obtained. Programming algorithms in MATLAB and tools in Linprog software are used to get the solution. The distribution of the food demand for laying hens and the random numbers are used in the model. The study shows that the distribution of monthly food demand for laying has a normal distribution, the monthly average amount (unit: 30 kg) of production from January to December. The minimum total cost average for 12 months is Baht 62,329,181.77. Therefore, the production planning can reduce the cost by 14.64% from real cost.

Keywords—Animal food, Stochastic linear programming, Production planning, Demand Uncertainty.

I. INTRODUCTION

THAILAND has large industries producing animal food. Agricultural products and related waste products are used in local animal food. The animal food factories operate from small to large and large sizes. However, the animal food demand fluctuates showing in Fig. 1, the production index in 2012 and 2013 [1]. Thus proper production planning is crucial for the manufactures and stakeholders. Aggregate planning is the process of development, analysis, and scheduling. It consists of sales forecasting, production level, and inventory level. An appropriate production planning brings about accurate demand forecast which can reduce the production cost.

Fig. 1 Monthly production value index of animal food in 2012-2013 showing fluctuation

For proper aggregate production planning, the producers need to know the true demand, so that the monthly amount of products may not exceed demand or may not fall behind the demand. If the amount of the product falls behind the demand, there will be some extra cost, for example, overtime expense, cost of raw material for urgent order, or penalty for late shipment. If the amount of the products exceed the demand, there will be some extra holding cost.

Reference [2] propose a forward algorithm for a solution to the dynamic version of the economic lot size model: allowing the possibility of demands for a single item, inventory holding charges, an id setup costs to vary over N periods, a minimum total cost inventory management scheme which satisfies known demand in every period was desired. Disjoint planning horizons were shown to be possible which eliminate the necessity of having data for the full N periods. Reference [3] present that if the demand and inventory cost are concave functions, the dynamic programming can be used for aggregate product planning. Reference [4] introduce an alternative to the backorder algorithm of [2]. When demand changes according to time, and is uncertain in each period of time, [5] propose the dynamic programming in which the assumptions are that there are no production capacity and no backorder to solve the problem of aggregate production planning. Using dynamic programming and stochastic integer programming, [6] study inventory system that the demand is discrete uncertain, and has a probability distribution. Reference [7] study aggregate production planning in which the continuous and uncertain demand changes over time and has a uniform distribution. Reference [8] modify the work of
[7], using a normal distribution instead of the uniform distribution.

The study of production planning motivated us to study the nature of planning in an animal food factory in Thailand which faces a problem of monthly uncertain demand in order to obtain a realistic or effective production planning. This research adopts the model proposed by [8], [9] by assuming that the demand changes over time, is continuous and uncertain, and has a normal distribution. The model is applied to an animal food factory in Nakhonratchasima province of Thailand.

This paper is organized as follows. Section II briefly describes the methodology and the application. In Section III, the result of the study is presented. Lastly, in section IV and V the discussion and conclusion are drawn.

II. METHODOLOGY AND APPLICATION

Data: The monthly data are collected from January, 1990 to December, 2013. They consist of the order amount, production cost, holding cost, and backorder cost which is penalty and overtime cost.

Models: The Kolmogorov-Smirnov goodness of fit test is employed to test if the distribution of the order amount is normal distribution. A stochastic method for dynamic linear programming problems with monthly uncertainty demand consists of an objective function and the constrains on that function. The objective function to be minimized is the expectation of the sum of production costs, holding costs, and backorder costs. The main constrain is that the sum of the backorder level at 1 step before the current period, the production level at the current period, and the backorder at the current period deducted by the holding level at 1 step before the current period and the holding level at the current period is equal to the current demand. The decision variables are the production and inventory levels and the backorder and holding levels, which are non-restricted variables at each period. The inventory level is the difference of the backordering and holding levels. The stochastic method for dynamic linear programming problems can be expressed in the mathematical form as follows. Objective function is defined as (1). The objective function is estimated by the discrete function as follows defined as:

\[ \text{Min} \sum_{t=1}^{T} c_t P_t + \sum_{t=1}^{T} g_t(k) I^-_t + h_t(k) I^+_t, \forall t, \forall k \]  

(1)

Constrains defined as (2)-(3):

\[ I^-_{t-1,k} - I^-_{t,k} + P_t + I^+_t - I^+_t = D_{t,k}, \forall t, \forall k \]  

(2)

\[ I^+_t, I^-_t \geq 0, \forall t, \forall k \]

\[ P_t \geq \min(D_{t,k}), \forall t, \forall k \]

\[ D_t \sim N(\mu, \sigma) \]  

(3)

Assumptions:

1. The order amount or demand is a continuous and uncertain variable whose monthly value is in the known interval.
2. The distribution of the demand in each period has a normal distribution.
3. The demand arises at the beginning of the period and can be different.
4. The amount of products produced at the beginning of period and all will be delivered.
5. At the beginning of period 1, there are no products in stock and at the end of the process there could be either products sold out or left.
6. Backorder is allowed.

Decision variables and Parameters

For period \( t, t = 1, 2, 3, \ldots, 12 \), and alternative \( k, k = 1, 2, 3, \ldots \),

Decision variables:

\[ P_t = \text{production level in period } t \]

\[ I^+_t = \text{amount of holdings at the beginning of period } t \]

\[ I^-_t = \text{amount of backorders at the beginning of period } t \]

Parameters:

\[ D_{t,k} = \text{demand in period } t \text{ under the alternative } k \]

\[ c_t = \text{production cost per unit in period } t \]

\[ g_t = \text{backorder cost per unit in period } t \]

\[ h_t = \text{holding cost per unit in period } t \]

\[ \text{Prob}_k = \text{probability that } D_{t,k} \text{ will occur} \]

\[ g_{t,k} = \text{average backorder cost per unit in period } t \text{ under the alternative } k \]

\[ g_{t,k} = \text{average backorder cost per unit in period } t \text{ under the alternative } k \]
\[ h_{t,k} = \text{Prob}_t \times h = \text{average holding cost per unit in period } t \text{ under the alternative } k \]

\[ T = \text{number of periods} \]
\[ K = \text{number of alternatives} \]

Parameter Estimation: We calculate the mean and variance of the monthly demand data, production cost, backorder cost, and holding cost. Then generate the monthly demand data, 10,000 random numbers for each month, from a normal distribution whose mean and variance equal to the calculated values of mean and variance. The total number of alternatives are equal to 10,000 raised to the power of 12. We choose a generated number randomly from each month. A random data set contains 12 alternatives. The number of random data sets used in the model are 2, 12, 22, 32, 42, ..., 10,000. The stochastic method for a dynamic linear programming is employed to those data. Programming optimization via Linprog in MATLAB are performed to find the best solution.

### III. RESULT

The calculated mean and variance of the monthly demand data, production cost, backorder cost, and holding cost are shown in Table I. The results of the demand data examination using Kolmogorov-Smirnov statistic via a statistical software show that the monthly demand has a normal distribution. Fig. 2 shows the minimum cost convergence after using 40,000 alternatives. We use 100 sets of random 50,000 alternatives for data analysis. The proper production planning, the best solution, is shown in Table II. The average minimum cost is 62,329,181.77 Baht with 171,124.13 standard deviation. The real cost is 71,455,666.72 Baht; therefore the production planning can reduce the cost by 14.64%.

### IV. DISCUSSION

Aggregate production planning is concerned with the determination of production, inventory, and work force levels to meet fluctuating demand requirements over a planning horizon that ranges from six months to one year. When the demand is continuous and uncertain, stochastic dynamic linear programming can be handled effectively by making use of approximating continuous problems through discretization. Since all manner of things surrounding us appear to have a normal distribution, assuming that the demand has a normal distribution seems to be more reasonable than other kinds of distributions. When the number of the constrain alternatives increase, the minimum cost converges to the optimal solution. In practice, the limitation of the computer capacity limits the number of constrain alternatives. In this study 50,000 alternatives are sufficient to get the best solution. This model can be applied to other kinds of industrial products.

### V. CONCLUSION

A stochastic linear programming model for aggregate planning in which the optimum production or minimum cost could be obtained is used to investigate the distribution of food demand for animal food and the optimum amount of that food production at minimum cost. The demand data of food for laying hens were collected from January, 1990 to December, 2013 from a factory in Nakhonratchasima province of Thailand. The study shows that the distribution of monthly food demand for laying has a normal distribution. The minimum total cost average for 12 months is Baht 62,329,181.77.

#### TABLE II

<table>
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<tr>
<th>Month</th>
<th>Production (Unit: 30 kg)</th>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>13013.74</td>
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<td>4</td>
<td>13196.18</td>
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</table>

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#### REFERENCES


