On The Design of Robust Governors of Steam Power Systems Using Polynomial and State-Space Based $H_{\infty}$ Techniques: A Comparative Study

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Abstract—This work presents a comparison study between the state-space and polynomial methods for the design of the robust governor for load frequency control of steam turbine power systems. The robust governor is synthesized using the two approaches and the comparison is extended to include time and frequency domains performance, controller order, and uncertainty representation, weighting filters, optimality and sub-optimality. The obtained results are represented through tables and curves with reasons of similarities and dissimilarities.

Keywords—Robust control, load frequency control, steam turbine, $H_{\infty}$-norm, system uncertainty, load disturbance.

I. BACKGROUND AND MOTIVATION

POWER system stability can be defined as that property of a power system that enables it to remain in state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance. The quality of power supply must meet certain minimum standard requirements with special attention being given to the constancy of frequency [1], [2]. Our main concern in work paper concerns the above issue. Most universal method of electric generation is accomplished using thermal generation, and the most common machine for this production is the steam turbines. In the world most of the generation is powered by steam-turbine-driven generators [2]. Poor balancing between generated power and demand can cause the system frequency to deviate away from the nominal value, and create inadvertent power exchanges between control areas. to avoid such a situation, load frequency controllers (LFC) are designed and implemented to automatically balance between generated power and the demand power in each control area [1], [3], [4].

The problem of load frequency control has been investigated by many researchers. In [5] speed governors have been designed based on PID techniques with different philosophies because of its simplicity and ease of implementation. Fuzzy sliding mode controller for LFC has been designed in [6] to account for the system’s parameters variations and the governor backlash. Genetic algorithm (GA) is a global search optimization technique. The researchers in [7] used GA for tuning the control parameters of the proportional-integral (PI) control subject to the $H_{\infty}$ constraints in terms of LMI. Modern control techniques have been reported in [8], [9] in which a load frequency controller for power systems has been designed using LQR techniques. The work in [10] investigated the design problem or robust load frequency controller using LMI methods for solving the $H_{\infty}$ control problem. This paper is organized as follows: Section II describes the modeling of steam turbine system, Section III presents the design formulation for the $H_{\infty}$ governor using polynomial and state-space approaches, Section IV is devoted to the performance evaluation of the robust governors. The conclusions are summarized in Section V.

II. STEAM TURBINES AND SPEED GOVERNING SYSTEM

A steam turbine converts stored energy of high pressure and high temperature steam into mechanical energy, which is in turn converted into electrical energy by the generator. The heat source for the boiler supplying the steam may be a nuclear reactor or a furnace fired by fossil fuel (coal, oil, or gas) [1], [2]. A typical mechanical-hydraulic speed governing system consists of a Speed Governor (SG), a Speed Relay (SR), a Hydraulic Servomotor (SM), and Governor-Controller Valves (CVs). In steam turbine-generator system, the governing is accomplished by a speed transducer, a comparator, and one or more force-stroke amplifiers. Fig. 1 depicts the complete system block diagram of a steam turbine generator [2].

Speed Governor (SG):

$$S_G = \frac{1}{R}$$ (1)

where $R$ is the steady-state speed regulation. The value of $R$ determines the steady-state speed load characteristic of the generating unit.

Speed Relay (SR):

$$S_R(s) = \frac{1}{sT_{SR} + 1}$$ (2)

where $T_{SR}$ is the time constant of the speed relay.
Servo Motor (SM):

\[ S_M(s) = \frac{1}{sT_{SM} + 1} \优点(P) \]

where \( T_{SM} \) is the time constant of the servo motor.

Steam Turbine (ST):

\[ S_T(s) = \frac{\Delta P_m}{\Delta P_i} = \frac{F_H(sT_{CO} + 1)(sT_{RH} + 1) + F_P(sT_{CO} + 1)}{(sT_{CH} + 1)(sT_{CO} + 1)(sT_{RH} + 1)} \]

where \( T_{CO}, T_{RH}, T_{CH} \) are the time constants for the cross over, re heater, and steam chest respectively.

Machine Dynamics (MD):

\[ \Delta P_m = \frac{1}{K_D + sT_M} \Delta \omega_r \]

where \( T_M \) is the mechanical starting time, \( \Delta P_m \) is the incremental change of mechanical power, \( \Delta P_i \) is the incremental change of load power, \( \Delta \omega_r \) is the incremental change of angular speed of the synchronous generator.

The model of the complete steam turbine system has been derived with typical values of the listed in Table I [1], [2], [11]. This model is applicable to a tandem-compound single reheat turbine of fossil-fuelled units.

III. ROBUST GOVERNOR DESIGN PROCEDURE

A different configuration for the problem of LFC of steam turbine has been proposed in this work as shown in Fig. 4.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_D )</td>
<td>Damping factor</td>
<td>2</td>
<td>pu</td>
</tr>
<tr>
<td>( T_M )</td>
<td>Mechanical starting time</td>
<td>8</td>
<td>sec</td>
</tr>
<tr>
<td>( F_{IP} )</td>
<td>IP turbine power fraction</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>( F_{LP} )</td>
<td>LP turbine power fraction</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>( F_{HP} )</td>
<td>HP turbine power fraction</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>( T_{CO} )</td>
<td>Crossover time constant</td>
<td>0.4</td>
<td>sec</td>
</tr>
<tr>
<td>( T_{SR} )</td>
<td>Speed relay time constant</td>
<td>0.1</td>
<td>sec</td>
</tr>
<tr>
<td>( T_{SM} )</td>
<td>Servomotor time constant</td>
<td>0.2</td>
<td>sec</td>
</tr>
<tr>
<td>( T_{CH} )</td>
<td>Steam chest time constant</td>
<td>0.25</td>
<td>sec</td>
</tr>
<tr>
<td>( T_{RH} )</td>
<td>Reheater time constant</td>
<td>7</td>
<td>sec</td>
</tr>
<tr>
<td>( P_{V_{max}} )</td>
<td>Maximum valve position</td>
<td>1</td>
<td>pu</td>
</tr>
<tr>
<td>( P_{V_{min}} )</td>
<td>Minimum valve position</td>
<td>0</td>
<td>pu</td>
</tr>
<tr>
<td>( f_o )</td>
<td>Speed disturbance bandwidth</td>
<td>0.5-2</td>
<td>Hz</td>
</tr>
</tbody>
</table>

In the proposed configuration the controller (governor) is placed in the feed forward path in contrast to the conventional governor in which the controller is positioned in the feed backward path. In what follows the above proposed configuration together with the robustness tools will be used to setup the problem within the framework of the \( H_\infty \) design methodology using the polynomial methods.

A. Formulation of the Design Problem: A Polynomial Approach

Fig. 5 represents the mixed sensitivity problem for steam turbine plant which includes the performance shaping filters (\( V(s) \) and \( W_1(s) \)) and the uncertainty filters \( W_2(s) \) where additive uncertainty is used to compensate for neglected dynamics which is represented as unstructured uncertainty through \( W_2(s) \), \( \Delta \)Stable unknown transfer function.
design of the shaping filters is highly depends on the model at hand and certain considerations have to be taken in the design of these shaping filters like uncertainty, high frequency roll-off; and integral control.

Fig. 5 Mixed Sensitivity Configuration

\[ G(s) = \frac{N(s)}{D(s)} \]
\[ V(s) = \frac{M(s)}{D(s)} \]
\[ W_1(s) = \frac{A_1(s)}{B_1(s)} \]
\[ W_2(s) = \frac{A_2(s)}{B_2(s)} \]

The mixed sensitivity problem schematized above is the problem of minimizing the \( H_\infty \) norm of the closed-loop transfer function matrix [12]-[13]:

\[ T_{zw} = \begin{bmatrix} W_1 S V \\ W_2 R V \end{bmatrix} \]

Now the problem can be stated as:

Design a stabilizing controller such that the \( H_\infty \) -norm \( \| T_{zw} \|_\infty \) of the above closed-loop transfer function is minimized, i.e. \( \| T_{zw} \|_\infty \leq \gamma \). Knowing that \( T_{zw} \) is the closed-loop transfer matrix defined as [12], [13]:

\[ T_{zw} = F_1(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1} P_{21} \]  \( \gamma \) is the solution of the \( H_\infty \) optimization, and \( F_1(P,K) \) lower linear fractional transformation. In the SISO case the \( H_\infty \) -norm of \( T_{zw} \) is given by [14], [15]:

\[ \| T_{zw} \|_\infty = \sup_{\omega \in (-\infty,\infty)} \left[ \| W_1(j\omega)S(j\omega)V(j\omega) \|_+ \right] \]

where sup is the supremum or the least upper bound. With suitably chosen weighting filters matrices \( W_1 \), \( W_2 \) and \( V \) a suitably chosen shaping matrix. The selection of the weighting filters is not an easy task for a specific design problem and often involves ad hoc, and fine-tuning. It is very hard to give a general formula for the weighting filters that will work in every case. Finally, the selection of the uncertainty weighting filter depends on the dynamics of the system and the nominal model chosen. After a thorough analysis, the weighting filters are found to be:

\[ W_1(s) = \frac{x + 0.85}{s} \]
\[ W_2(s) = \frac{(272s + 0.08)(s + 80)}{(1000s + 80)^2 160} \]
\[ V(s) = \frac{M_{min}(s)}{D_{min}(s)} \frac{(s + 2)(0.72 + 1.2s + s^2)}{(0.25s + 1)(7s + 1)(8s + 2)} \]

B. Formulation of the Design Problem: A State-Space Approach

The block diagram of the augmented plant including the performance and uncertainty weighting filters together with plant is shown in Fig. 6 where an output multiplicative uncertainty is assumed in the system as indicated by the uncertainty weighting filter \( W_1(s) \) and the input \( d_1 \). In symbolic notations:

\[ z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \quad w = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \quad P(s) = \begin{bmatrix} w \\ u \end{bmatrix}, \quad P_{31}(s), P_{22}(s) \]

According to these definitions, the open loop generalized plant can be obtained as [16], [17]:

\[ P(s) = \begin{bmatrix} 0 & 0 & 0 & W_G1 \\ 0 & 0 & 0 & W_U \\ W_p G_2 & -W_p G_2 & W_p W_3 D_3 & W_p G_3 G_2 \\ G_2 & -G_2 & G_2 D_3 & G_3 G_2 \end{bmatrix}: \]

\[ A = \begin{bmatrix} A_1 & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \]
The control input \( u \), and the measured output \( y \) are related to the \( K(s) \) as follows:

\[
u(s) = K(s) y(s)
\]

The weighted closed-loop transfer function matrix \( T_{2w} \) is given by [16], [17]:

\[
\begin{bmatrix}
    \Delta & W_U & W_P \\
    W_U & W_D & W_P \\
    W_P & W_D & W_P
\end{bmatrix}
\]

It is obtained by connecting the measured output \( y \) with the control input \( u \) through the controller \( K(s) \).

Now the problem has been set up and the last step to do before the \( H_\infty \) controller design is the selection of the weighting filters which is highly depending on the application and usually the optimum set of weighting filters is reached by an ad hoc procedure. After a deep analysis, these weighting filters are selected as:

\[
W_P(s) = \frac{s + 0.29}{s + 2.9 \times 10^4}, \quad W_U(s) = \frac{1}{10}, \quad W_I(s) = \frac{4500s + 1067}{s + 3333}
\]

The problem has been solved via both the state-space and the polynomial approaches and the obtained results are evaluated in the following section.

IV. SIMULATIONS AND RESULTS

Concerning time domain performance, the polynomial approach is evidently faster than the state-space approach as shown in Fig. 7, where the transient response of the output frequency deviation \( \Delta \omega_r \) is drawn using both approaches when the system is subjected to sudden increase in the load of 0.04 p.u (i.e. \( \Delta P_L = 0.04 \)). As shown in Fig. 7, polynomial approach has less settling time (10.4 sec) with overshoot of peak 1.48*10^{-3} at 4.7 sec as compared to state-space approach (20.3 sec). This means that the polynomial approach satisfies time domain specifications and gives better results than the state-space approach. The reason behind this improved and fast response lies in the flexibility added to the design using the polynomial approach, whereas partial pole-placement process enables the designer to get some control on the location of some of the closed-loop poles and the ability to determine nature of the response through placing the poles in different places in the open left half s-plane. Table II shows the time response of the output frequency deviation (\( \Delta \omega_r \)).

The orders of the resulting controllers using both approaches are shown in Table III. It can be concluded that with improper weighting filter \( W_I(s) \) used and applying the design on the original model \( G(s) \), the polynomial approach gives a controller with an order always less than at least by one than the order of its counterpart using the state-space approach. While the gain in the order reduction will be two if the weighting filter \( W_I(s) \) is constant (i.e. \( W_I(s) = 1/c \)). Furthermore, applying the design procedure on the nominal model \( G_{nom}(s) \) with improper weighting filter \( W_I(s) \), the gain in the reduction will be at least two, while order reduction will be three if \( W_I(s) \) used in the design is a constant.

Moreover, the zeros of the designed controller in the state-space approach includes the poles of the weighting filter \( W_U(s) \) and \( W_I(s) \), and the stable open-loop poles of the original plant \( G(s) \). While its poles includes the poles of \( W_P(s) \). In the polynomial approach, the poles of the controller includes the poles of \( W_I(s) \). However, its zeros involves the zeros of \( M(s) \) that represent the open-loop poles of the original plant. Regarding the frequency domain requirements of the system, the polynomial approach exhibits obvious reductions in the gain and phase margins from their corresponding values in the state-space approach. Table IV lists the values using both approaches. Finally yet importantly, increasing or decreasing the bandwidth is more flexible with the polynomial approach than the state-space approach. The reason behind this is that the closed-loop bandwidth is tightly related to radius of the desired complex conjugates poles that are pre-assigned through a partial pole placement process in the polynomial approach. While in the state-space approach, the bandwidth of the system is determined through the selection of the frequency \( \omega_B \) which is the cutoff frequency of the performance filter \( W_P \).
Finally, Integral control can be designed easily in the polynomial approach than the state-space approach. The reason is state-space approach doesn't accept a pole on $j\Omega$ axis in the weighing functions or the plant $G(s)$, while there is no such limitation in the polynomial approach. Furthermore, high frequency roll-off can be obtained by using improper weighting filters in the design procedure. This can be done in the polynomial approach by using improper $W_2(s)$ which is not allowed in the state-space approach.

### Table II

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<tbody>
<tr>
<td>Settling time (sec)</td>
<td>23.7</td>
<td>20.3</td>
<td>7.06</td>
<td>10.4</td>
</tr>
<tr>
<td>Undershoot</td>
<td>$8.58 \times 10^{-3}$</td>
<td>$8.46 \times 10^{-3}$</td>
<td>$5.56 \times 10^{-3}$</td>
<td>$4.27 \times 10^{-3}$</td>
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### Table III

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<tbody>
<tr>
<td>Formula</td>
<td>$\text{ord}(G) + \text{ord}(W_p) + \text{ord}(W_u) + \text{ord}(W_i)$</td>
<td>$\text{ord}(G) + \text{ord}(1/W_2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Order after reduction</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
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### Table IV

<table>
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<tbody>
<tr>
<td>Phase Margin</td>
<td>78°</td>
<td>72°</td>
<td>37°</td>
<td>32°</td>
</tr>
<tr>
<td>Gain Margin (dB)</td>
<td>25</td>
<td>56</td>
<td>8.3</td>
<td>11</td>
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<tr>
<td>Phase crossover frequency rad/sec</td>
<td>2.68</td>
<td>3.91</td>
<td>3.58</td>
<td>2.375</td>
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<tr>
<td>Gain crossover frequency rad/sec</td>
<td>0.227</td>
<td>0.26</td>
<td>1.4</td>
<td>0.965</td>
</tr>
</tbody>
</table>

### V. Conclusion

In this research, a comparison study is introduced to solve the problem of the load frequency control for the steam turbines of the power systems. Both approaches result in a proposed robust controller which ensures both robust stability and robust performance and satisfy time and frequency domains specifications but it is important to stress that better results, with respect to the transient response characteristics are obtained by using the polynomial approach while state-space approach achieves slightly better results in the frequency domain than the polynomial approach. Finally, problem formulation and satisfying the design requirements is much easier to do with the polynomial approach than with the state-space one.

### Acknowledgment

The author thanks Baghdad University for the direct help and support for this work.

### References