Loudspeaker Parameters Inverse Problem for Improving Sound Frequency Response Simulation

Y. T. Tsai, Jin H. Huang

Abstract—The sound pressure level (SPL) of the moving-coil loudspeaker (MCL) is often simulated and analyzed using the lumped parameter model. However, the SPL of a MCL cannot be simulated precisely in the high frequency region, because the value of cone effective area is changed due to the geometry variation in different mode shapes, it is also related to affect the acoustic radiation mass and resistance. Herein, the paper presents the inverse method which has a high ability to measure the value of cone effective area in various frequency points, also can estimate the MCL electroacoustic parameters simultaneously. The proposed inverse method comprises the direct problem, adjoint problem, and sensitivity problem in collaboration with nonlinear conjugate gradient method. Estimated values from the inverse method are validated experimentally which compared with the measured SPL curve result. Results presented in this paper not only improve the accuracy of lumped parameter model but also provide the valuable information on loudspeaker cone design.

Keywords—Inverse problem, cone effective area, loudspeaker, nonlinear conjugate gradient method.

I. INTRODUCTION

An important acoustical characteristic of moving-coil loudspeakers (MCL) operated in a free-field or an half-space free-field condition is the sound pressure level (SPL) transfer response between electrical input voltage and the sound pressure output generated at an on-axis reference point in the far field [1], [2]. The frequency response and the distortion sound pressure output generated at an on-axis reference point in the half-space free-field condition is the sound pressure level (SPL). The frequency response and the distortion sound pressure output generated at an on-axis reference point in the half-space free-field condition is the sound pressure level (SPL).

The typical MCL is shown in Fig. 1, consists of a magnetic system (magnet under yoke and polar piece) and a vibration system (diaphragm and voice coil). The magnetic system of MCL transfers electrical-to-magnetic force to drive the voice coil, and that a diaphragm suspension system is used to improve the acoustic radiation mass and resistance. The cone effective area is changed due to the geometry variation in different mode shapes during the loudspeaker unit vibration. The experimental study of non-linear vibrations in a loudspeaker cone was made by Zhang et al. [5]; the experimental results show that non-linear vibrations in a thin shell are related to bending resonance. The amplitude and phase measurements have been made of the mechanical motion of different points on the cone diaphragm for various critical frequencies [6]. The behavior of a loudspeaker diaphragm beyond the piston range of operation has previously only been investigated using analytic techniques such as the Finite Element Method [7]. The study of sound reproduction quality of loudspeaker [8] is highly dependable on the vibration and radiation properties of the loudspeaker cone. However, accurate estimations of the cone effective area, without the need for expensive equipment and complex operational processes, are still being designed, developed, and improved. This work helps to fill certain gaps in these efforts.

Having elicited the interest of many scientists in recent years due to their wide range of applications, the inverse problems of differential equations now constitute a discipline between mathematics and engineering. Inverse problems have been successfully applied to identify the surface temperature [9]-[11], Heat Conduction [12] and geometric shape [13], which are either not measurable or too difficult to measure. The proposed method involves the following four calculation procedures: (i) direct problem, for solving both the voice-coil displacement and current simultaneously from the transduction equations; (ii) adjoint problem, wherein the Lagrange equations derived from the transduction equations are applied to obtain the gradient of the objective function; and (iv) sensitivity problem, for obtaining the appropriate step length of the CGM. All unknown parameters can be obtained using these procedures.

This study is organized as follows. Section II presents the mathematical framework of MCL. Section III contains the description of the construction of the presented method and its calculation steps. The experiment is presented in Section IV. Finally, related work is summarized in Section V.

II. THE MATHEMATICAL FRAMEWORK OF MOVING-COIL LOUDSPEAKER

The typical MCL is shown in Fig. 1, consists of a magnetic system (magnet under yoke and polar piece) and a vibration system (diaphragm and voice coil). The magnetic system of MCL transfers electrical-to-magnetic force to drive the voice coil, and that a diaphragm suspension system is used to improve the acoustic radiation mass and resistance. The cone effective area is changed due to the geometry variation in different mode shapes during the loudspeaker unit vibration. The experimental study of non-linear vibrations in a loudspeaker cone was made by Zhang et al. [5]; the experimental results show that non-linear vibrations in a thin shell are related to bending resonance. The amplitude and phase measurements have been made of the mechanical motion of different points on the cone diaphragm for various critical frequencies [6]. The behavior of a loudspeaker diaphragm beyond the piston range of operation has previously only been investigated using analytic techniques such as the Finite Element Method [7]. The study of sound reproduction quality of loudspeaker [8] is highly dependable on the vibration and radiation properties of the loudspeaker cone. However, accurate estimations of the cone effective area, without the need for expensive equipment and complex operational processes, are still being designed, developed, and improved. This work helps to fill certain gaps in these efforts.

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generate vibration, causing the voice-coil to suspend and begins to vibrate. Parameters in the electrical domain include input voltage signal $e(t)$, electrical inductance $L_e$, and resistance $R_e$. Meanwhile, electroacoustic parameters in the mechanical domain include mechanical stiffness $K_m$, mechanical mass $M_m$, and mechanical resistance $R_m$. In addition, the force factor $Bl$ converts magnetic force to mechanical force between the electrical and mechanical domains, $M_{a-rad}$ and $R_{a-rad}$ represent the acoustic radiation mass and acoustic radiation resistance of air loading. These parameters are essential in the operation of the MCL lumped parameter model.

The governing equations of the MCL can be expressed as

$$Bli(t) = M_m\ddot{x}(t) + R_m\dot{x}(t) + K_m x(t)$$

$$L_e\dddot{x}(t) + R_e\dot{x}(t) + Bl\ddot{x}(t) = e(t)$$

where $u(t) = S_d dx(t)/dt$ is the volume velocity. If the far-field pressure radiated by a baffled piston, the simplified model of $M_{a-rad}$ and $R_{a-rad}$ are given by,

$$M_{a-rad} = 0.85 \rho_i a / S_d$$

$$R_{a-rad} = 0.5 \rho_i c / S_d$$

where $a$ is the radius of cone, $\rho_i (=1.29 kg/m^3)$ is the sound pressure density in air, $c (=343 m/s)$ is the sound speed in air. Subscribing (3) and (4) into (1) and (2), then (1) can be rewritten as,

$$Bli(t) = (M_m + 1.0965 \rho S_d) \dddot{x}(t)$$

If the given parameters $(M_m, R_m, K_m, Bl, R_e, L_e, a, S_d)$ and the initial conditions of the displacement $x(t)$, current $i(t)$, and volume velocity $u(t)$ are known, the solutions for voice-coil displacement and current for (2) and (5) are the direct problems and the solutions are direct solutions. In contrast, within any given time interval $t \in (0, t_f)$, if input voltage $e(t)$, voice-coil displacement $x(t)$, and current $i(t)$ are known, the solutions for electroacoustic parameters for (2) and (5) are the inverse problems and the solutions are inverse solutions. Inverse solutions are the focus of this study, and the calculation steps are discussed in the next section.

III. INVERSE METHOD

A. Nonlinear Conjugate Gradient Method

Through the measured value $i_{mea}(t)$ and estimated value $\hat{i}(t)$, the objective function $J$ can be defined as

$$J(w) = \int_0^t [i(t; w) - i_{mea}(t)]^2 dt$$

where $w$ is a unknown vector to be determined. The above equation reveals that when the objective function $J$ is at minimum value, the estimated value $\hat{i}(t)$ will approach the measured value $i_{mea}(t)$. Solving the unknown parameters in direct problem as it gradually moves toward the minimum value will obtain the solution for an optimal set through iterations. Therefore, the nonlinear conjugate gradient method is designed to optimize by repeated iteration and which leads to objective function minimization. The iterative equation is

$$\mathbf{w}^{(k+1)} = \mathbf{w}^{(k)} - \beta^{(k)} \mathbf{p}^{(k)}$$

Here the superscript $k$ represents the $k^{th}$ iteration, $\beta^{(k)}$ denotes the $k^{th}$ search step length, and $\mathbf{p}^{(k+1)}$ is the $(k+1)^{th}$ search direction with decreased value:

$$\mathbf{p}^{(k+1)} = \nabla J^{(k)} + \gamma^{(k)} \mathbf{p}^{(k)}$$

where $\nabla J^{(k)}$ represents the gradient of the objective function at the $k^{th}$ iteration. The conjugate gradients update parameter $\gamma^{(k)}$ is given by

$$\gamma^{(k)} = \|\nabla J^{(k)}\| / \|\nabla J^{(k-1)}\| = \sum_{j=0}^{N} \left( \frac{\partial J^{(0)}}{\partial \gamma_j} \right)^2 / \sum_{j=0}^{N} \left( \frac{\partial J^{(k-1)}}{\partial \gamma_j} \right)^2$$

Note that, when descending direction does not take into account $\gamma^{(k)} \mathbf{p}^{(k)}$, then $\mathbf{p}^{(k+1)} = \nabla J^{(k)}$ in (8). At this point, CGM will degenerate into steepest decent method.

During the convergence process of CGM, the voice-coil displacement $x(t)$, the gradient of objective function $\nabla J$, and the step length $\beta$ must be solved. They are respectively the solutions of the direct problem, adjoint problem, and sensitivity problem, which will be explained in following subsections.

B. Solving Adjoint Problem for $\nabla J$

Using Lagrange multiplier method to multiply the direct problem by a Lagrange multipliers $\lambda(t)$, and substituting it into the objective function in (6) to obtain a new objective function. The adjoint problem is established to solve the Lagrange multipliers $\lambda(t)$ as
\[
\dot{\dot{\lambda}}(t) + \ddot{\dot{\lambda}}(t) + K_s \dot{\lambda}(t) = Bl \dot{\lambda}(t) \\
- L_0 \dot{\lambda}(t) + R_0 \dot{\lambda}(t) - Bl \dot{\lambda}(t) = -2(\dot{\omega}(t) - i_{\text{mea}}(t))
\]

with the final conditions
\[
\lambda(\omega) = 0 \quad \text{and} \quad \dot{\lambda}(\omega) = 0
\]

where \( \dot{\lambda} = \dot{\dot{\lambda}} \).

Therefore, the gradient of the objective function can be obtained as [17],
\[
\nabla J = \left[ \frac{\partial J}{\partial M_m}, \frac{\partial J}{\partial R_m}, \frac{\partial J}{\partial K_m}, \frac{\partial J}{\partial B_l}, \frac{\partial J}{\partial R_e}, \frac{\partial J}{\partial L_e} \right]^T
\]

where
\[
\frac{\partial J}{\partial M_m} = \int_0^\omega \lambda(t) \dot{x}(t) dt, \quad \frac{\partial J}{\partial R_m} = \int_0^\omega \lambda(t) \dot{x}(t) dt, \\
\frac{\partial J}{\partial K_m} = \int_0^\omega \lambda(t) \dot{x}(t) dt, \quad \frac{\partial J}{\partial B_l} = \int_0^\omega \lambda(t) \dot{x}(t) dt, \\
\frac{\partial J}{\partial R_e} = \int_0^\omega \lambda(t) \dot{x}(t) dt, \quad \frac{\partial J}{\partial L_e} = \int_0^\omega \lambda(t) \dot{x}(t) dt
\]

Once \( \nabla J \) is solved, the search direction \( \mathbf{P}^{(k+1)} \) is readily solved.

C. Sensitivity Problem

After confirming the search direction \( \mathbf{P} \), the step length \( \beta \) of (7) must be determined. The step length \( \beta \) can be obtained as [17],
\[
\beta = \frac{\int_0^\omega (\dot{\omega}(t) - i_{\text{mea}}(t)) \delta \dot{x}(t) dt}{\int_0^\omega \delta \dot{x}(t) dt}
\]

where \( \delta \dot{x}(t) \) can be solved by following equations,
\[
\dot{M}_m \ddot{\delta x}(t) + \dot{R}_m \dot{\delta x}(t) + \ddot{K}_m = -B_l \delta \dot{x}(t)
\]

Once \( \nabla J \) is solved, the search direction \( \mathbf{P}^{(k+1)} \) is readily solved.

D. Results and Discussion

The loudspeaker is excited by a sweep-tone test signal with 2 Vrms generated from the KLIPPEL analyzer system [1]. The RMS displacement \( x_{\text{mea}}(t) \) can be obtained by [18],
\[
x_{\text{mea}} = \frac{P_{\text{rms}}}{\rho_0 S_d f}
\]

Fig. 2 A two-inch MCL placed on the fixture of the Klippel analyzer system.

First, the laser vibrometer was used for measuring the center displacement \( x_{\text{mea}}(t) \) in cone piston motion at resonance frequency (210Hz). The values of estimated parameters at resonance frequency are listed in Table I. Then for determining the values of parameters \( (M_m, \dot{R}_m) \) versus the frequencies, the estimated displacement \( \dot{x}(t) \) can be solved by [17] to solve the differential equations for obtaining the estimated \( \dot{x}(t) \). The unknown parameters \( (M_m, \dot{R}_m, B_l, L_e, R_e) \) can be solved by proposed inverse method.

In (15) and (16), \( \delta \mathbf{w} \) is the search direction \( \mathbf{P} \) of its iteration.
The inverse method for estimating the electroacoustic parameters with considering the acoustic radiation mass, resistance and the cone effective area versus frequencies are presented in this study. The calculation steps for inverse method address the direct problem for solving the equations, adjoint equations for solving the gradient, sensitivity problem for solving the step length, CGM for solving the search direction. The accuracy of the proposed model was ascertained by comparing the measured and estimated SPL result. Through the measurement, the results show that inverse solutions can be estimated in high agreement. The proposed method is useful not only in parameter evaluation but also in the diagnosis of loudspeaker cone vibration, which is of great importance in quality control.

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