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Statistical Estimation of Spring-back Degree Using Texture Database

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***Abstract*—**Using a texture database, a statistical estimation of spring-back was conducted in this study on the basis of statistical analysis. Both spring-back in bending deformation and experimental data related to the crystal orientation show significant dispersion. Therefore, a probabilistic statistical approach was established for the proper quantification of these values. Correlation was examined among the parameters $F(x)$ of spring-back, $F(x)$ of the buildup fraction to three orientations after 92° bending, and $F(x)$ at an as-received part on the basis of the three-parameter Weibull distribution. Consequent spring-back estimation using a texture database yielded excellent estimates compared with experimental values.

***Keywords*—**Bending, Spring-back, Database, Crystallographic Orientation, Texture, SEM-EBSD, Weibull distribution, Statistical analysis.

I. Introduction

THE authors have focused on fine-grained materials and laser-cut materials. Moreover, they have experimentally investigated the transition of texture in each material synthesis step and bending step using SEM-EBSD and XRD based on such a background [1], [2]. Measurement data of crystallographic orientation have been accumulated in a database with spring-back data in bending [3].

This paper addresses the statistical estimation of spring-back based on statistical analysis using this crystallographic orientation database. Because the experimental data of spring-back in bending and crystallographic orientation both have dispersion, a stochastic method for quantifying these properly has been established. This epoch-making approach can estimate spring-back based on the texture database accumulated to date without newly carrying out bending of a processing-resistant material.

II. Database Overviews and Crystallographic Orientation

Previous studies have compiled property data with great dispersion to experimental data such as fatigue and heat treatment properties of metallic materials, into a database for statistical work [4]. This paper applies this view to texture analysis. To date, the authors have accumulated experimental data of spring-back and measurement data of the texture accompanying bending into a database. We conduct statistical analysis using this and estimate the spring-back of fine-grained materials to elucidate their deformation properties.

Texture data have been acquired in the following procedure and have been stored in a database. Crystallographic orientation analysis is conducted on the ND plane at the bending section of a fine-grained material specimen of thickness t=1.0mm before and after bending using an SEM-EBSD instrument. Die conditions in bending are a punch tip radius R=0.6 mm, a punch tip angle of 88°, die width V=6 mm, die shoulder radius R=1.5 mm, and a die angle of 88°. The dies and specimens are mounted to a general-purpose universal tester in our laboratory. Then V-bending is conducted at room-temperature in air. The external angle of a specimen is 92° when V-bending is conducted until a specimen makes intimate contact with a die. A spring-back indicator is used for spring-back measurement. It performs real-time measurement of bending angle or spring-back in the accuracy of 0.01°(36") by mounting it between dies. It was reported in our previous paper [1] that the crystallographic orientation transition occurs in both the material that synthesis steps by large strain deformation and in the bending step in the bending of fine-grained materials, the subject of measurement of this study, and the orientation tend to accumulate on the last stable orientation determined for crystal lattice type and stress mode.

The numerical data of Eulerian angles obtained using the SEM-EBSD analysis were analyzed using a crystallographic orientation tabulation program coded with Visual Basic 6.0. The crystallographic orientation distribution on the ND plane was computed quantitatively. An IPF was divided into 10 partitions along the radial and θ directions, respectively, to a total of 55 areas. Then 10 areas adjacent to [001], [101], and [111] of each vertex were defined respectively as $A_{[001]}$, $A_{[101]}$, and $A_{[111]}$, which represented buildup fraction to each orientation and which was expressed in percentages: analyzed points on an IPF were processed as (1). This study treated analyzed points on 25 areas other than this as random orientation, and did not append them to the database. They were not used in the later statistical analysis either. This study adopted such a simplified handling of texture because its principal objective was the establishment of a database system and statistical analysis applying it. Nevertheless, modification of the data input algorithm of the database will enable us to extract typical textures such as Cube texture, Goss texture, or Fiber texture in the bcc structure, and will enable us to subdivide the areas corresponding to high-precision expression of texture.

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$$A_{[001]} = \frac{A_{[001]}}{A_{[001]} + A_{[101]} + A_{[111]}} \times 100\%$$
$$A_{[101]} = \frac{A_{[101]}}{A_{[001]} + A_{[101]} + A_{[111]}} \times 100\%$$
$$A_{[111]} = \frac{A_{[111]}}{A_{[001]} + A_{[101]} + A_{[111]}} \times 100\%$$
$$\therefore A_{[001]} + A_{[101]} + A_{[111]} = 100\% \tag{1}$$

The database accepted and accumulated data pairs that combined $A_{[001]}$, $A_{[101]}$, and $A_{[111]}$, the buildup fraction of crystallographic orientation to the three above-described orientations, and the quantitative data of spring-back. At this time, 75data pairs and 22.3MB of texture data and spring-back data have been inputted into the database. Most dominant are the data of pure Cu single crystal materials (axial orientation [111]) and pure Cu polycrystalline materials (C1020), which consist of a total of 37 series. Data of nonferrous metals and alloys total 60 series, including 25 series of pure Al single-crystal materials (axial orientation [110]). It also contains 13 series of data of steel materials (SUS316, SPCC, SS400, and NFG).

III. Statistical Analysis Technique Using the Weibull Cumulative Distribution Function

The statistical analysis technique proposed in this paper inputs $A_{[001]}$, $A_{[101]}$, and $A_{[111]}$ obtained from an IPF using three-parameter Weibull distribution functions [5], and estimates spring-back statistically.

A. Cumulative Distribution Function of Three-Parameter Weibull Distribution

Statistical analysis using a three-parameter Weibull distribution has proved useful for arrangement of experimental data with dispersion. This study is the first trial to conduct statistical analysis applying three-parameter Weibull distribution to the quantitative data of crystallographic orientation.

The cumulative distribution function of a three-parameter Weibull distribution $F(x)$ is expressed in (2):

$$F(x) = 1 - \exp\left\{ -\left(\frac{x-c}{b} \right)^a \right\} \tag{2}$$

where a is a shape parameter, b is a scale parameter, and c is a location parameter. The three-parameter Weibull distribution is so designated because a function is constituted by these three parameters. The Weibull distribution has two forms: the general two-parameter form and three-parameter form added by location parameter c. Because the latter, the three-parameter form, can approximate a complex distribution shape with much dispersion more precisely, it was adopted for the present study.

Statistical analysis software STANAD (Statistical Analysis Software for Material Strength Database for Reliability Design; The Society of Materials Science, Japan) was used for the graphical representation and function expression of a three-parameter Weibull distribution. The software works as an add-on to Microsoft Excel 2003.

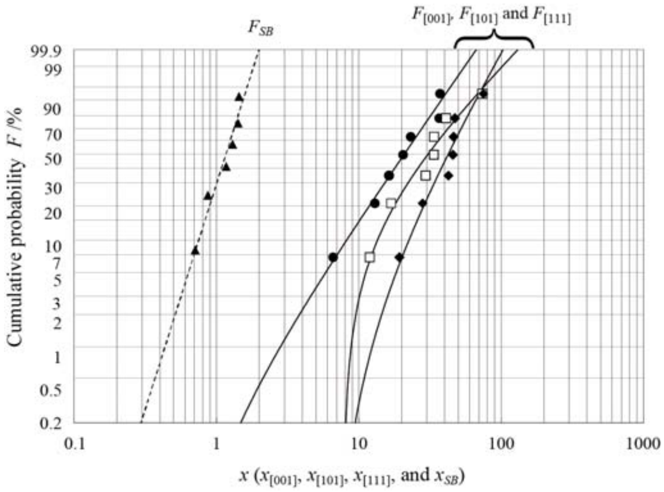
In plotting the three-parameter Weibull distribution on a Weibull probability paper using STANAD, the horizontal axis x was determined using the symmetric ranking method shown in (3).

$$x = \frac{i-0.5}{n} \times 100\% \tag{3}$$

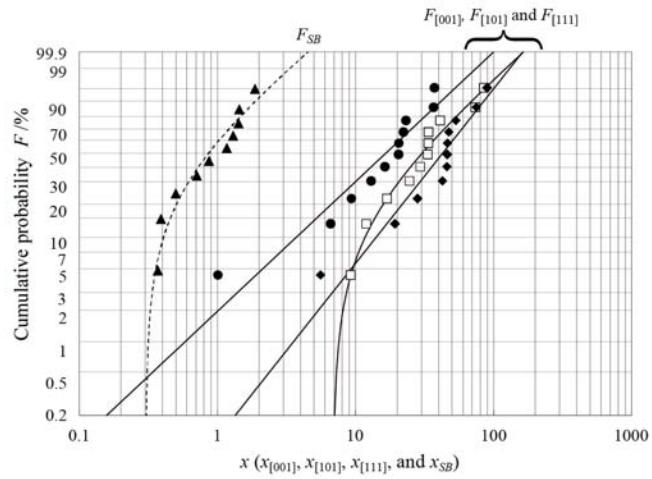
B. Statistical Estimation Procedure of Spring-back

The algorithm of spring-back estimation using the cumulative distribution function of three-parameter Weibull distribution is presented below. Cumulative distribution functions of three-parameter Weibull distribution $F_{[001]}$, $F_{[101]}$, and $F_{[111]}$ are provided by (4), assuming a buildup fraction of the crystallographic orientation to three orientations $A_{[001]}$, $A_{[101]}$, and $A_{[111]}$ as mutually independent functions. Spring-back in the experimental data of this material is also approximated by a Weibull distribution as another function F_{SB}. In all, four cumulative distribution functions are presented on a Weibull probability scale. Cumulative distribution functions $F_{[001]}$, $F_{[101]}$, and $F_{[111]}$ are functions of $x_{[001]}$, $x_{[101]}$, and $x_{[111]}$, where $x_{[xyz]}$ is only a realigned form of buildup fraction $A_{[xyz]}$ of the crystallographic orientation to three orientations in the order of (3), so that A and x are in concordance. $x_{[xyz]}$ in $F_{[xyz]}$ is buildup fraction of crystallographic orientation, whereas x_{SB} in F_{SB} is spring-back obtained by experiment. The proposal in this report is therefore characterized by the fact that two types of functions x with originally different meaning displayed along the identical axis enable us to make statistical estimation of spring-back.

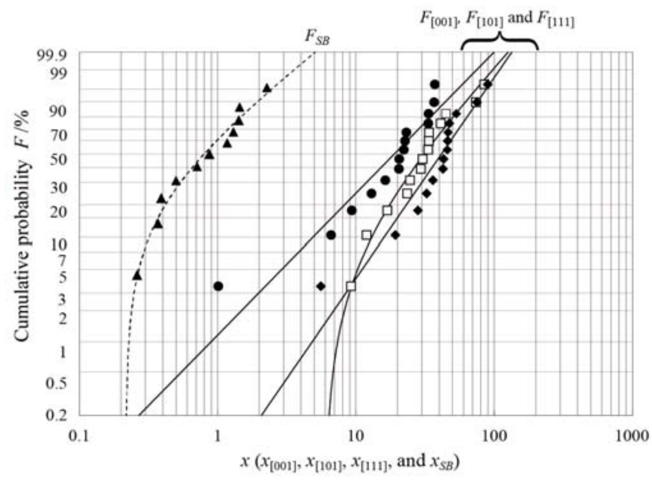
$$[001];\ F_{[001]}(x_{[001]}) = 1 - \exp\left\{ -\left(\frac{x_{[001]}-c}{b} \right)^a \right\}$$
$$[101];\ F_{[101]}(x_{[101]}) = 1 - \exp\left\{ -\left(\frac{x_{[101]}-c}{b} \right)^a \right\}$$
$$[111];\ F_{[111]}(x_{[111]}) = 1 - \exp\left\{ -\left(\frac{x_{[111]}-c}{b} \right)^a \right\}$$
$$F_{[001]} + F_{[101]} + F_{[111]} = 100\%$$
$$Springback;\ F_{SB}(x_{SB}) = 1 - \exp\left\{ -\left(\frac{x_{SB}-c}{b} \right)^a \right\} \tag{4}$$



(a) Weibull distributions for pure Cu single- and poly-crystal



(b) Weibull distributions for f. c. c. stravture materials



(c) Weibull distributions for all materials

Fig. 1 3-parameters Weibull distribution for spring-back degree data and texture data [6]

Because the sum of buildup fractions of crystallographic orientation to the three orientations $A_{[001]}$, $A_{[101]}$, and $A_{[111]}$ is 100% as described above, the sum of $F_{[001]}$, $F_{[101]}$, and $F_{[111]}$ which express these as functions on Weibull probability paper is also 100%.

Each cumulative distribution function of $F_{[001]}$, $F_{[101]}$, and $F_{[111]}$ is constituted by buildup fractions to each orientation $x_{[001]}$, $x_{[101]}$, and $x_{[111]}$. Because each x value is known from the database, this fraction is multiplied as the weight of a cumulative distribution function; (5) yields a $\overline{F}$. Regarding this $\overline{F}$ equivalent to F_{SB} on a Weibull probability scale enables us to ascertain x_{SB} inversely from the cumulative distribution function of F_{SB}, so that spring-back can be estimated uniquely.

$$\begin{gathered} F_{[001]} \times x_{[001]} + F_{[101]} \times x_{[101]} + F_{[111]} \times x_{[111]} = \overline{F} \\ \overline{F} \rightarrow (identification) \rightarrow F_{SB} \rightarrow (estimation) \rightarrow x_{SB} \end{gathered} \tag{5}$$

IV. Statistical Analysis Technique Using the Weibull Cumulative Distribution Function

Fig. 1 [6] displays distribution properties as an example of analysis using a three-parameter Weibull distribution. Panels (a)–(c) show Weibull distribution properties at a bending angle of 92°, searched and extracted from the following (a)–(c), respectively: (a) Data of pure Cu single-crystal and polycrystalline data in the database (7 series out of all the 75 series corresponding to this), (b) Data of fcc metals in the database (11 series), and (c) Data of any metallic material in the database (14 series).

Fig. 1 (a) shows that $x_{[001]}$, $x_{[101]}$, $x_{[111]}$, and seven extracted data of x_{SB} can be expressed respectively by the cumulative distribution function of the three-parameter Weibull distribution of $F_{[001]}$, $F_{[101]}$, $F_{[111]}$, and F_{SB}. These four cumulative distribution functions $F(x)$ are shown as in (6). Each buildup fraction $x_{[001]}$=21.9%, $x_{[101]}$=34.4%, and $x_{[111]}$=43.5% are weighed and multiplied to each cumulative distribution function $F_{[001]}$, $F_{[101]}$, and $F_{[111]}$, to compute $\overline{F}$. This $\overline{F}$ is regarded as equivalent to F_{SB}; x_{SB} was inversely determined from the cumulative distribution function of F_{SB}, so that x_{SB}=1.19° was estimated. This estimated value was +0.04° and +3.5% of error to 1.15°, the average of experimentally obtained results obtained by actual bending of the material, which thereby demonstrated that this technique can predict experimentally obtained results well. This procedure was also applied to extracted data of (b) and (c), and calculation and comparison were conducted. The error of estimate from experimentally obtained result was 8.2% at the maximum. Consequently, results suggest that the technique proposed in this paper is effective in the spring-back prediction of various processing-resistant materials. Its application and deployment are fully expected in the future.

$$
\begin{aligned}
&[001];\ F_{[001]}(x)=1-\exp\left\{-\left(\frac{x-0.50}{24.4}\right)^{1.93}\right\}\\
&[101];\ F_{[101]}(x)=1-\exp\left\{-\left(\frac{x-7.80}{29.6}\right)^{1.53}\right\}\\
&[111];\ F_{[111]}(x)=1-\exp\left\{-\left(\frac{x-6.62}{41.7}\right)^{2.30}\right\}\\
&F_{[001]}(x)+F_{[101]}(x)+F_{[111]}(x)=100\%\\
&F_{[001]}\times 21.9\%+F_{[101]}\times 34.4\%+F_{[111]}\times 43.5\%=\overline{F}\\
&Springback;\ F_{SB}(x)=1-\exp\left\{-\left(\frac{x-0}{1.26}\right)^{4.27}\right\}
\end{aligned}
\tag{6}
$$

V. Conclusions

This study has applied a texture database to the bending of fine-grained materials, and conducted quantitative estimate of spring-back using three-parameter Weibull cumulative distribution function. The results that were obtained are shown below.

1. A texture database compiled by the present authors was applied in this study. The spring-back estimate algorithm was proposed using a statistical analysis technique with a three-parameter Weibull distribution.
2. The three-parameter Weibull distribution was applied to statistical analysis of crystallographic orientation for the first tim
3. $x_{[001]}$, $x_{[101]}$, $x_{[111]}$, and extracted data of x_{SB} can be displayed with $F_{[001]}$, $F_{[101]}$, $F_{[111]}$, and F_{SB}, the cumulative distribution functions of the three-parameter Weibull distribution.
4. Each parameter of F_{SB} of spring-back and $F_{[001]}$, $F_{[101]}$, $F_{[111]}$, buildup fractions to the three orientations after 92° bending, are correlated based on the three-parameter Weibull distribution. Spring-back can be estimated from the texture database. Predicted values well reproduce the experimental values.
5. The error of estimate result from experimentally obtained result for various materials was 8.2% at the maximum. Consequently, results suggest that the proposed technique in this paper is effective for spring-back prediction of various processing resistant materials. Its application and deployment are fully expected in the future.

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