

Effect of Magnetic Field on Mixed Convection Boundary Layer Flow over an Exponentially Shrinking Vertical Sheet with Suction

S. S. P. M. Isa, N. M. Arifin, R. Nazar, N. Bachok, F. M. Ali, I. Pop

Abstract—A theoretical study has been presented to describe the boundary layer flow and heat transfer on an exponentially shrinking sheet with a variable wall temperature and suction, in the presence of magnetic field. The governing nonlinear partial differential equations are converted into ordinary differential equations by similarity transformation, which are then solved numerically using the shooting method. Results for the skin friction coefficient, local Nusselt number, velocity profiles as well as temperature profiles are presented through graphs and tables for several sets of values of the parameters. The effects of the governing parameters on the flow and heat transfer characteristics are thoroughly examined.

Keywords—Exponentially shrinking sheet, magnetic field, mixed convection, suction.

I. INTRODUCTION

THE problems of flow and heat transfer in the boundary layers of a continuous stretching/shrinking surface have attracted considerable attention of researchers due to their numerous applications in industrial manufacturing processes. Some of the applications are extraction of polymer sheets, paper production, hot rolling and glass-fiber production. Sakiadis [1], [2] first initiated the study of boundary layer flow over a stretched surface. The problems in [1] and [2] are extended to discuss the various aspects of flow and heat transfer characteristics by many researchers [3]-[5]. The effect of suction or injection on the boundary layer flow and heat transfer over a continuously stretched surface has motivated the works of Fox [6] and Chen and Char [7]. Later, Gupta and Gupta [8] emphasized that the stretching of the sheet may not necessarily be linear. The study of exponential variations of stretching velocity and temperature distributions in the flow of stretching surface has been initiated by Magyari and Keller [9]. The extended work of Magyari and Keller [9] have been reported by several researchers such as Elbashbeshy [10], Khan and Sanjayanand [11], Sanjayanand and Khan [12], Sajid and Hayat [13], and Partha et al. [14].

S. S. P. M. Isa is with the Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia (corresponding author: phone: +603-89467939; e-mail: suzi_isa@yahoo.com).

N. M. Arifin, N. Bachok, F. M. Ali are with the Department of Mathematics, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia (e-mail: norihanarifin@yahoo.com).

R. Nazar is with the School of Mathematical Science, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia (e-mail: rnm@ukm.edu.my).

I. Pop is with the Department of Mathematics, Babes-Bolyai University, R-400084 Cluj-Napoca, Romania. (e-mail: popm.ioan@yahoo.co.uk).

Recently, the boundary layer flow induced by shrinking sheet has gained considerable interest. For the shrinking sheet flow, the fluid attracted towards a slot and the vorticity generated at the shrinking sheet is not confined within a boundary layer and a steady flow is not possible unless adequate suction is applied at sheet [15]. The pioneering study of the flow due to the shrinking sheet was first observed by Miklavčič and Wang [16]. Some of the investigators focused their work on the effect of magnetic field on the boundary layer flow and heat transfer over a shrinking sheet [17]-[23]. The first attempt of studying the problem of boundary layer flow induced by an exponentially shrinking sheet was made by Bhattacharyya [24]. In the earlier study, Bhattacharyya [24] obtained the dual solutions when the mass suction parameter s satisfies the condition $s > 2.266684$, and consequently for $s < 2.266684$ the flow has no solution. The work by Bhattacharyya [24] is extended by Rohni et al. [25], by adding the effect of buoyancy force. Rohni et al. [25] found that the presence of buoyancy force would contribute to the existence of triple solutions to the flow and heat transfer.

Motivated by the analyses of Rohni et al. [25], we investigate the effects of magnetic field on the flow and heat transfer characteristics of a viscous fluid over an exponentially shrinking sheet with variable temperature distribution. We employed similarity transformation to reduce the governing nonlinear boundary layer equations to nonlinear ordinary differential equations. The numerical solutions are obtained by using the shooting method. The effects of the physical parameters on the velocity profiles, temperature profiles, skin friction coefficient and local Nusselt number are analysed.

II. MATHEMATICAL FORMULATIONS

Consider the two-dimensional incompressible, viscous and electrically conducting fluid over an exponentially permeable shrinking surface (see Fig. 1), with the effect of magnetic field. The x -axis runs along the shrinking surface in the direction opposite to the sheet motion and the y -axis is perpendicular to it. It is assumed that the surface temperature is T_w and the temperature of the fluid at infinity is T_∞ , where $T_w > T_\infty$ corresponds to the assisting flow case (heated sheet) and the presence of opposing flow case is when $T_w < T_\infty$ (cooled sheet). A transverse magnetic field is assumed to be applied in the y -axis. Under the assumption of Boussinesq and boundary layer approximations, the flow and heat transfer

problems are governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B^2(x)}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

where u and v are the components of velocity in the x and y directions, respectively, $\nu = \mu/\rho$ is the kinematic viscosity, μ is the viscosity, ρ is the fluid density, g is the acceleration due to gravity, β is the thermal expansion coefficient, σ is the electrical conductivity, α is the thermal diffusivity and T is the temperature of the fluid. In (2) we choose the special form of magnetic field $B(x) = B_0 e^{(x/2L)}$.

The appropriate boundary conditions are

$$\begin{aligned} \text{At } y = 0: \\ u = u_w(x) = -U_w e^{(x/L)}, \\ v = v_w(x), \\ T_w(x) = T_\infty + T_0 e^{(2x/L)} \end{aligned} \quad (4)$$

$$\begin{aligned} \text{As } y \rightarrow \infty: \\ T_w(x) = T_\infty + T_0 e^{(2x/L)}, \\ u \rightarrow 0, \\ T \rightarrow T_\infty \end{aligned}$$

where $U_w > 0$ is the velocity of the shrinking surface and $v_w(x) < 0$ is the wall mass suction velocity.

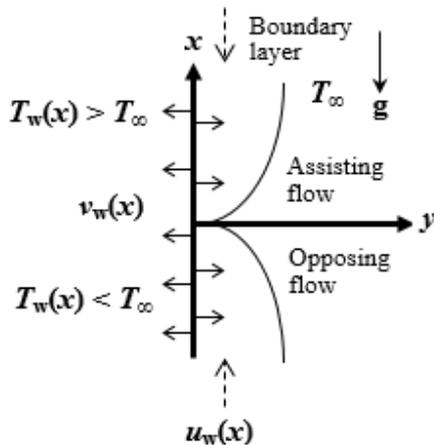


Fig. 1 Physical model and coordinate system

We introduce new similarity variables as follows:

$$\begin{aligned} \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \\ \eta &= y \left(\frac{U_w}{2\nu L} \right)^{1/2} e^{(x/2L)}, \\ u &= U_w e^{(x/L)} f'(\eta), \\ v &= - \left(\frac{U_w \nu}{2L} \right)^{1/2} e^{(x/2L)} \\ &\quad [f(\eta) + \eta f'(\eta)] \end{aligned} \quad (5)$$

By substituting (5) into (1)–(3), we obtain nonlinear ordinary differential equations:

$$f''' + ff'' - 2(f')^2 - H^2 f' + 2\lambda\theta = 0 \quad (6)$$

$$(1/\text{Pr})\theta'' + f\theta' - 4f'\theta = 0 \quad (7)$$

and (4) reduce to boundary conditions:

$$\begin{aligned} \text{At } \eta = 0: \\ f'(\eta) = -1, \\ f(\eta) = s, \\ \theta(\eta) = 1 \end{aligned} \quad (8)$$

$$\begin{aligned} \text{As } \eta \rightarrow \infty: \\ f'(\eta) \rightarrow 0, \\ \theta(\eta) \rightarrow 0 \end{aligned}$$

where $H^2 = 2\sigma B_0^2 L / \rho U_w$ is the Hartman number, $\text{Pr} = \nu/\alpha$ is the Prandtl number, $s > 0$ is the suction parameter and $\lambda = Gr/\text{Re}^2$ is the constant mixed convection parameter, where $Gr = g\beta_T T_0 L^3 / \nu^2$ is the Grashof number and $\text{Re} = U_w L / \nu$ is the Reynolds number. It should be noticed that $\lambda > 0$ corresponds to assisting flow, $\lambda < 0$ corresponds to opposing flow and $\lambda = 0$ corresponds to forced convection flow.

The physical quantities of interest in the present problem are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are given by

$$\begin{aligned} C_f &= \frac{\mu}{\rho U_w^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \\ Nu_x &= \frac{L}{T_w - T_\infty} \left(- \frac{\partial T}{\partial y} \right)_{y=0} \end{aligned} \quad (9)$$

Substituting (5) into (9), we get

$$\begin{aligned} C_f (2\text{Re}_x)^{1/2} e^{(-3x/2L)} &= f''(0), \\ Nu_x \left(\frac{2}{\text{Re}_x} \right)^{1/2} e^{(-x/2L)} &= -\theta'(0) \end{aligned} \quad (10)$$

III. RESULTS AND DISCUSSION

Equations (6) and (7) subject to (8) were solved numerically using the shooting method. The numerical values of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ as well as the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ are obtained for various values of the Hartmann number H , the suction parameter s , and the mixed convection parameter λ . To assess the accuracy of the numerical method, we compare the present results with those obtained by Bhattacharyya [24] and Rohni et al. [25] in Table I. In this table, the comparison is made for the non-magnetic case $H = 0$, non-buoyant flow $\lambda = 0$, and when $Pr = 1$. This table shows the comparison for the value of the critical suction parameter s_c and singularity point of suction parameter s_a . The values of s_c , which connect the upper branch and lower branch are depicted from the graphs of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ against s . Discontinuity points in the lower branch, which determined the singularity points are obtained from the variations of $-\theta'(0)$ versus s . As a conclusion, the present values of s_c and s_a are in good agreement with those obtained by Bhattacharyya [24] and Rohni et al. [25]. Therefore, the good comparison gives us much confidence in our theoretical study and numerical computation. In this paper, we focus on the problem of mixed convection in the case of when the flow is opposing and assisting with the effect of magnetic field.

The effect of magnetic field and suction on the flow and heat transfer in the case of when the flow is opposing ($\lambda < 0$) have been depicted in Figs. 2–5. The variation of the dimensionless skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ against s when $Pr = 1$ and $\lambda = -0.5$ are displayed in Figs. 2 and 3. In this case, four solutions exist. The first and second solutions are combined at the critical point s_c , whereas the third solution continues until it reaches $s = 0$ (impermeable surface). The fourth solutions occur at large values of the suction parameter s .

Next we tabulate the values of the skin friction coefficient and local Nusselt number for various values of s for $\lambda = -0.5$ in Table II. It can be seen from Table II that larger H and s values imply higher values of $f''(0)$ and $-\theta'(0)$ for the first solution. The values of $f''(0)$ and $-\theta'(0)$ are consistently reduced by the presence of magnetic field in the second, third, and fourth solutions.

Figs. 4 and 5 show the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles when $\lambda = -1$. From this figure, we notice that among all the profiles, the third solution has the largest magnitude of velocity, at the point of near to the shrinking wall. Then, negative velocity $f'(\eta)$ in all profiles tend to achieve a constant value, namely zero. The temperature $\theta(\eta)$ of fourth solution profile is initially decreasing and goes to a certain negative value, then for larger η the temperature starts to increase.

To study the flow and heat transfer characteristics of the assisting flow case ($\lambda > 0$), we plot the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ against s when $\lambda = 1$ in Figs. 6 and 7. Similar to the case of opposing flow, four profiles are obtained. The critical point s_c is an intersection point between the second and third solution profiles. The first and fourth solutions continue until $s = 0$ for $f''(0)$ and $-\theta'(0)$.

The values of $f''(0)$ and $-\theta'(0)$ for several values of s when $\lambda = 1$ are tabulated in Table III. From Table III, it is clear that the skin friction coefficient increases with the increase of magnetic field in the first, second and third solutions. On the other hand, as H increases, the value of $f''(0)$ decreases for the fourth solution. The values of the local Nusselt number increase with the increase of the rate of magnetic field, which is displayed in Table III. This statement is true for the first, second and fourth solutions. When we consider the third solution, the values of the local Nusselt number always decrease with increasing H . Further, Table III shows that the values of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ increase with an increase of the rate of suction.

The graphs of the velocity profiles $f'(\eta)$ are depicted in Fig. 8. From this figure, the velocity increases with an increase in η for the first solution. The velocity profiles $f'(\eta)$ of the second and third solutions are initially decreasing at the point near to the shrinking wall. For larger value of η , these two solutions start to increase until they approach zero value. Fig. 9 depicts the variations on the temperature profiles. From Fig. 9, the temperature at a point for the fourth solution is found to increase initially, but it decreases for a large value of η . The temperature $\theta(\eta)$ of second solution profile is decreasing near to the shrinking sheet, then the temperature starts to increase significant little away from the shrinking sheet.

The illustrations of $f''(0)$ and $-\theta'(0)$ versus mixed convection parameter λ by adding the effects of suction are shown in Figs. 10 and 11, respectively. In Fig. 10, when the convection tends to become the assisting flow case, all the profiles show the increment in the values of the skin friction coefficient. Some profiles show that the local Nusselt number increases or decreases infinitely when λ closes to 0. The negative values of the skin friction coefficient $f''(0)$ show the occurrence of reverse flow; which means that there is a velocity overshoot in the boundary layer. Heat transfer from the wall to the ambient fluid is in the case of $-\theta'(0) > 0$, whereas reverse heat flow is indicated by $-\theta'(0) < 0$. From Figs 10 and 11, four solutions exist for the variations of $f''(0)$ and $-\theta'(0)$ in the case of $\lambda \neq 0$. But, dual solutions only occur when $\lambda = 0$ in the variations of the local Nusselt number.

TABLE I
 COMPARISON OF CRITICAL SUCTION PARAMETER s_c AND SINGULARITY POINT
 OF SUCTION PARAMETER s_a

	Parameters	
	s_c	s_a
Bhattacharyya [24]	2.266684	-
Rohni et al. [25]	2.2665	Between 2.3378 and 2.3379
Present	2.26662	Between 2.33773 and 2.33776

TABLE II
 VALUES OF SKIN FRICTION COEFFICIENT AND LOCAL NUSSELT NUMBER
 FOR OPPOSING FLOW CASE

s	$f''(0)$		$-\theta'(0)$	
	$H = 0.28$	$H = 0.3$	$H = 0.28$	$H = 0.3$
	()	{ }	{ }	{ }
3.6	2.8475 (-1.7097) {-4.9553} [2.6356]	2.8555 (-1.7113) {-5.0030} [-0.7361]	2.7435 (-5.1319) {1.6824} [8.8545]	2.7448 (-5.2423) {1.6320} [5.9416]
3.7	2.9784 (-2.0471) {-5.3349} [2.6580]	2.9860 (-2.0486) {-5.3839} [-0.7320]	2.8794 (-6.1313) {1.7448} [9.7626]	2.8805 (-6.2529) {1.6940} [6.8006]
3.8	3.1064 (-2.4069) {-5.7441} [2.6131]	3.1135 (-2.4085) {-5.7943} [-0.6967]	3.0117 (-7.2491) {1.8122} [10.7240]	3.0126 (-7.3827) {1.7611} [7.7951]
3.9	3.2318 (-2.7904) {-6.1843} [2.4624]	3.2386 (-2.7920) {-6.2355} [-0.6166]	3.1411 (-8.4961) {1.8839} [11.7213]	3.1419 (-8.6423) {1.8327} [8.9526]
4.0	3.3551 (-3.1985) {-6.6569} [2.1036]	3.3616 (-3.2002) {-6.7088} [-0.4698]	3.2679 (-9.8834) {1.9594} [12.6865]	3.2687 (-10.0430) {1.9080} [10.3084]

* () Second solution, { } Third solution and [] Fourth solution

TABLE III
 VALUES OF SKIN FRICTION COEFFICIENT AND LOCAL NUSSELT NUMBER
 FOR ASSISTING FLOW CASE

s	$f''(0)$		$-\theta'(0)$	
	$H = 0.28$	$H = 0.3$	$H = 0.28$	$H = 0.3$
	()	{ }	{ }	{ }
3.6	3.9287 (-1.2402) {-1.2077} [0.5836]	3.9338 (-1.2318) {-1.1308} [0.2153]	2.8980 (4.3046) {2.0941} [0.1568]	2.8986 (4.4399) {1.8855} [0.3907]
3.7	4.0138 (-1.5867) {-1.6475} [0.6728]	4.0188 (-1.5796) {-1.5899} [0.2564]	3.0161 (5.0442) {2.0933} [-0.2097]	3.0166 (5.1648) {1.9294} [-0.0199]
3.8	4.1001 (-1.9527) {-2.1323} [0.8328]	4.1050 (-1.9468) {-2.0892} [0.3324]	3.1334 (5.8033) {2.1264} [-0.6312]	3.1339 (5.9190) {1.9899} [-0.3924]
3.9	4.1875 (-2.3411) {-2.6564} [1.1156]	4.1923 (-2.3363) {-2.6246} [0.4586]	3.2500 (6.6046) {2.1764} [-1.1236]	3.2505 (6.7198) {2.0587} [-0.8603]
4	4.2758 (-2.7538) {-3.2169} [1.7771]	4.2806 (-2.7500) {-3.1944} [0.6641]	3.3659 (7.4614) {2.2362} [-1.7288]	3.3664 (7.5786) {2.1321} [-1.3992]

** () Second solution, { } Third solution and [] Fourth solution

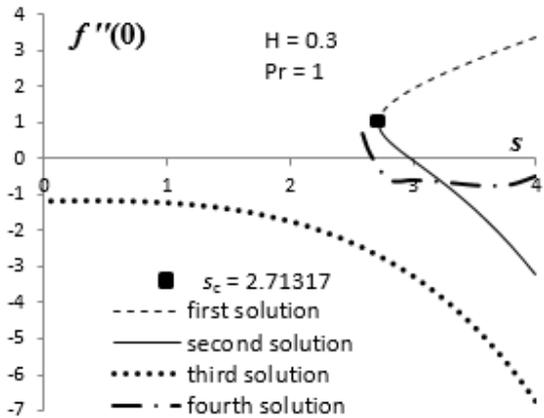


Fig. 2 Variation of $f''(0)$ with the suction parameter s when $\lambda = -0.5$

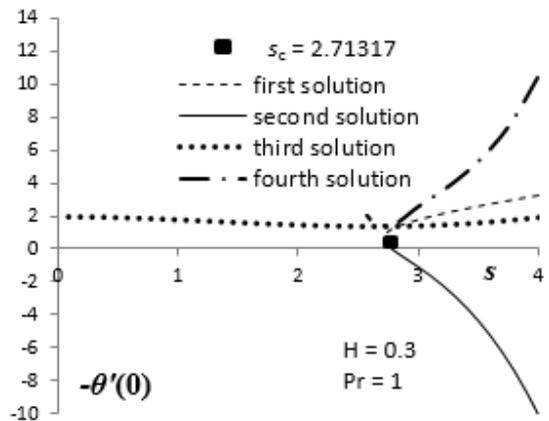


Fig. 3 Variation of $-\theta'(0)$ with the suction parameter s when $\lambda = -0.5$

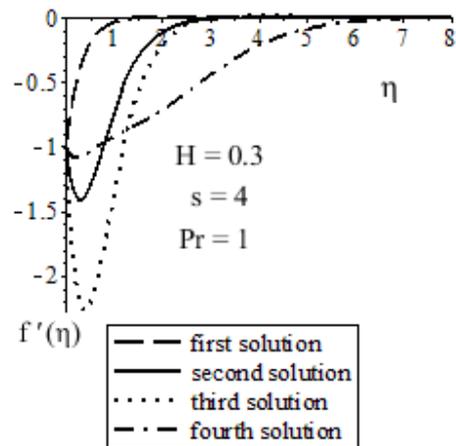


Fig. 4 Velocity profiles when $\lambda = -1$

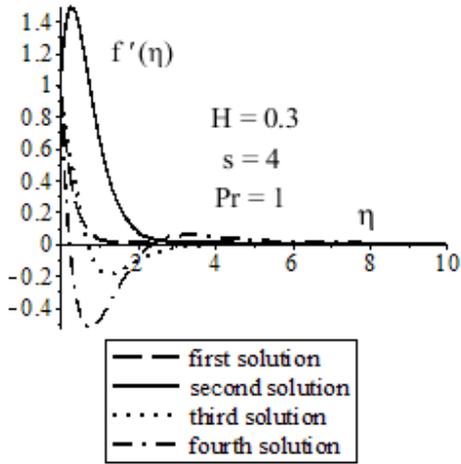


Fig. 5 Temperature profiles when $\lambda = -1$

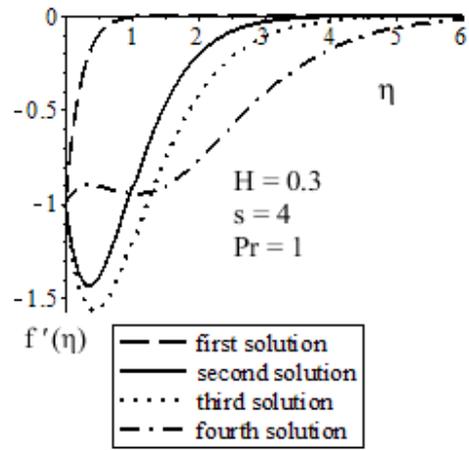


Fig. 8 Velocity profiles when $\lambda = 1$

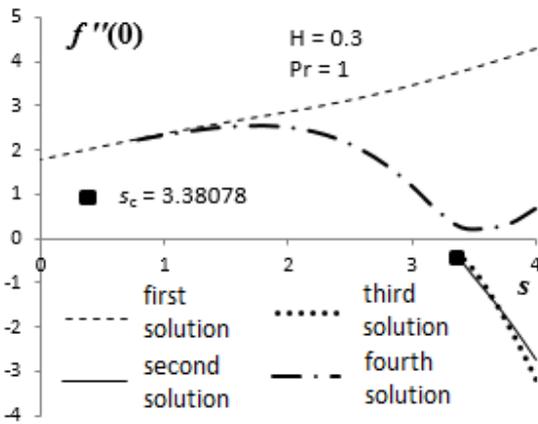


Fig. 6 Variation of $f''(0)$ with the suction parameter s when $\lambda = 1$

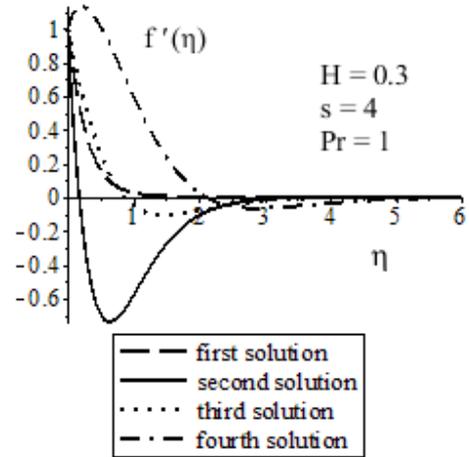


Fig. 9 Temperature profiles when $\lambda = 1$

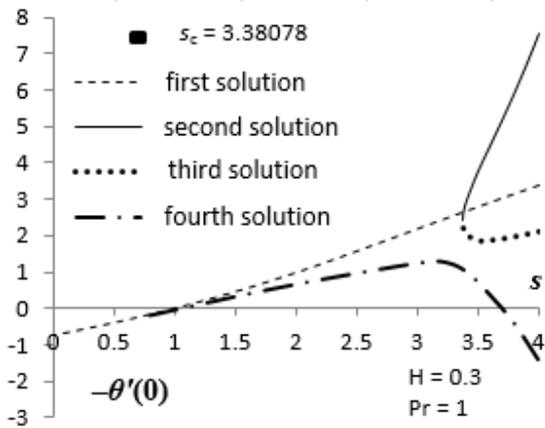


Fig. 7 Variation of $-\theta'(0)$ with the suction parameter s when $\lambda = 1$

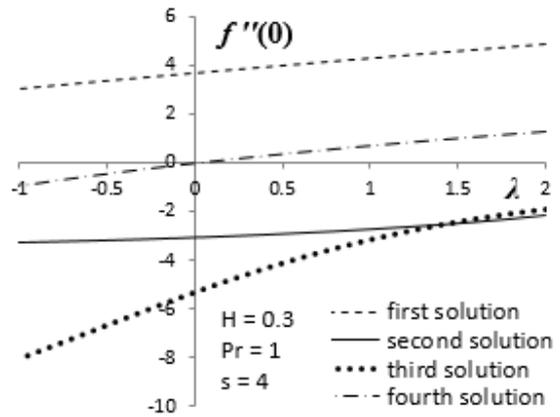


Fig. 10 Variation of $f''(0)$ with mixed convection parameter λ

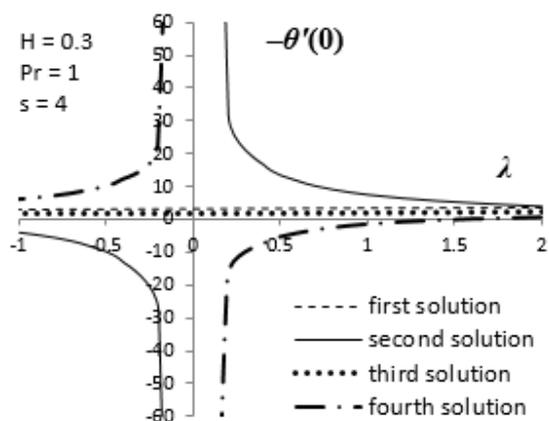


Fig. 11 Variation of $-\theta'(0)$ with mixed convection parameter λ

IV. CONCLUSION

We have studied the problem of mixed convection boundary layer flow over an exponentially shrinking sheet with suction in the presence of magnetic field. The governing partial differential equations are converted into ordinary differential equations by similarity transformation, which is then solved numerically using the shooting method. Numerical results for the skin friction and the local Nusselt number as well as the velocity and temperature profiles are shown in tables and graphs for some values of the governing parameters. Four (multiple) solutions are found for this exponentially shrinking sheet problem.

ACKNOWLEDGMENT

This work was supported by the research grant (FRGS) from Ministry of Education, Malaysia.

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