Analysis of Linear Equalizers for Cooperative Multi-User MIMO Based Reporting System

S. Hariharan, P. Muthuchidambaranathan

Abstract—In this paper, we consider a multi user multiple input multiple output (MU-MIMO) based cooperative reporting system for cognitive radio network. In the reporting network, the secondary users forward the primary user data to the common fusion center (FC). The FC is equipped with linear equalizers and an energy detector to make the decision about the spectrum. The primary user data are considered to be a digital video broadcasting - terrestrial (DVB-T) signal. The sensing channel and the reporting channel are assumed to be an additive white Gaussian noise and an independent identically distributed Raleigh fading respectively. We analyzed the detection probability of MU-MIMO system with linear equalizers and arrived at the closed form expression for average detection probability. Also the system performance is investigated under various MIMO scenarios through Monte Carlo simulations.

Keywords—Cooperative MU-MIMO, DVB-T, Linear Equalizers.

I. INTRODUCTION

Currently insufficiency in radio frequency spectrum has a serious setback because the present spectrum schemes cannot further hold an increase in the number of users. Therefore, CR (cognitive radio) technique is the likely solution to improve spectrum utilization by spectrum sharing. CR is an active wireless communication system that is aware of its location and access the idle radio frequency spectrum when the Primary User (PU) who holds the right to use the spectrum is idle [1]. The users who access the idle/unused spectrum opportunistically is secondary user's (SU) and the SU has to detect the existence of the PU, for that, SU has to sense the spectrum every so often. The challenge in sensing the spectrum is SU should minimize the interference to the PU and should efficiently sense the spectrum holes

Cooperative spectrum sensing improves the detection probability under multipath fading, shadowing and receiver uncertainty by utilizing spatial diversity and it is well discussed in [2]-[5]. Recent research shows much interest in using MIMO for cognitive radio systems. Introducing MIMO system will bring spatial diversity at both transmitter and receiver and achieves diversity gain to improve the error performance and transmits multiple copies of data independently to improve the capacity. Lots of studies are carried out in employing MIMO system in the CR network to improve the detection performance. In [6]-[10], the authors analyzed the performance of SU equipped with multiple antennas to sense the PU signal. Authors in [11]-[13] analyzed spectrum sensing by deploying centralized MIMO system for reporting. In [11], [12] local decision made by SU is reported to the FC and decision fusion is performed by FC to make the final decision wherein [13] soft data fusion is performed.

The IEEE 802.22 standard is aimed at using cognitive radio techniques to utilize the unused television broadcast service spectrum to get broadband access in the rural and remote areas. The spectrum sensing techniques and performance analysis of different sensing schemes for IEEE 802.22 WRAN standard over fading channels is discussed in [14]-[18]. For fixed Broadband Wireless Access (BWA) applications, multiple antenna technologies have significant benefits [6], [19]. Thus the use of cooperative MIMO system can meet a considerable increase in the performance WRAN system [15], [16], [20], [21].

The FC combines the signal received from different SU's using conventional diversity combining schemes like Equal Gain Combining (EGC), Maximum Ratio Combining (MRC) and optimal soft combining [5] and performs energy detection (ED) which is simple and low complex. The ED does not require any information about the PU signal. The statistical and the performance analysis of ED over multipath fading channels are discussed in [22], [23]. In [12], the authors proposed cooperating spectrum sensing and a realistic reporting channel with the MMSE detector at FC to perform decision fusion. In [24], [25] the authors studied transceiver design techniques with MMSE and ZF equalizer to improve the data rate for CR system.

In this paper, we consider a cooperative MU-MIMO based reporting system for IEEE802.22 standard where the PU signal is DVB-T data. We assume the sensing channel as Additive white Gaussian channel and IID Rayleigh reporting channel. The SU forwards the sensed DVB-T signal to the FC on a Time division multiple access basis and the FC performs equalization to minimize the effects due to channel uncertainties and estimates the actual transmitted data by means of Minimum Mean Square Error (MMSE) and Zero forcing (ZF) equalizers to improve the detection performance reasonably over the spectrum of interest. To make the decision on the presence of DVB-T signal FC performs energy detection. We derive a closed form expression for the average detection probability of FC over IID Rayleigh for multiple SU with MMSE and ZF equalizer. Finally, we analyze the detection performance of various multiple antenna scenarios.

The rest of this paper as follows: Section II discusses about the proposed system model. Section III evaluates the average

S.Hariharan is with Electronics and Communication Engineering Department, National Institute of Technology, Tiruchirappalli - 620015, India (e-mail: 408110004@nitt.edu).
P. Muthuchidambaramanathan is with Electronics and Communication Engineering Department, National Institute of Technology, Tiruchirappalli - 620015, India (e-mail: mathuc@nitt.edu).

International Scholarly and Scientific Research & Innovation 8(5) 2014 830 ISSN:000000091950263
detection probabilities of FC over IID Rayleigh for MMSE and ZF. While Section IV outlines the simulation results, Section V concludes the paper.

II. SYSTEM MODEL

![Fig. 1 Cooperative - MIMO reporting system](image)

We consider a CR network of single primary user, K secondary user with single antenna to sense the primary user signal and N_T transmitting antenna to report to the FC and a FC with N_R receiving antenna and equipped with Linear equalizer and energy detector as shown in the Fig. 1. The sensing channel is assumed to be an AWGN for analysis and the reporting channel experiencing IID Rayleigh fading. The signal received at K^{th} SU is given by,

\[ R_k^{(n)} = \left( \begin{array}{c} W_k^{(n)} \\ s_k^{(n)} + w_k^{(n)} \end{array} \right); \mathcal{H}_0 \]

where \( s_k^{(n)} \sim \mathcal{N}(0,1) \) is n^{th} PU sample received at K^{th} SU, \( w_k^{(n)} \sim \mathcal{N}(0,1) \) is the noise signal and both assumed to be circularly symmetrical complex random variable, 1 ≤ k ≤ K and 1 ≤ n ≤ N.

The signal received at the FC is given by,

\[ R_{fc} = \sum_{k=1}^{K} \sum_{n=1}^{N} W_k^{(n)}; \mathcal{H}_0 \]

where \( R_{fc} \) is \([N_R \times N]\); \( W_k^{(n)} \) is \([N_R \times N]\); \( R_k^{(n)} \) is \([N_R \times 1]\) and \( H_k^{(n)} \) is \([N_R \times N_T]\) vectors respectively from K^{th} SU.

The FC performs energy detection and the output of the detector when \( H_k^{(n)} \) is an AWGN channel is given by,

\[ Y = \sum_{l=1}^{N_R} \sum_{j=1}^{N_T} |R_{fc}(l,j)|^2 \]

where Y is the observed energy value at FC. The sum of squares of N Gaussian random variable with zero mean and unit variance follows a central chi-square (\( \chi^2 \)) distribution with 2N degrees of freedom under \( \mathcal{H}_0 \) and noncentral (\( \chi^2 \)) distribution with 2N degrees of freedom and 2N\( \gamma \) non centrality parameter under \( \mathcal{H}_1 \). Adding L IID noncentral (\( \chi^2 \)) variable with 2N degrees of freedom and non-centrality parameter 2N\( \gamma \) will result in another noncentral (\( \chi^2 \)) variable with 2NL degrees of freedom and non-centrality parameter 2NL\( \gamma \). where \( \gamma \) is the instantaneous SNR [22], [23], [26] and L = N_R \times N_T is the number of diversity branches. If K independent SU with N_T transmitting antenna and the FC with N_R antennas, then the total diversity paths will be K \times N_T \times N_R [27], [28] therefore the degrees of freedom and non centrality parameter of the central and noncentral (\( \chi^2 \)) variable will be given as 2NK_L and 2NK_L\( \gamma \) respectively. The Probability Density Function (pdf) of Y can be written as,

\[ f_Y(y) = \frac{1}{\Gamma(NK_L/2) \Gamma(NK_L\gamma/2)} y^{NK_L-1} e^{-2yN\gamma} \]

where \( \Gamma(\cdot) \) is gamma function and \( I_\nu(\cdot) \) is the \nu^{th} order modified Bessel function of the first kind [22], [23].

\[ P_d = \text{Pr}(Y > (\lambda|\mathcal{H}_1)) = Q_{NK_L}(\sqrt{2NK_L\gamma}, \sqrt{\lambda}) \]

where \( \lambda \) is the decision threshold, \( \Gamma(\cdot, \cdot) \) denotes the incomplete gamma function and \( Q_m(a,b) \) is the generalized Marcum Q-function.

A. Average Detection Probability analysis of ZF Equalizer

The FC estimates the signal from K^{th} SU using the weight matrix of ZF and is given by,

\[ W_{zf} = [H_k^{(n)}]^\dagger \]

where \( [H] = (H^HH)^{-1}H \) is the pseudo inverse of the matrix \( H \) [27]. The estimated PU data at FC is given by,

\[ \hat{R}_k^{(n)} = R_{fc}^{(n)} W_{zf} \]

The output of the energy detector under \( \mathcal{H}_1 \) is given by,

\[ Y_{zf} = \sum_{k=1}^{K} \sum_{n=1}^{N} \| \hat{R}_k^{(n)} \|^2 \]

The \( P_d \) of the FC with ZF is obtained as in (5) over an AWGN channel is given by,

\[ P_{d,zf} = \text{Pr}(Y > (\lambda|\mathcal{H}_1)) = Q_{NK_L}(\sqrt{2NK_LY_{zf}}, \sqrt{\lambda}) \]

where \( Y_{zf} \) is the instantaneous SNR of the ZF equalizer.
To obtain a closed form expression of an average detection probability the conditional \( P_d \) over the AWGN channel is convolved with the output SNR distribution. The \( P_f \) of (6) will be the same for any fading channel condition as it is considered noise only criteria and independent of SNR. The pdf of output SNR \( \gamma_{zf} \) of the ZF equalizer over IID Rayleigh with K SU’s is given by [30],

\[
f(\gamma) = \frac{KN_T}{\bar{\gamma}(N_r-KN_T)^!} \left( \frac{KN_T}{\bar{\gamma}} \right)^{N_k-1} \left( 1 - \gamma \right)^{N_k-1-N_rK} \left( 1 - \gamma \right)^{N_r-KN_T} \left( \gamma - \bar{\gamma} \right) \tag{11}\]

where \( \bar{\gamma} \) is the average SNR of each diversity branch. The average detection probability at FC can be obtained by,

\[
\bar{P}_{d, sf} = \int_0^\infty f(\gamma_{zf}) Q_{NKL}(\sqrt{2NKL\gamma_{zf}}, \sqrt{x}) d\gamma_{zf} \tag{12}\]

The integration in (12) can be solved by using [31, eqn. 29] while making the change of variable, \( M = NKL, x = \sqrt{\gamma_{zf}}, \frac{\gamma_{zf}^2}{\bar{\gamma}} = \frac{KN_T}{N_r-KN_T}, p = 2(N_r-KN_T), a = \sqrt{2NKL} \) and \( b = \sqrt{x} \). Finally the average detection probability \( \bar{P}_{d, sf} \) at FC with multiple SU is expressed as in (13), where \( \xi = \frac{2(N_r-KN_T)+1}{2} \), \( F_1(\xi, \ldots, \xi) \) is the confluent hyper geometric function [29], [30], [32]-[34], where \( G_{M-1} \) can be evaluated and obtained as in [23], [29], [31].

### B. Average Detection Probability analysis of MMSE Equalizer

In the case of MMSE, the weight matrix is given by

\[
W_{mmse} = \left[ H_k^{(n)} [H_k^{(n)} + I_n\sigma_0^2]^{-1} \right]^N \tag{14}\]

where \( I_n \) is identity matrix, \( \sigma_0^2 \) is the noise variance, \( [H]^{H} \) is Hermitian transpose of matrix the H. The estimated PU data at FC is given by

\[
\hat{R}_{k, mmse}^{(n)} = [R_{zf}]_k^{(n)} W_{mmse} \tag{15}\]

The output of the energy detector under \( H_t \) is given by,

\[
Y_{mmse} = \sum_{k=1}^{K} \sum_{n=1}^{N} \| \hat{R}_{k, mmse}^{(n)} \|^2 \tag{16}\]

The \( P_d \) of the FC with MMSE is obtained as in (5) over an AWGN channel is given by,

\[
P_{d, mmse} = Pr \{ Y > (\lambda[H_t]) \} = Q_{NKL}(\sqrt{2NKL Y_{mmse}}) \tag{17}\]

\[
P_{d, mmse} = \int_0^\infty N_h (N_r + K - 1) \left( \frac{q^{N_{sa}}}{1 + q^{Y_{mmse}}} \right) (Y_{mmse})^{N_k-1} |C|^{-N_{sa}} \times F_0(N_r + K, T, S) Q_{NKL}(\sqrt{2NKL Y_{mmse}}, \sqrt{x}) dY_{mmse} \tag{22}\]

The average detection probability \( P_{d, mmse} \) at FC with multiple SU can be obtained from (23).

### III. RESULTS AND DISCUSSION

The DVB-T signal of bandwidth 8 MHz operating in 2K mode with QPSK modulation and a code rate of 1/16 is considered. The sensing channel is assumed as AWGN and the reporting channel experiences a Rayleigh fading. The Threshold is obtained from (6) to meet the given false alarm probability of 0.01. The number of Monte Carlo realizations used for calculating the probability of detection is 1000.

We consider the study of performance of different multiple antenna scenario for reporting, In Figs. 2 and 3 we analyzed the performance of ZF and MMSE equalizers respectively, with a single SU for reporting and the sensing time is 15 ms. The observation shows 1x4 system and the MMSE has better performance and also as the number of receiving antenna increased at FC has a better detection probability.

The multiple SU analysis is shown in Figs. 4 and 5 with different multiple antenna scenario and the number of SU considered for analysis is K=5 and sensing time is 15 ms. The observations show MMSE and 1x4 system has better performance similar to single SU case also the MMSE with 1x4 system achieves an interesting detection probability,
according to [35] the minimum requirement for practical implementation of WRAN system the SU have to detect the DVB-T signal at -22.2 dB SNR with detection probability is about 90% and the minimum probability of false alarm is 0.1.

In Fig. 4 the detection probability of 98% is achieved about -23 dB. Thus, by employing multiple antenna at FC in a centralized cooperative reporting system we met the minimum requirement to implement WRAN system practically and also the proposed system shows better performance at lower SNR levels using the energy detector.

**IV. CONCLUSION**

In this paper we have discussed MU-MIMO spectrum sensing based reporting system for detecting DVB-T signal. The SU forwards the received DVB-T signal to the FC and we assume that the reporting channel has no bandwidth limitation and channel state information available at FC. The FC estimates the received PU data with linear equalizers and performs energy detection to make the final decision. The average detection probability of FC with linear equalizer under multiple SU case operating over an IID Rayleigh reporting channel and the AWGN sensing channel is obtained. Finally the performance of the proposed system under different multiple antenna scenario was analyzed through Monte Carlo simulations. As the simulation results met the minimum requirement to implement WRAN system practically, in future the studied system model will be tested on a real time wireless platform.

**REFERENCES**


S. Harilaran received his B.Eng. Degree in Electronics and Communication Engineering from Periyar University, Salem, Tamilnadu, India, in 2004, and the M.Eng. Degree in Communication Systems from Anna University, Chennai, India, in 2006. Currently he is pursuing his Ph.D. in Wireless Communication Networks in the Department of Electronics and Communication Engineering, National Institute of Technology (NIT), Tiruchirappalli, India. He is a recipient of Canadian Commonwealth Scholarship Award-2011 for Graduate Student Exchange Program in the Department of Electrical and Computer Engineering, University of Saskatchewan, Saskatoon, SK, Canada. His research interests include cooperative spectrum sensing, statistical signal processing, orthogonal frequency division multiplexing (OFDM), multiple-input and multiple-output (MIMO) systems and vector quantization.

P. Muthuchidambaranath received his B.Eng. Degree in Electronics and Communication Engineering from Government College of Technology, Coimbatore, India, in 1992, the M.Eng. Degree in Microwave and Optical Engineering, from A.C. College of Engineering and Technology, Karaikudi, India, in 1994. He obtained his Ph.D. degree in optical communication from the National Institute of Technology (NIT), Tiruchirappalli, India. He is a recipient of Indian Commonwealth Scholarship Award-2011 for Graduate Student Exchange Program in the Department of Electrical and Computer Engineering, University of Saskatchewan, Saskatoon, SK, Canada. His research interests include wireless communications, and optical communications. He published his research papers in refereed international journals, and international and national conferences. He is an author of the textbook "Wireless Communications" (published by Prentice Hall of India).