Spectral Broadening in an InGaAsP Optical Waveguide with \( \chi^{(3)} \) Nonlinearity Including Two Photon Absorption

Keigo Matsuura, Isao Tomita

Abstract—We have studied a method to widen the spectrum of optical pulses that pass through an InGaAsP waveguide for application to broadband optical communication. In particular, we have investigated the competitive effect between spectral broadening arising from nonlinear refraction (optical Kerr effect) and shrinking due to two photon absorption in the InGaAsP waveguide with \( \chi^{(3)} \) nonlinearity. The shrunk spectrum recovers broadening by the enhancement effect of the nonlinear refractive index near the bandgap of InGaAsP with a bandgap wavelength of 1490 nm. The broadened spectral width at around 1525 nm (196.7 THz) becomes 10.7 times wider than that at around 1560 nm (192.3 THz) without the enhancement effect, where amplified optical pulses with a pulse width of \( \sim 2 \) ps and a peak power of 10 W propagate through a 1-cm-long InGaAsP waveguide with a cross-section of 4 (\( \mu m \))^2.

Keywords—InGaAsP Waveguide, \( \chi^{(3)} \) Nonlinearity, Spectral Broadening.

I. INTRODUCTION

The rapid increase in data traffic in recent years has been demanding a broad bandwidth to perform multiplex communication in optical fibers, i.e., broadband optical communication [1], [2]. To implement this multiplex communication, several light sources that can generate multiple laser wavelengths, such as distributed-feedback (DFB) laser arrays [3], [4] and supercontinuum light sources [5], [6], have been developed. However, the low device yield of DFB laser arrays with equal-spacing wavelengths is not suitable for practical use, and supercontinuum light sources with precisely-spaced wavelengths are costly because of their dispersion-controlled, tapered optical fibers that are designed specially for the spectral broadening. Instead of these devices, we study an InGaAsP nonlinear optical waveguide to produce a broad spectrum using optical Kerr effect via \( \chi^{(3)} \) nonlinearity, where the InGaAsP waveguide possesses four orders of magnitude larger nonlinear refractive index \( n_2 \) than that of SiO\(_2\) in optical fibers, e.g., those used in the supercontinuum light sources. Several \( n_2 \) values of typical nonlinear optical materials are shown in Table I [7], [8]. Here, \( \alpha_0 \) is the intrinsic optical loss for those materials, and PU-STAD stands for Poly-Urethane containing Symmetrically substituted Tris-Azo Dye. Since semiconductor laser diodes (LDs) and semiconductor optical amplifiers (SOAs) for 1.55-\( \mu m \) optical communication are made of InGaAsP [9], [10], the InGaAsP waveguide is compatible with them and is suitable for integrating all of them on the same chip [11], [12]. A broad spectrum in the InGaAsP waveguide seems to be easily obtained through the same principle of the supercontinuum light sources, but actually, two photon absorption in the waveguide impedes spectral broadening. In what follows, we investigate the competitive effect between the spectral broadening originating from the nonlinear refractive index and the shrinking due to the two photon absorption in the InGaAsP waveguide.

II. NUMERICAL ANALYSIS PROCEDURE

A. Basic Equation Describing the Phenomena

When amplified optical pulses (e.g., amplified pulses from an InGaAsP laser with an InGaAsP amplifier) start to pass through the InGaAsP waveguide, self-phase modulation via \( \chi^{(3)} \) nonlinearity [13] also starts to broaden the spectrum of the optical pulses. But, optical losses in the waveguide, such as the intrinsic optical loss and the two-photon-absorption loss, reduce the pulse power and restrict the size of the spectral broadening. These processes can be analyzed by the following nonlinear Schrödinger equation [13]:

\[
\frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial \tau^2} + \frac{\alpha_0}{2} A = \frac{i \omega_0 n_2}{c A_{\text{eff}}} |A|^2 A - \frac{\alpha_2}{2 A_{\text{eff}}} |A|^2 A, \tag{1}
\]

where \( z \) is the coordinate along the waveguide, \( \tau = t - z/v_g \) is the time measured from the moving frame with a velocity of \( v_g \), \( A = A(z, \tau) \) is the amplitude of the optical pulses, \( \beta_2 = d^2 \beta(\omega)/d\omega^2 \) is the group velocity dispersion, \( \alpha_0 \) is the intrinsic optical loss, \( n_2 \) is the nonlinear refractive index, \( \alpha_2 \) is the two-photon-absorption coefficient, \( A_{\text{eff}} \) is the effective cross-section of the waveguide, \( \omega_0 \) is the center frequency of the optical pulses, and \( c \) is the velocity of light in vacuum.
B. Simplification of the Basic Equation

Equation (1) is simplified for InGaAsP because the dispersion term \((i/2)\beta_2 \partial^2 A / \partial \tau^2\) is ignored when compared with the nonlinear term, e.g., \(i(\omega_0 n_2/c A_{\text{eff}})|A|^2 A\), by the following reason. Here, note that the term \((\alpha_2/2 A_{\text{eff}})|A|^2 A\) is on the same order of magnitude as \(i(\omega_0 n_2/c A_{\text{eff}})|A|^2 A\) and cannot be neglected. As an input pulse, if we employ

\[
A(z, \tau) = A_0 e^{-\left(\frac{\tau^2}{\tau_0^2}\right)} e^{i k z - i \omega_0 \tau},
\]

where \(A_0\) and \(\tau_0\) are constants and \(k\) is a wavenumber, then the ratio \(\eta\) between the nonlinear term and the dispersion term is of the form

\[
\eta = \frac{\text{The nonlinear term}}{\text{The dispersion term}} = \frac{n_2 \omega_0 A_0^2 \tau_0^{2}}{c \beta_2 A_{\text{eff}}},
\]

Substituting typical values \(n_2 \approx 6 \times 10^{-16} \text{ m}^2/\text{W}, A_0^2 \approx 1 \text{ W}, A_{\text{eff}} \approx 4 (\mu\text{m})^2, \beta_2 \approx 1 \times 10^{-24} \text{ s}^2/\text{m}, \tau_0 \approx 1 \text{ ps}, \omega_0 \approx 2\pi \times 193.5 \text{ THz},\) and \(c = 3 \times 10^8 \text{ m/s}\) for (3), we obtain \(\eta \approx 6 \times 10^2\), which means that the dispersion term can be omitted safely in (1).

C. Calculation of the Output Spectrum

To calculate the spectrum of the output pulses in the waveguide, we insert \(A(z, \tau) = |A(z, \tau)| e^{i \theta(z, \tau)}\) into (1) excluding the dispersion term, where \(|A(z, \tau)|\) is the amplitude of the pulses and \(\theta(z, \tau)\) is their phase. We then obtain decoupled equations:

\[
\frac{\partial I(z, \tau)}{\partial \tau} = -\alpha_0 I(z, \tau) - \alpha_2 I^2(z, \tau),
\]

\[
\frac{\partial \theta(z, \tau)}{\partial z} = \frac{\omega_0}{c} n_2 I(z, \tau),
\]

where \(I(z, \tau) = |A(z, \tau)|^2 / A_{\text{eff}}\). Integrating (4) with respect to \(z\), we obtain

\[
I(z, \tau) = \frac{I_0(\tau) e^{-n_2 \alpha_0 z}}{\alpha_0 + n_2 I_0(\tau)(1 - e^{-\alpha_0 z})},
\]

where \(I_0(\tau) = I(z = 0, \tau)\) is the pulse waveform at \(z = 0\). Substituting (6) for (5) and integrating (5) from \(z = 0\) to \(L\) (the total waveguide length), we have

\[
\theta(L, \tau) = \frac{\omega_0}{c} \frac{n_2}{\alpha_2} \ln \left(1 + \frac{n_2 I_0(\tau)}{\alpha_0} \frac{1 - e^{-n_2 \alpha_0 L}}{1 - e^{-\alpha_0 L}}\right).
\]

Finally, we calculate the spectrum \(S(\omega)\) of the output pulses by using the following Fourier transformation:

\[
S(\omega) = \left| \int_{-\infty}^{\infty} A(L, \tau) e^{-i(\omega - \omega_0) \tau} d\tau \right|^2
\]

\[
= \left| \int_{-\infty}^{\infty} |A(L, \tau)| e^{i \theta(L, \tau)} e^{-i(\omega - \omega_0) \tau} d\tau \right|^2.
\]

Here, FFT in C is used in computing (8) [14], and obtained numerical results are shown in the next section.
near the bandgap of InGaAsP, as shown in Fig. 2 [8]. This effect is also observed at other experiments and is utilized for nonlinear switching devices [15], [16]. According to the experiment by Darwish et al. [8], although $\text{Im} \{\chi^{(3)}\} \propto \alpha_2$ is independent of the input wavelength at the measured region of 1510 – 1560 nm, $\text{Re} \{\chi^{(3)}\} \propto n_2$ is dependent on the wavelength and is enhanced as the wavelength is shortened. Using the relations $n_2 = (3/4cn^2\varepsilon_0)\text{Re} \{\chi^{(3)}\}$ and $\alpha_2 = (3\omega_0/2\varepsilon_0n^2\varepsilon_0)\text{Im} \{\chi^{(3)}\}$ ($n$: the InGaAsP refractive index, $\varepsilon_0$: the permittivity in vacuum), we can rewrite (7) as

$$\theta(L, \tau) = \frac{1}{2} \text{Re} \{\chi^{(3)}\} \ln \left(1 + \alpha_2 I_0(\tau) \frac{1 - e^{-\alpha_2 L}}{\alpha_0} \right).$$

A calculated spectrum with (11) for the center frequency $\omega_0$ shifted to 196.7 THz (1525 nm) is shown by the solid line in Fig. 3, where $n_2$ is $2.9 \times 10^{-16}$ m$^2$/W, which is 5 times larger than the dashed line at 192.3 THz (1560 nm). The dashed line in Fig. 3 is the same one as the solid line in Fig. 1, which is placed for comparison in the spectral width. We can see that owing to the $n_2$-enhancement effect, a 10.7 times wider spectral width is obtained when the solid and dashed lines in Fig. 3 are compared at the full width at half maximum (FWHM). If $\omega_0$ is shifted to 198.7 THz (1510 nm), $n_2$ becomes $9.2 \times 10^{-16}$ m$^2$/W. In this case, the spectral width seems to be much more enhanced, but actually, it is limited to $< 10$ nm ($< 1.3$ THz) because of the narrow effective wavelength range ($< 10$ nm) for the large $\text{Re} \{\chi^{(3)}\}$ or $n_2$, as seen in Fig. 2.

Finally, we examine the dependence of the spectral width on the input power. In laboratory experiments, achieving a peak power of 10 W is possible by use of a mode-locked laser with an Er-doped fiber amplifier, but is not easily realized with LDs and SOAs.

For decreased power, Fig. 4 depicts the dependence of the spectral width $W$ (FWHM) on the peak power $P_{\text{peak}}$ for $\omega_0 =$ 196.7 THz (1525 nm) with $n_2 = 2.9 \times 10^{-16}$ m$^2$/W and $\alpha_2 = 2.3 \times 10^{-10}$ m$^2$/W that is normalized by the spectral width $W_0$ (FWHM) for $\omega_0 = 192.3$ THz (1560 nm) with $n_2 = 0.58 \times 10^{-16}$ m$^2$/W and $\alpha_2 = 2.3 \times 10^{-10}$ m$^2$/W. We can see that the magnification factor $W/W_0$ is not linear with respect to the peak power $P_{\text{peak}}$, which is due to two photon absorption, and that, for instance, at a small peak power of $P_{\text{peak}} \approx 2$ W [17], 4.7 times magnification of $W/W_0$ is obtained.

![Fig. 3 Recovery of spectral broadening by the $n_2$-enhancement effect. The solid line represents the case of the center frequency $\omega_0$ shifted to 196.7 THz (1525 nm), where $n_2$ is $2.9 \times 10^{-16}$ m$^2$/W, which is 5 times larger than $n_2 = 0.58 \times 10^{-16}$ m$^2$/W at 192.3 THz (1560 nm), indicated by the dashed line. Here, $\alpha_2$ is $2.3 \times 10^{-10}$ m$^2$/W for both cases.](image)

![Fig. 4 Relation between the peak power $P_{\text{peak}}$ of the input pulse and the spectral width $W$ with $n_2 = 2.9 \times 10^{-16}$ m$^2$/W that is normalized by the spectral width $W_0$ with $n_2 = 0.58 \times 10^{-16}$ m$^2$/W, where $\alpha_2$ is $2.3 \times 10^{-10}$ m$^2$/W for both cases.](image)

### IV. CONCLUSION

We have investigated spectral broadening for amplified optical pulses that propagate through a 1-cm-long InGaAsP waveguide with a cross-section of 4 (μm)$^2$ via $\chi^{(3)}$ nonlinearity. Although the InGaAsP waveguide has a four orders of magnitude larger nonlinear refractive index $n_2$ than that of SiO$_2$ used in optical fibers of supercontinuum light sources, we have seen that two photon absorption in the
InGaAsP waveguide considerably reduces the spectral width. But, using the enhancement effect of $\tau_2$ near the bandgap of InGaAsP, we have observed that spectral broadening is recovered and that the obtained spectral width at 196.7 THz (1525 nm) is 10.7 times broader than that without the enhancement effect at 192.3 THz (1560 nm), where the input pulse has a width of $\sim 2$ ps and a peak power of 10 W. But, since the 10-W peak power is fairly large in achieving with LDs and SOAs, we have examined the spectral-width dependence on reduced peak power, and for a peak power of 2 W, we have obtained a magnification factor of 4.7.

ACKNOWLEDGMENT

The authors would like to thank the Institute of National Colleges of Technology of Japan for financial support. They also thank Dr. E. Shiraki for helpful comments on this research.

REFERENCES


Keigo Matsuura received Associate’s degree from Gifu National College of Technology in 2013. In 2013, he entered Advanced Course of Electronic System Engineering, Gifu National College of Technology to earn Bachelor of Engineering. At this course, he is currently studying nonlinear optical phenomena in III-V semiconductor devices for device applications.

Isho Tomita received Ph.D. in physical science from Waseda University in 2000. In 2000, he joined NTT R&D center and studied nonlinear optical devices, such as ferroelectric and semiconductor wavelength converters. He also studied technologies to generate multi-wavelength laser beams aiming for broadband optical communication and to generate far-infrared laser beams from semiconductors for THz application. He won the 2003 Young Researcher Award of the Institute of Electronics, Information and Communication Engineers of Japan for the development of multi-wavelength light sources. Since 2011, he has been Associate Professor at Department of Electrical and Computer Engineering, Gifu National College of Technology. He is a member of the Japan Society of Applied Physics and the Institute of Electronics, Information and Communication Engineers of Japan.