Abstract—This paper presents the application of finite dynamic programming, specifically the "Markov Chain" model, as part of the decision making process of a company in the cosmetics sector located in the vicinity of Bogotá DC. The objective of this process was to decide whether the company should completely reconstruct its wastewater treatment plant or instead optimize the plant through the addition of equipment. The goal of both of these options was to make the required improvements in order to comply with parameters established by national legislation regarding the treatment of waste before it is released into the environment. This technique will allow the company to select the best option and implement a solution for the processing of waste to minimize environmental damage and the acquisition and implementation costs.

Keywords—Decision making, Markov chain, optimization, wastewater.

I. INTRODUCTION

WATER is fundamental for life. Its preservation and care are necessary to guarantee our survival. Water is a valuable and scarce asset, which makes its appropriate use and recycling obligatory. This is not just because of its essential role in our lives, but also due to the legal requirements that establish criteria which is becoming stricter in order to obtain increased and improved levels of purification of water that has been contaminated by industrial processes [1]. In this sense the implementation of treatment systems for industrial waste has become an evident need among the majority of companies in Colombian industry, especially the cosmetics sector, given that the wastewater released into the environment has a high level of pollutants.

One of the systems that is most frequently employed for the treatment of wastewater in these types of industries is a physical-chemical plant as an alternative tube end technology [2], [3]. This article covers a case study in a cosmetics company in Bogotá D.C., which despite having a wastewater treatment plant (WTP), the company’s continued growth and the need to undertake sufficient additions and improvements to the plant to guarantee the minimal levels of pollutants that are released into the environment.

"Wastewater can be defined as water that comes from the water storage system of a population after having been modified for diverse use in domestic, industrial and community activities..." [4]

Wastewater results from the combination of liquids and solid waste transported by water that comes from residences, offices, commercial buildings and institutions. Wastewater is also produced by industrial and agricultural activities, as well as subterranean, surface or rain water that can also eventually be combined with wastewater.

Industrial wastewater comes from any industrial activities in which water is used in the processes of production, transformation or manipulation, such as liquid waste, water for production processes and drainage water. Liquid waste comes directly from the manufacturing of all types of products. It consists of a water-based solution with a distinct concentration of the product that is being manufactured [5].

Wastewater from the cosmetic industry has a high organic level due to the different materials used in the creation of these products, which translates to an increased chemical oxygen demand (COD). This wastewater is characterized by a high presence of solid waste in suspension, fats, oils and detergents [6], [7]. An especially important characteristic in this type of water is the high COD that is generally related to a high presence of non-biodegradable material and as a result doesn’t respond well to conventional biological treatment.

The treatment of wastewater incorporates physical, chemical and biological processes, which treat and remove physical, chemical and biological pollutants introduced by human use, in this case the industrial use of water. The objective of the treatment is to produce clean water or treated effluent that is reusable in the environment, as well as a residual solid, or what is also called sludge, as part of the final treatment of wastewater [8]-[13].

The industrial wastewater can be treated within the site where it is generated (for example, with septic tanks or other purification methods) or collected and transported through a network of tubes and pumps to an external treatment plant. This is often because the wastewater requires specialist treatment processes, as is the case in this study.

The initial treatment processes are typically referred to as primary treatment or treatment of suspension material. The operations to eliminate this type of contamination are usually
the first to be carried out, due to the fact that the suspended particles can be inconvenient for later processes [14]. The most habitual operations include roughing, which consists of eliminating the larger particles using bars in the area in which the water circulates that are later cleaned with a rake. Sedimentation is a physical operation that takes advantage of gravity so that the densest particles drop to the bottom of the water. Another technique used is filtration, which is an operation in which water passes through a porous medium with the objective of eliminating the highest quantity of material that is suspended. Flotation is a physical operation that consists of generating small air bubbles that compact the particles present in the water and are subsequently lifted to the surface, where they are collected and taken out of the system [15].

The secondary treatment or treatment for dissolved material includes operations such as precipitation, which eliminates undesirable substances through the addition of reactive chemicals that form insoluble compounds. Electro-chemical processes pass an electric current through water that contains electrolytes and provokes an oxidation and subsequent reduction of the undesired material. A third process is the absorption method that attracts soluble material to the surface of a solid, which is generally made of activated carbon.

The third level of treatment or biological treatment includes different processes that use microorganisms and eliminate undesirable material. Bacteria are the most commonly used microorganisms in anaerobic, aerobic, or anoxic processes.

II. METHODOLOGY

A Markov Chain technique was used, which takes its name from the Russian mathematician Andrei Markov, is a succession of similar tests or observations in which each test has the same number of possible finite results and in which the probability of the result of each test depends on the result of the previous test and not on any other previous result [16].

The Markov Chain has been used in industry to solve different types of problems, especially those associated with decision making processes regarding investment, such as the previous test and not on any other previous result [16]. The Markov Chain has been used in industry to solve different types of problems, especially those associated with decision making processes regarding investment, such as equipment or infrastructure replacement [17]-[21].

A Markov Chain is a sequence of random variables, X1,X2,X3,..., with the value of Xn being the state of the process at the time n. If the distribution of conditional probability is Xn+1 in previous states, then it a function of Xn alone, and so:

\[ P(X_{n+1}=x_{n+1}|X_n=x_n,X_{n-1}=x_{n-1},...,X_1=x_1) = P(X_{n+1}=x_{n+1}|X_n=x_n) \]  

in which Xn is the state of the process in the instant i. The state in t+1 only depends on the state t and not on the previous evolution of the system.

A. Markov Chains in Continuous Time

If instead of considering a discreet sequence X1, X2,..., Xn,... within indexed in the group of natural numbers, the random variables are considered to be Xt with t varying on a continuous interval of real numbers, then this is continuous passage of time. For these type of time chains, the Markov property is expressed in the following manner [22]:

\[ P(X(t_{n+1})=x_{n+1}|X(t_n)=x_n,...,X(t_i)=x_i) = P(X(t_{n+1})=x_{n+1}|X(t_n)=x_n) \]  

so that:

\[ t_{n+1}>t_n>t_{n-1}>...>t_1 \]  

A Markov Chain continues with a finite number of states and can be defined in a stochastic matrix, such as:

\[ P(t_1,t_2)=[P_i(T_1,T_2)]_{i,j}=I,...,N, \]  

\[ P_i(T_1,T_2) = P(X(t_i)=j | X(t_j)=i), 0 \geq t_1 < t_2 \]  

The chain is homogenous if P(t1,t2) = P(t1-t2). For a Markov Chain in continuous homogenous time and with a finite number of states the infinitesimal generator can be defined as:

\[ Q = \lim_{h \to 0} \frac{P(h)-I}{h} \]

and can be expressed as a stochastic matrix by:

\[ P(t) = e^{Qt} = \sum_{n=0}^{\infty} \frac{Q^n t^n}{n!} \]  

III. PROBLEM STATEMENT

The company that is the subject of this study has difficulties in complying with environmental laws related to residual wastewater, given that when the services of an external laboratory were obtained to collect and preserve effluent samples, it was found that the results were below the minimum parameters demanded by current legislation. In line with the social commitment of the company, it requires its Wastewater Treatment Plant (WTP) to meet minimum environmental standards so that it doesn’t affect the body of water to which wastewater is released.

Currently, and based on results obtained in the measurements, the company has to decide between two alternatives to improve the efficiency of its WTP, which should be rated between High (H), Medium (M) and Low (L) levels of efficiency. The first possible decision focuses on improving the plant through the inclusion of new pieces of equipment that contribute to improving the plant’s efficiency. The second alternative is the acquisition of a new plant that complies with the wastewater treatment efficiency parameters, so that the mass or concentration of removed waste and the mass of waste that remains in the effluent meet the minimum levels established by law.

Below is a graphic description of the current WTP processes:
With the goal of determining the best alternative for the results of the industrial wastewater treatment undertaken by the company, a dynamic finite programming model was applied to two periods, taking into account the efficiency of the process and the two alternatives established that will in this case be the objectives of the exercise: the optimization and improvement of the WTP or the complete reconstruction of the WTP.

IV. RESULTS

The Markov decision making model consists of transition phases, states, actions, rewards and probabilities. Taking a course of action generates a reward and determines the state of the following decision through a function of the transition probability [23]. This type of process has frequently been used by companies to make decisions that have a significant level of uncertainty [24]-[28]. The process represents reliability in the method that is used and the results that are obtained.

In the current study, and once the results were analyzed, it can be observed that the company needs to undertake a reconstruction of its residual water treatment plant, given that the costs for this are lower for this decision and the decision also complies with the objective of minimizing damage to the ecosystem and meets the standards established by national environmental laws.

The current situation of the company, which is presenting continual growth in its sales that has led to higher levels of production, requires a new plant that guarantees the treatments of new quantities of effluent that the company generates. Not complying with the legislation will result in fines that would cost more to pay than the proposed investment.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PROBABILITIES AND COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision probability</td>
<td>Cost in millions of pesos</td>
</tr>
<tr>
<td>Optimize the WTP</td>
<td>Pij</td>
</tr>
<tr>
<td>0.4</td>
<td>0.35</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Total</td>
<td>0.3</td>
</tr>
<tr>
<td>Reconstruction of WTP</td>
<td>Pij</td>
</tr>
<tr>
<td>WTP</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>ITERATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>V_{i1}</td>
</tr>
<tr>
<td>i</td>
<td>K=1</td>
</tr>
<tr>
<td>High</td>
<td>1 (0.3<em>200)+(0.25</em>100)+(0.45*150)</td>
</tr>
<tr>
<td>Med</td>
<td>2 (0.4<em>250)+(0.35</em>275)+(0.25*180)</td>
</tr>
<tr>
<td>Low</td>
<td>3 (0.4<em>170)+(0.4</em>200)+(0.2*170)</td>
</tr>
</tbody>
</table>

| i | K=2 | V_{i2} |
| High | 1 (0.3*280)+(0.3*160)+(0.4*120) | 180 |
| Med | 2 (0.25*250)+(0.35*185)+(0.4*198) | 206.45 |
| Low | 3 (0.5*300)+(0.3*165)+(0.2*140) | 227.5 |

| i | K=3 | V_{i1} | F_{1(K)} | K* |
| High | 1 152.5+(0.3*180)+(0.25*241.2)+(0.45*227.5) | 360.48 | 386.93 | 2 |
| Med | 2 241.25+(0.4*180)+(0.35*241.25)+(0.25*227.5) | 442.38 | 414.70 | 1 |
| Low | 3 182+(0.4*170)+(0.4*241.25)+(0.2*227.5) | 382.08 | 424.93 | 2 |

| i | K=4 | V_{i2} |
| High | 1 180+(0.3*180)+(0.3*241.25)+(0.4*227.5) | 386.93 |
| Med | 2 206.45+(0.25*180)+(0.35*241.25)+(0.4*227.5) | 414.70 |
| Low | 3 227.5+(0.5*180)+(0.3*241.25)+(0.2*227.5) | 424.93 |
V. DISCUSSION

The results obtained through the use of the finite dynamic programming are optimal in the sense that they minimize the uncertainty of decision making processes, based on probabilities evident in events that have been recently observed. In this sense it is possible that other mathematic models arrive to conclusions that are different to those identified by the authors of this study, even when this doesn’t devalue the use of the technique used in this study.

Contrasting the different options available for the potential investment maximizes and guarantees the achievement of these objectives. As a result, this study provides a grounded option for the company that can be used as a basis of their decision to acquire a new WTP. These decisions are subject to the availability of resources to the company, an aspect that wasn’t taken into account in the development of the model. To date, there are financial options that could validate the decision suggested by the authors of this study.

Even though this study focuses on a cosmetic products company, and in the existing literature no studies were found that are specifically related to the case presented in this paper, different variables have been defined for the application of Markov chains to decision making processes in industry. Leff, Dada and Graves [29] worked on the application of the Markov chains to decision making processes in industry. The application of Markov chains generates a range of options for the solution of problems through the analysis of random variable patterns that form a stochastic process.

For the case of the specific study, the options to either improve or reconstruct the wastewater treatment plant were a decision that the organization hadn’t taken due to the high level of uncertainty involved. The costs associated with either decision were so different that a mathematical simulation was carried out to minimize any qualms they might have had. As a result, the work that was carried out allowed the company to achieve its objectives.

VI. CONCLUSIONS

In the last few decades and as a result of the development of computational tools, the analysis for decision making processes for investment using mathematical models has become generalized, allowing organizations to carry out big investments with tranquility, knowing that the risks have been both analyzed as well as minimized. In the case of the current study, it is evident how this type of tool favors industry in general.

Taking this into account, it has been proved that the specific needs of industry in this area can be satisfied with the application of finite dynamic programming, specifically Markov chains. The application of Markov chains generates a range of options for the solution of problems through the analysis of random variable patterns that form a stochastic process.

REFERENCES

Oscar Vega Camacho was born in Bogotá – Colombia in 1972. Industrial Engineer from “Francisco José de Caldas” University in 1999 and a specialization in production engineering from the same University. Is Master in business administration MBA from Externado University of Colombia and currently developing a Ph.D. in business, economy and society at Alicante University of Spain.

Andrea Vargas Guevara and Ellery Rowina Ariza are industrial engineers.