

# PAPR Reduction of FBMC Using Sliding Window Tone Reservation Active Constellation Extension Technique

V. Sandeep Kumar, S. Anuradha

**Abstract**—The high Peak to Average Power Ratio (PAR) in Filter Bank Multicarrier with Offset Quadrature Amplitude Modulation (FBMC-OQAM) can significantly reduce power efficiency and performance. In this paper, we address the problem of PAPR reduction for FBMC-OQAM systems using Tone Reservation (TR) technique. Due to the overlapping structure of FBMC-OQAM signals, directly applying TR schemes of OFDM systems to FBMC-OQAM systems is not effective. We improve the tone reservation (TR) technique by employing sliding window with Active Constellation Extension for the PAPR reduction of FBMC-OQAM signals, called sliding window tone reservation Active Constellation Extension (SW-TRACE) technique. The proposed SW-TRACE technique uses the peak reduction tones (PRTs) of several consecutive data blocks to cancel the peaks of the FBMC-OQAM signal inside a window, with dynamically extending outer constellation points in active (data-carrying) channels, within margin-preserving constraints, in order to minimize the peak magnitude. Analysis and simulation results compared to the existing Tone Reservation (TR) technique for FBMC/OQAM system. The proposed method SW-TRACE has better PAPR performance and lower computational complexity.

**Keywords**—FBMC-OQAM, peak-to-average power ratio, sliding window, tone reservation Active Constellation Extension.

## I. INTRODUCTION

**F**ILTER Bank Multicarrier with offset quadrature amplitude Modulation (FBMC-OQAM) has attracted increasing attention recently, owing to its high spectrum potency, low side lobes and strength to the narrow-band interference [1]-[4]. Just like different multicarrier systems like OFDM, a basic drawback of FBMC-OQAM systems is that the high peak-to-average power ratio (PAPR) of the signal, that degrades the potency of a high power amplifier. Over the past decade, numerous PAPR reduction techniques for OFDM are proposed, among which tone reservation (TR) [5] - [8] attracted a lot of attention. The TR technique is straightforward, effective and it causes no interference to the Information signal as a result of the similarity between the FBMC-OQAM and OFDM systems, it is natural to think about using TR to scale back the PAPR of FBMC-OQAM

signals. However, FBMC-OQAM signals have a very totally different signal structure compared with the OFDM signals: signals of adjacent information blocks overlap with

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each other for FBMC-OQAM systems, whereas they're independent for OFDM systems. Therefore, directly applying TR schemes of OFDM systems to FBMC-OQAM systems is not effective. In this paper, we propose a sliding window tone reservation Active Constellation Extension (SW-TRACE) technique for the PAPR reduction of FBMC-OQAM signals, called SW-TRACE technique.

## II. SYSTEM MODEL

At the transmitter of a typical FBMC-OQAM system [9], the complex input symbols are written as

$$X_m^n = a_m^n + jb_m^n, 0 \leq n \leq N-1, 0 \leq m \leq \infty \quad (1)$$

where  $N$  is a positive integer.  $a_m^n$  and  $b_m^n$  are the real and imaginary parts of the  $m^{th}$  symbol on the  $n^{th}$  tone, respectively. The  $m^{th}$  symbols on all tones form a data block  $X_m = [X_m^0, X_m^1, \dots, X_m^{N-1}]^T$ . The in-phase and quadrature components are staggered in time domain by  $T/2$ , where  $T$  is the symbol period. Then, the symbols are passed through a bank of transmission filters and modulated using  $N$  tone modulators whose carrier frequencies are  $1/T$ -spaced apart.

Generally speaking, we are more concerned with the reduction of the PAPR of the continuous-time FBMC-OQAM signal. However, most existing PAPR reduction schemes can only be implemented for discrete-time signals. To approximate the true PAPR of the signal, the FBMC-OQAM signal is sampled with sampling period  $T/F$ , where  $F = LN$  and  $L$  is the oversampling factor. It was known in [10] that the PAPR of the sampled signal approximates to the true PAPR of the continuous-time signal very well for OFDM signals when  $L \geq 4$ .

The discrete modulated baseband signal  $s[n]$  of FBMC can be expressed based on the complex modulation symbol  $X_m[k]$  at the  $k^{th}$  subcarrier during the  $m^{th}$  time slot as

$$s[n] = \sum_{m=0}^{\infty} \sum_{k=0}^{N-1} (\theta_k \Re X_m[k] p_0[n - mN] + \theta_{k+1} \Im X_m[k] p_0[n - mN - \frac{N}{2}]) \exp(jk(n - mN) \frac{2\pi}{N}) \quad (2)$$

where

$$\theta_k = \begin{cases} 1, & \text{if } k \text{ is even} \\ j, & \text{if } k \text{ is odd} \end{cases} \quad (3)$$

$j = \sqrt{-1}$  and  $N$  represents the number of available subcarriers. The overlapping ratio of consecutive symbols

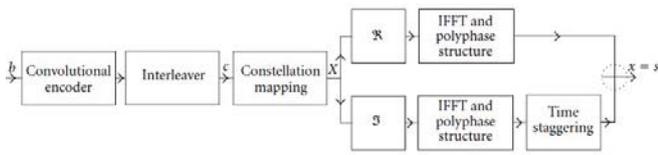


Fig. 1 FBMC transmitter

is strongly related to the length of the prototype filter. For simplicity the filter is designed with an impulse response of length  $K * N$ , meaning that the symbol duration is stretched and K symbols are overlapping in the time domain to prevent data rate loss.

The block diagram of an FBMC transmitter can be seen in Fig. 1. The bit stream  $b$  is encoded to the coded bit stream  $C$ , then the bits are mapped to complex symbols  $X(1)$  according to the modulation alphabet  $A$ . Finally (2) is implemented computationally efficiently using an IFFT and a polyphase decomposition of the modulated prototype filters for the real and imaginary parts. Then the two output signals are time staggered and add

To reduce PAPR, the TR method adds a structured vector  $C = [C_0, C_1, \dots, C_{Z-1}]$  (where  $Z$  is the size of the PRT set) to the data-bearing signal to produce a new composite signal expressed as

$$a = X + C \quad (4)$$

To achieve distortion less data transmission, the data symbol vector  $X$  and the reserved symbol vector  $C$  are constrained to lie into disjoint frequency subcarriers, i.e.,

$$X_m + C_n = \begin{cases} X_n, & n \in D \\ C_n, & n \in D^c \end{cases} \quad (5)$$

where  $D$  denotes the index set of the data-bearing subcarriers and  $D^c$  represents the index set of the PRCs. With this design approach, at the receiver the information symbols are simply recovered by selecting the outputs with indices in the set  $D$ , requiring no extra operation. In addition, no side information requires transmission.

$$C_m^n = 0, \text{ for } n \in D^c, X_m^n = 0, \text{ for } n \in D \quad (6)$$

A TR scheme selects proper  $C_m$  so that the peak power of the signal is greatly reduced, where  $C_m = [C_m^0, C_m^1, \dots, C_m^{N-1}]$ . Since the OFDM signals of the adjacent data blocks do not overlap, all TR schemes for OFDM signals independently determines  $C_m$  for each data block  $m$ . Therefore, directly applying the TR schemes of OFDM systems to FBMC-OQAM systems is thus ineffective due to the reason that signals of adjacent FBMC-OQAM data blocks overlap with each other.

### III. SLIDING WINDOW TR TECHNIQUE FOR THE FBMC-OQAM SYSTEM

This section presents a sliding window TR technique [15] to reduce the PAPR of the FBMC-OQAM signal.

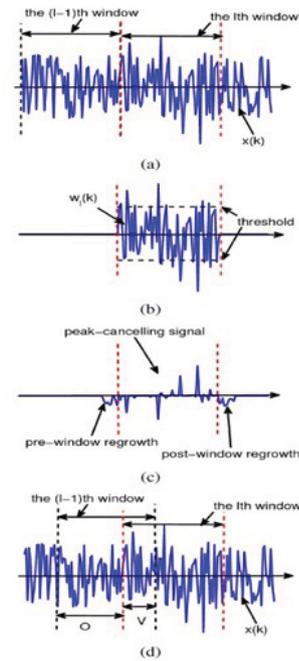


Fig. 2 Decomposition of the sliding window TR technique. (a)  $x(k)$ . (b)  $w_l(k)$ . (c) out of window regrowth. (d) overlapping window SW-TR method

#### A. Sliding Window Tone Reservation (SW-TR)

The basic procedure of the proposed SW-TR technique is: firstly use the PRTs of several consecutive data blocks to cancel the peaks of the FBMC-OQAM signal inside a window, and then slide the window when the threshold of peak or the maximum number of iterations is reached. In this paper, we denote the length of the window by  $W = w * F$ , where  $w$  is an integer that could be used to adjust the length of the window. Due to the overlapping structure of the FBMC-OQAM signal, there are  $P$  adjacent FBMC-OQAM data blocks overlapping with one sliding window in time domain, while the  $P$  data blocks are denoted as the  $G(l)^{th}$  data block,  $(G(l) + 1)^{th}$  data block, ...,  $(G(l) + P - 1)^{th}$  data block, respectively.  $G(l)$  and  $P$  can be readily determined when  $A$  and  $w$  are given. Obviously,  $C_{G(l)}, C_{G(l)+1}, \dots, C_{G(l)+P-1}$  contribute to the peak reduction of the signal in the  $l^{th}$  window. The SW-TR algorithm presented below determines  $C_{G(l)}, C_{G(l)+1}, \dots, C_{G(l)+P-1}$  for the  $l^{th}$  window, and slide the window to the next position.

**step-1:** After the  $(l-1)^{th}$  window is well processed, extract the signal in the  $l^{th}$  window from  $x[k]$  (Fig.2(a)) as shown in Fig. 2(b), and denote it by the following sequence

$$w_l(k) = \begin{cases} x_k, & (l-1)w \leq k \leq lw - 1 \\ 0, & \text{else} \end{cases} \quad (7)$$

**step-2:** Similar to [10], the objective is to clip the amplitude of the signal with a predefined threshold  $B$  in Fig. 2(b)

$$\hat{w}_l(k) = \begin{cases} w_l[k], & |w_l[k]| \leq B \\ B \exp(j\angle(w_l[k])), & |w_l[k]| > B \end{cases} \quad (8)$$

where  $\angle w_l[k]$  is the phase of  $w_l[k]$ . The threshold  $B$  affects the PAPR reduction performance and is always obtained from simulation results. The expected clipping signal corresponding to (8) is  $f_l[k] = \hat{w}_l[k] - w_l[k]$ , i.e.,

$$\hat{w}_l(k) = \begin{cases} 0, & |w_l[k]| \leq B \\ (B - |w_l[k]|) \exp(j\angle(w_l[k])), & |w_l[k]| > B \end{cases} \quad (9)$$

Though  $f_l(k)$  can cancel the peak of the signal to the predefined threshold, it brings interference to the data tones and degrades the bit error rate performance of the system. Therefore, it is better to approximate the clipping signal  $f_l(k)$  to  $\hat{f}_l(k)$ , which only has nonzero signal on the reserved tones. Similar to [11], it needs several iterations to obtain  $C_{G(l)}, C_{G(l)+1}, \dots, C_{G(l)+P-1}$  that produce  $\hat{f}_l(k)$ . The iteration is quite the same as that in the OFDM system [11], the only difference is that the TR technique of the FBMC-OQAM system utilizes the reserved tones of several data blocks rather than those of a single data block. If the maximum number of iterations is reached or the threshold level  $B$  is obtained, the iteration stops.

Then, the peak-canceling signal is

$$\hat{f}_l(k) = \begin{cases} \sum_{m=G(l)}^{G(l)+P-1} c_m[k], & (G(l) - 1)F \leq k \leq (G(l) + P - 1 + A)F \\ 0, & \text{else} \end{cases} \quad (10)$$

where  $c_m[k]$  is the time domain sequence corresponding to  $C_m$

**step-3:** replace  $x[k]$  with

$$x[k] \leftarrow x[k] + \hat{f}_l[k] \quad (11)$$

**step-4:** Slide the window, i.e., let  $l \leftarrow l + 1$ , and got to Step 1.

Since the proposed SW-TR scheme uses the peak reduction tones of several consecutive data blocks to cancel the peaks of the FBMC-OQAM signal inside a window, compared with traditional TR scheme, an extra processing delay of  $(A-1)T$  is incurred ( $A$  represent no succeeding data blocks that are overlapping), where  $AT-1$  is length of the filter and  $T$  is the symbol interval.

#### B. Overlapping Sliding Window to Control the out of Window Peak Regrowth

Obviously, the length of the nonzero part of  $\hat{f}_l[k]$  is larger than that of the sliding window as shown in Fig. 2(c). The out-of-window part of  $\hat{f}_l[k]$  can be represented by

$$\phi_l(k) = \begin{cases} \hat{f}_l[k], & -\text{inf} \leq k \leq (l-1)W - 1 \text{ or } lW \leq k \leq \text{inf} \\ 0, & \text{else} \end{cases} \quad (12)$$

The part of  $\phi_l^1[k]$  before the  $l^{\text{th}}$  window,

$$\phi_l^1[k] = \begin{cases} \hat{f}_l[k], & -\text{inf} \leq k \leq (l-1)W - 1, \\ 0, & \text{else} \end{cases} \quad (13)$$

Overlaps with the signal in the previous window, which could cause regrown peaks and damage the performance of the

proposed scheme. On the other hand, the part of  $\hat{f}_l[k]$  after the window is not a serious issue, since it can be handled by the clipping of the succeeding windows. In this subsection, we propose an overlapping sliding window scheme to control the pre-window peak regrowth, which overlaps the adjacent sliding windows. The length of the overlapping part of two adjacent windows is denoted as  $V$ . In other words, the starting points of two adjacent windows have an offset of  $O$ , where  $O = W - V$ , which is shown in Fig. 2(d). With the overlapping sliding windows, (7) in the SW-TR algorithm is replaced with

$$w_l[k] = \begin{cases} x[k], & (l-1)O \leq k \leq (l-1)O + W - 1, \\ 0, & \text{else} \end{cases} \quad (14)$$

The other steps of the algorithm do not need to be changed. The reason that the overlapping sliding window scheme can help to control the out-of-window regrowth could be explained as follows. After the clipping of the  $(l-1)^{\text{th}}$  window, the peak signal in the overlapping part of the  $(l-1)^{\text{th}}$  and  $l^{\text{th}}$  window is already lower than or approximates to the threshold  $B$ . In other words, the peaks to be clipped in the  $l^{\text{th}}$  window are mainly distributed over the part that is not overlapped with the  $(l-1)^{\text{th}}$  window (the part on the right hand side of the  $l^{\text{th}}$  window as shown in Fig.2(c)). Consequently, the energy of the peak-canceling signal  $f_l[k]$  is mainly distributed in the right hand side of the  $l^{\text{th}}$  window and post-window section.

Therefore, the energy of  $\phi_l^1[k]$  is smaller than that of the method without overlapped window, which leads to less prewindow peak regrowth.

#### IV. ACTIVE CONSTELLATION EXTENSION (ACE)

The basics of this method were investigated for OFDM signals in [13]. The fundamental idea is that once clipping and demodulation, the data symbols value (i.e., the constellation points position) can be altered in a way that the Euclidian distance between the constellation points is increased. In the case of  $X_k^{\text{new}}$  for indices  $k \in N_z$  the values are reset to 0. The data symbols either be reset to their original value  $X_k$ , retain the new (clipped) value  $X_k^c$ , or obtain the new value with a mapping algorithm. During this mapping algorithm a projection method is employed, which is demonstrated for 4-QAM and 16-QAM in Fig. 3 and 4, respectively.

In case of 4-QAM the values  $X_k^c$  originating from the constellation point marked by gray color may fall in the following regions:

- (i) A: values which fall in this region remain unaltered.
- (ii) B: the values are orthogonally projected onto the borderlines of the regions B and A.
- (iii) C: values in this region are reset to their original values  $X_k$ .

For 16-QAM the same rules apply to the four corner constellation points as for 4-QAM. If the clipped symbols originate from the interior constellation symbols marked with black color, they are reset to their original values. For the values  $X_k^c$  originating from the kind of side constellation points marked by gray color in Fig.4 (i.e., symbol constellation border but not corner points) may fall in the following regions:

- (i) D: values falling in this region are reset to their

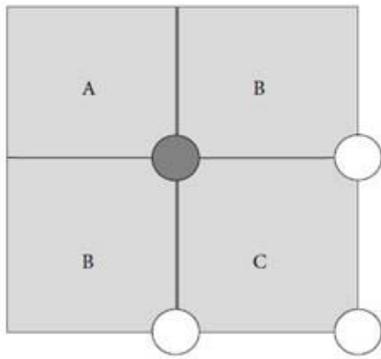


Fig. 3 Constellation diagram and ACE decision regions for the marked symbol of 4-QAM

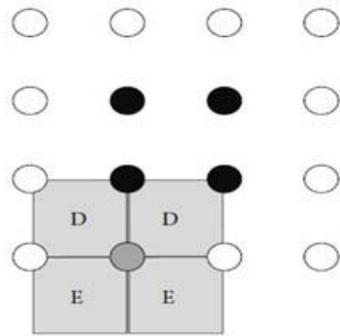


Fig. 4 Constellation diagram and ACE decision regions for the marked symbol of 16-QAM

original values  $X_k$ .

(ii) E: the values are orthogonally projected onto the borderline of the two E regions. Further are techniques for the processing of the clipped symbols are found in [13]. This method reduces PAPR more effectively than the TR scheme while still retaining the BER. On the other hand, this method has a high computational complexity; furthermore it significantly increases the average power of the signal and prevents the use of a soft decision based demodulation in the receiver.

#### V. JOINT IMPLEMENTATION OF SW-TR AND ACE

Since SW-TR and ACE methods may be operated on different subcarriers (having indexes  $N_R$  and  $N_Z$ ), these methods can be applied simultaneously. The joint application of these methods may lead to further improved PAPR reduction performance. However, their simultaneous application also combines their disadvantages and raises the computational complexity at the same time. A similar idea using a signal model slightly different from (8) for OFDM was presented in [14].

#### VI. SIMULATION RESULTS

In this section, simulation results are presented to investigate the PAPR reduction performance of the proposed SW-TRACE

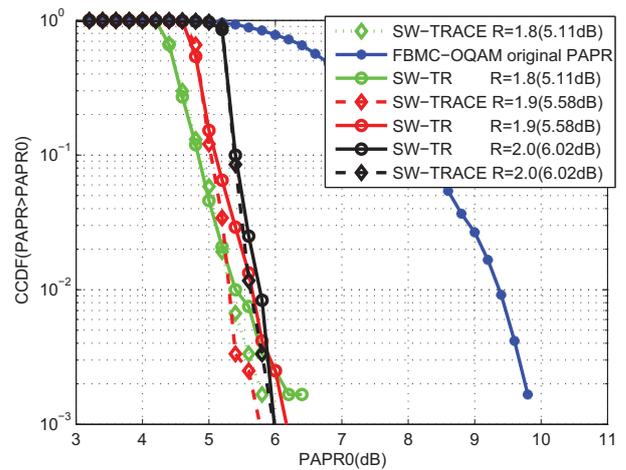


Fig. 5 PAPR Performance of the SW-TRACE method with different threshold R, W=3F and V=2F

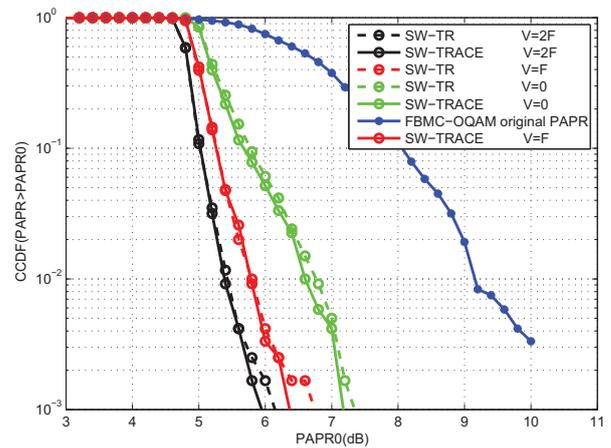


Fig. 6 PAPR Performance of the SW-TRACE method with W=3F and different V, R=5.58dB

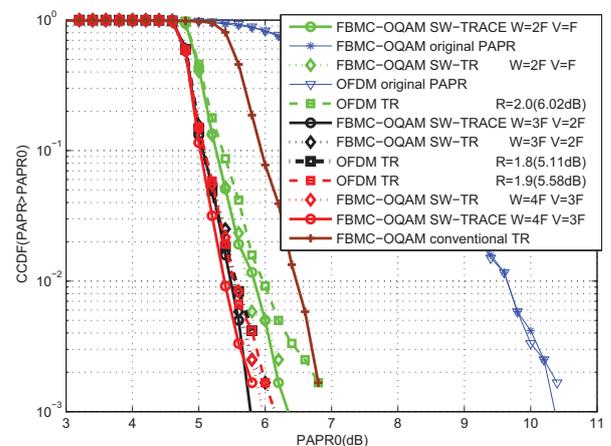


Fig. 7 PAPR performance of the SW-TR method with different W and V, R=5.58dB

technique. During simulation FBMC-OQAM system employs 64 tones, where 56 tones are used for data and eight tones are reserved as PRTs, i.e.,  $Z=8$ . All data tones are QAM modulated. The oversampling factor is  $L=1$ . The PRT tones are selected randomly, since it is known that randomly generated PRT sets and interleaved PRT sets on average [12]. The number of the iterations for the TR schemes is 50 in the simulations. The length of the prototype filter  $h[k]$  is chosen to be  $4F-1$ , i.e., its time duration is about four times of  $T$ . Thus, a FBMC-OQAM data block overlaps with four succeeding data blocks, i.e.,  $A = 4$ . Complementary cumulative distribution function (CCDF) is employed as measurement of PAPR reduction performance in the simulations.

Fig. 5 shows the PAPR performance of the proposed SW-TRACE technique with different thresholds. The curve FBMC-OQAM original PAPR represents the performance of the FBMC-OQAM system without TR. The length of the sliding window is  $W = 3F$ , and the length of the overlapping part is set as  $V = 2F$ . We normalize threshold  $B$  with the power of the signal, and the normalized threshold is denoted as  $R = B/\sqrt{E[|x[K]|^2]}$ . It is observed that the PAPR performance of the SW-TR technique varies with different  $R$ . Among all simulated thresholds, the best PAPR reduction performance in the CCDF range from 0.01 to 0.001 is achieved with  $R = 1.9(5.58\text{dB})$ . Therefore, we fixed  $R = 5.58\text{dB}$  in the following simulations for the SW-TR technique. In simulation of Fig. 6, the length of the overlapping part of two adjacent sliding windows,  $V$ , is varied. The length of the sliding window is  $W = 3F$ , and the length of the overlapping part is set as  $V = 2F$ ,  $V = F$  and  $V = 0$ , respectively, where  $V = 0$  means sliding windows without overlapping. As shown in Fig. 6, the corresponding PAPR at CCDF of 0.001 is 5.9dB, 6.4dB and 7.3dB, respectively. Therefore, it is concluded that overlapping sliding windows is quite effective for the proposed SW-TRACE technique.

Fig. 7 compares the PAPR reduction of the proposed SW-TR technique with different  $W$ . The length of the overlapping part is set as  $V = W-F$ . It is obvious that the best performance is achieved with the largest  $W$ ,  $W = 4F$  and  $V = 3F$  among all simulated parameters. Compared with the PAPR performance of the Conventional TR(C-TR) and SW-TR method for the FBMC-OQAM system, the SW-TRACE method outperforms almost 0.4dB by SW-TR and 2.4dB by C-TR, at CCDF = 0.001, with  $W = 4F$  and  $V = 3F$ . In addition, we compare the PAPR performance of the following two systems: 1) the FBMC-OQAM system with the proposed SW-TR technique; 2) an OFDM system with TR technique, which has the same subcarrier number, PRT tone number, and modulation type with those of the FBMC-OQAM system. As shown in the simulation results, we can draw the conclusion that, the PAPR reduction performance of the proposed SW-TRACE method for the FBMC-OQAM system is even better than that of the TR method for the OFDM system.

## VII. CONCLUSION

Due to the signal structure difference between FBMC-OQAM and OFDM systems, the TR technique

for OFDM systems is not suitable for the FBMC-OQAM system. In this paper methods of SW-TR and ACE and their joint use were investigated for PAPR reduction of FBMC-OQAM systems. The simulation results show that the proposed SW-TRACE technique is effective in reducing the PAPR of the FBMC-OQAM signal.

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