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Possibilistic Aggregations in the Investment Decision Making

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***Abstract*—**This work proposes a fuzzy methodology to support the investment decisions. While choosing among competitive investment projects, the methodology makes ranking of projects using the new aggregation OWA operator – AsPOWA, presented in the environment of possibility uncertainty. For numerical evaluation of the weighting vector associated with the AsPOWA operator the mathematical programming problem is constructed. On the basis of the AsPOWA operator the projects' group ranking maximum criteria is constructed. The methodology also allows making the most profitable investments into several of the project using the method developed by the authors for discrete possibilistic bicriteria problems. The article provides an example of the investment decision-making that explains the work of the proposed methodology.

***Keywords*—**Expert evaluations, investment decision making, OWA operator, possibility uncertainty.

I. Introduction

IN the environment of market economy and competition, investments are exposed to the risk of loss, especially in the sphere of crediting. Hence the issue of increasing the effectiveness of credit policies and lowering credit risks becomes very important [12], [16]-[18].

When there is little or no objective data to make an investment decision, experienced experts are commissioned to solve the problem. In that case, the knowledge and intellectual activity of experts yield expert data [20]. Thus, the analysis of investment projects involves experts' evaluations that may become dominant in the decision making process. Experts' qualitative (verbal) evaluations can be correctly processed by applying possibility analysis [2], [22] and the fuzzy-set approach [1], [5], [6]-[8], [12]-[20].

Very useful approach for decision making under uncertainty is the use of the ordered weighted averaging (OWA) operator, which was introduced by R. R. Yager in [24]. The OWA operator has been studied and applied in a wide range of problems [21], [23]-[25], including the problem of investment decisions ([11] and others).

In [8]-[10] the probabilistic generalization of the OWA operator - POWA is presented. Along with probabilistic generalization we propose the possibilistic generalization of the OWA operator - AsPOWA. For a numerical evaluation of the weighting vector associated with the AsPOWA operator a mathematical programming problem is constructed.

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In this paper the AsPOWA operator in investment decision making to compare investment projects and to make their ranking is used. The description of this approach is given in Section II.

In practice, the capital is frequently invested in several projects simultaneously, each of them requiring a different credit amount. At the same time, the total investment amount is predetermined and fixed. In such cases, it becomes necessary to decide which of the projects and to what extent should share the initial investment amount. On the basis of the AsPOWA operator the projects' group ranking maximum criteria is constructed. Taking into account the levels of ranking of projects' group and also considering initial investment amount bicriteria discrete optimization problem [2]-[4], [16], [19] is applied for the most advantageous investment in several projects simultaneously. Thus, those projects are selected, which possess a maximum ranking of projects' group level and of gaining a maximum profit for the bank. The method is discussed in Section III.

The research of the authors resulted in a new methodology and, consequently, software package development. The software package, which is based on the combined approach, was used in investment tender and supported the decision making. In Section IV the authors provide an example clearly illustrating the work of the proposed methodology.

II. Possibilistic Aggregations in the OWA Operator

It is well recognized that intelligent decision making systems (IDMS) and technologies [20] have been playing an important role in improving almost every aspect of human society. In this type of problem the decision making person (DMP) has a collection $D=\{d_1,d_2,...,d_n\}$ of possible uncertain alternatives from which he/she must select one or some rank decisions by some expert's preference relation values. Associated with this problem as a result is a variable of characteristics, activities, symptoms and so on, acts on the decision procedure. This variable normally called the state of nature, which affects the payoff, utilities, valuations and others to the DMP's preferences or subjective activities. This variable is assumed to take its values (states of nature) in the some set $S=\{s_1,s_2,...,s_m\}$. As a result the DMP knows that if he/she selects d_i and the state of nature assumes the value s_j then his/her payoff (valuation, utility and soon) is a_{ij}. The objective of the decision is to select the "best" alternative, get the biggest payoff. But in IDMS the selection procedure becomes more difficult. In this case each alternative can be seen as corresponding to a row vector of possible payoffs. To make a choice the DMP must compare these vectors, a problem which generally doesn't lead to a compelling solution.

Assume d_i and d_k are two alternatives such that for all $j, j = 1,2,....,m \quad a_{ij} \geq a_{kj}$. In this case there is reason to select d_i. In this situation we shall say d_i dominates d_k $(d_i \succeq d_k)$. Furthermore, if there exists one alternative (optimal decision) that dominates all the alternatives then it will be optimal solution [23]. Facing the general difficulty of comparing vector payoffs we must provide some means of comparing these vectors. Our focus in this work is on the construction of aggregation operator F that can take a collection of m values and convert it into a single value, $F: R^m \Rightarrow R^1$. In [23] Yager introduced a class of mean aggregation operators called Ordered Weighed Averaging (OWA) operator.

Definition 1 [23]: An OWA operator of dimension m is mapping $OWA: R^m \Rightarrow R^1$ that has an associated weighting vector W of dimension m with $w_j \in [0;1]$ and $\sum_{j=1}^{m} w_j = 1,$ such that

$$OWA(a_1,....,a_m) = \sum_{j=1}^{m} w_j b_j \,, \tag{1}$$

where b_j is the jth largest of the $\{a_i\}, i = 1,2,...,m$.

In the role of uncertainty measure a possibility distribution is taken. So, we consider possibilistic aggregations based on the OWA operator. Therefore, we introduce the definition of a possibility measure [2]:

Definition 2: A possibility measure $-Pos$ on 2^S can be uniquely determined by its possibility distribution function $\pi: S \to [0,1]$ via the formula:

$$\begin{aligned} &\forall A \in 2^S \\ &Pos(A) = \max_{s \in A} \pi(s). \end{aligned} \tag{2}$$

Let S_m be the set of all permutations of the set $\{1,2,..,m\}$, Let $\{P_\sigma\}_{\sigma \in S_m}$ be the associated probabilities class [5] of a possibility measure $-Pos$. Then, we have the following connections between $\{\pi_i\}$ and $\{P_\sigma\}_{\sigma \in S_m}$:

$$\begin{aligned} &\forall \sigma \in S_m, \\ &P_\sigma(s_{\sigma(i)}) = \max_{\nu=1,i} \pi(s_{\sigma(\nu)}) - \max_{\nu=1,i-1} \pi(s_{\sigma(\nu)}) \,, \end{aligned} \tag{3}$$

for each $\sigma = (\sigma(1), \sigma(2),...,\sigma(m)) \in S_m$, which are called the associated probabilities [20].

Definition 3: An associated probabilistic OWA operator - AsPOWA of dimension m is mapping $AsPOWA: R^m \Rightarrow R^1$, that has an associated objective weighted vector W of dimension m such that $w_j \in [0,1]$ and $\sum_{i=1}^{m} w_j = 1,$ some possibility measure $Pos: 2^S \Rightarrow [0,1]$ with associated probability class $\{P_\sigma\}_{\sigma \in S_m}$, according to the following formula:

$$\begin{aligned} &AsPOWA(a_1, a_2,..., a_m) = \beta \sum_{j=1}^{m} w_j b_j + (1-\beta) \cdot \\ &M\left(\sum_{i=1}^{m} a_i P_\sigma(s_i) \Big/ \sigma \in S_m \right) = \beta \sum_{j=1}^{m} w_j b_j + \\ &+ (1-\beta) \cdot M\left(E_{P_{\sigma_1}}(a), E_{P_{\sigma_2}}(a),..., E_{P_{\sigma_k}}(a) \right) \end{aligned},$$

where b_j is the jth largest of the $\{a_i\}, i = 1,...,m.$; $E_{P_{\sigma_i}}(a)$ is a Mathematical Expectation of a with respect to associated probability P_{σ_i}, $k = m!$.

We will consider an AsPOWA operator for a mean function M: AsPOWAmean if M = *Mean* in the decision making procedure.

$$AsPOWAmean(a_1, a_2,..., a_m) = \beta \sum_{j=1}^{m} w_j b_j + (1-\beta) \cdot \left(\sum_{i=1}^{k} E_{P_{\sigma_i}}(a) \right) \Big/ k. \tag{4}$$

We obtain the components of vector W by solving following mathematical programming problem:

$$\begin{cases} \max \ H(W) = -\sum_{i=1}^{m} w_i \ln w_i \\ \sum_{i=1}^{m} w_i = 1; & (i) \\ 0 \leq w_i \leq 1, \quad i = 1,\ldots,m; & (ii) \\ \sum_{j=1}^{m} \frac{m-j}{m-1} w_j = \frac{\hat{\alpha}}{\beta} - \frac{1-\beta}{\beta} M\left\{ \sum_{j=1}^{m} P_{\sigma(j)} \cdot \frac{m-\sigma(j)}{m-1} \right\} & (iii) \end{cases} \tag{5}$$

where $\hat{\alpha} = Orness(W)$.

We perform the ranking of pair decisions: $d_i \succcurlyeq d_j$ if

$$AsPOWAmean\,(d_i) \geq AsPOWAmean\,(d_j),$$

where $\succcurlyeq$ is the ranking relation on *D*.

III. Problem of the Investment's Optimal Distribution

Assume that after evaluation the projects with AsPOWA operator, there are n ranking projects, and for each decision (project) d_j the ranking level $\delta_j = AsPOWAmean(d_j)$ of its choice is calculated. We consider the issue of possible financing of the projects in ℓ years.

Let's assume there are additional conditions for financing the projects. In particular, it is known that for financing of j th project $j \in \{1, 2, \ldots, n\}$ within i th year $i \in \{1, 2, \ldots, \ell\}$, a_{ij} monetary units are required, the profit received from implementation of j th project constitutes c_j monetary units, and b_i monetary amount is allocated to finance projects within i th year.

In practice, the amount of funding, as a rule, is insufficient to satisfy all projects. Therefore, it is supposed that for at least one $i \in \{1, 2, \dots, \ell\}$ the inequality $\sum_{j=1}^{n} a_{ij} > b_i$ is true.

Considering the listed restrictions, the question arises as to which of the projects should be financed to achieve maximum investment profits at the minimum risks. We offer the following solution of the problem.

If we introduce a Boolean variables x_j, $j \in \{1, 2, \dots, n\}$ by the following rule

$$x_j = \begin{cases} 1, & \text{if the j-th project is selected for finance} \\ 0, & \text{otherwise}, \end{cases}$$

Then we may present every selected projects' group by the vector $(x_1, x_2, ..., x_n)$. We obtain the following bicriteria Boolean linear programming problem:

$$\begin{cases} \max \sum_{j=1}^{n} \delta_j x_j , & (i) \\ \max \sum_{j=1}^{n} c_j x_j , & (ii) \\ \sum_{j=1}^{n} a_{ij} x_j \le b_i , i = 1, \dots, \ell , & (iii) \\ x_j = 0 \vee 1, \end{cases} \qquad (6)$$

where the criterion (i) represents the decision on the selection of the projects' group with the maximum level of ranking, the criterion (ii) represents the decision on selection of the group of projects giving the maximum profit, while the conditions (iii) corresponds to the financial constraints.

Thus, the objective functions will be:

1) $f_1 = \max \sum_{j=1}^{n} \delta_j x_j$ – selection the projects' group with the maximum ranking level;
2) $f_2 = \max \sum_{j=1}^{n} c_j x_j$ – selection the projects' group ensuring a maximum profit.

To solve this problem we often apply the method developed by the authors for discrete possibilistic bicriteria problems [16], [19].

On the other hand X is the set of all Boolean vectors satisfying the conditions of the bicriteria optimization problem. Then by considering the scalar optimization problem

$$\lambda f_1 + (1-\lambda) f_2 \to \max , \ (x_1, x_2, \dots, x_n) \in X , \ \lambda \in (0,1) , \qquad (7)$$

with conditions (iii), where λ is a weighted parameter, we can find, in the general case, some Pareto optima [2]-[4], [16], [19].

Thus, the bicriteria discrete optimization problem can be solved by linear convolution of criteria.

IV. An Example of the Application of Fuzzy Decision Making Approach

We have developed a software package supporting decision making for optimal credit granting. The decision making block consists of two main soft computing modules: the first provides the software platform for the application of the AsPOWA operator, and the second is used to solve a discrete optimization problem.

The software was tested on concrete data. The required information was provided by the group of 10 experts – expert commission – from the Bank of Georgia and filtered according to our demands after consultations with the managers of the Bank's crediting department.

A. Comparison and Ranking the Projects Using the AsPOWA Operator

In our example the set of alternatives (decisions) $D = \{d_1, d_2, ..., d_n\}$ represents all investment projects - applicants of granting credit.

Let us determine main s_k, $k = 1, \dots, 9$ states of nature, by which the group of all experts will score a candidate seeking a credit: s_1 - business profitability; s_2 - purpose of the credit; s_3 - pledge guaranteeing repayment of the credit; s_4 - credit amount (monetary value); s_5 - payment of interest; s_6 - credit granting date; s_7 - credit repayment date; s_8 - monthly payment of a portion of the principal and accrued interest (repayment scheme); s_9 -per cent ratio of the pledge to the credit monetary amount [13].

Let us assume, that the number of competitors d_i equals to four ($i = 1, 2, 3, 4$).

Suppose that the aggregate table of a_{ij} evaluations of competitors looks like Table I:

TABLE I
The Aggregate Table of a_{ij} Values

D	S								
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
d_1	0.6	0.4	0.1	0.5	0.2	0.6	0.3	0.5	0.2
d_2	0.7	0.1	0.2	0.6	0.8	0.3	0.2	0.3	0.4
d_3	0.5	0.8	0.3	0.3	0.5	0.1	0.3	0.2	0.6
d_4	0.1	0.5	0.4	0.3	0.2	0.4	0.4	0.6	0.6

In our case $m = 9$, $k = 9!$. We take $\beta = 0.3$ and $\hat{\alpha} = 0.6$.

The possibility distribution $\{\pi_i\}$ was calculated using algorithm for determining of membership levels of a fuzzy set [26]:

$$\pi(s_1) = 0.8, \pi(s_2) = 1.0, \pi(s_3) = 0.9, \pi(s_4) = 0.6,$$
$$\pi(s_5) = 0.7, \pi(s_6) = 0.5, \pi(s_7) = 0.4, \pi(s_8) = 0.6,$$
$$\pi(s_9) = 0.7.$$

The solution of the mathematical programming (6) gives us the following components of vector W (see (5)):

$$w_1 = 0.1316, w_2 = 0.1259, w_3 = 0.1205, w_4 = 0.1153,$$
$$w_5 = 0.1104, w_6 = 0.1056, w_7 = 0.1011, w_8 = 0.0968,$$
$$w_9 = 0.0927.$$

Using (3) and (4) we calculate values of the AsPOWAmean operator with respect to all possible decision $d_i, i = 1,...,4.$:

$$\{ d_1/0.3692, d_2/0.3868, d_3/0.4693, d_4/0.3999 \}. \quad (8)$$

If we have to find the unique decision, the final decision will be d_3, because the investment project of the competitor d_3 obtains the maximum value of the AsPOWA operator.

The ranking of possible decisions is following:

$$d_3 \succ d_4 \succ d_2 \succ d_1$$

B. Problem of the Optimal Distribution of Investment

Using (6) we will deal with the bicriteria discrete optimization problem allowing for the most profitable investments into a number of projects.

Bank considers an investment that totals to \$ 120 million over three years ($i = 1,2,3$), \$ 40 million a year ($b_i = 40$).

The values a_{ij} of investments, that are required for j th project in i th year, as well as the c_j magnitudes of profits from the realization of j th project during three years are shown in the following table (see Table II):

TABLE II
THE VALUES OF a_{ij} AND c_j

	Years	Projects			
		d_1	d_2	d_3	d_4
a_{ij}	1	10	6	14	12
	2	4	14	20	16
	3	10	10	20	14
c_j		25	20	35	30

Using (7), the information given in (8) and Table II, we solve (6)-(7) taking for value $\lambda = 0.5$. As a result, we obtain the following set of Boolean variables

$$\{ 1, 1, 1, 0 \}.$$

This means that only three projects - d_1, d_2, d_3 - receive the credit.

At the same time, investment over the years amounted as \$30 million in the first year, \$38 million in the second year and \$40 million in the third year will bring the bank a total profit of \$80 million in three years.

V. CONCLUSION

In this work our focus is directed on the construction of a new generalization of the aggregation OWA operator – AsPOWA in the possibilistic uncertainty environment.

Using the AsPOWA operator we developed the methodology for processing and synthesizing expert information and applied it to the problem of investment decision making. Based on this methodology we have developed software package for decision making which is used to identify high-quality investment projects and to make optimal investment in several of them.

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