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Generalized Maximum Entropy Method for Cosmic Source Localization

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Abstract**—**The Maximum entropy principle in spectral analysis was used as an estimator of Direction of Arrival (DoA) of electromagnetic or acoustic sources impinging on an array of sensors, indeed the maximum entropy operator is very efficient when the signals of the radiating sources are ergodic and complex zero mean random processes which is the case for cosmic sources. In this paper, we present basic review of the maximum entropy method (MEM) which consists of rank one operator but not a projector, and we elaborate a new operator which is full rank and sum of all possible projectors. Two dimensional Simulation results based on Monte Carlo trials prove the resolution power of the new operator where the MEM presents some erroneous fluctuations.

Keywords**—**Maximum entropy, Cosmic source, Localization, operator, projector, azimuth, elevation, DoA, circular array.

I. Introduction

SOURCE localization problem of radiating sources is an active field of research [1]-[6] because of the diversity of its applications such as radar, sonar, radio astronomy, geophysics, positioning systems and so on.

Localization of the wave field by means of DoA techniques has the advantage of no requirement of the synchronization between the sources and the sensors. In the last two decades, one of the famous proposed spectra for DoA problem is the maximum entropy principle [4] which is very performing when the signals of the radiating sources are complex identically distributed random processes which is correct model for cosmic sources [5]. In fact the MEM method has the advantage of high resolution power, in other words it breaks the Rayleigh limit angular resolution of the array which is inversely proportional to the array aperture, the second advantage is that it only requires the inversion of the inter spectral matrix to compute the angular spectrum. The origin of the MEM method is based on an optimization problem where the goal is to search for a vector that minimizes the output power of the array with a constraint that the first element of the target vector equals one. The resulting operator is dyadic product of the optimized vector. In this paper, we briefly describe the MEM mechanism and we present a generalization of the concept where we propose a new operator which is a full rank consisting of the summation of all possible projectors into the complement signal subspace. In the next section we describe the geometry of the problem and the statistical data model, in the third section we present the new formulation and we confirm its validity by some computer simulation in the last section.

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II. Statistical Data Model

In medium of propagation, suppose that we have P statistically independent sources generating a radiation during a time of observation, the resulting electric field [6] at point $\vec{r}$ is given by the partial differential equation as:

$$\Delta \sum_{i=1}^{P} \vec{E}_i(\vec{r},t) = \frac{1}{c^2}\frac{\partial^2 \sum_{i=1}^{P} \vec{E}_i(\vec{r},t)}{\partial t^2} \tag{1}$$

where Δ is the Laplacian operator: $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and c is the velocity of propagation. The solution for one source of the first equation is differentiable equation C^∞ which is given by:

$$\vec{E}_i(\vec{r},t) = \sum_{i=1}^{P} s_i(t) e^{-j(\omega t - \vec{k}_i . \vec{r})} \tag{2}$$

where $\vec{k}_i$ is the i^{th} wave vector. The geometry is illustrated in the Fig. 1 that represents the propagation of one source in (x, y) plane:

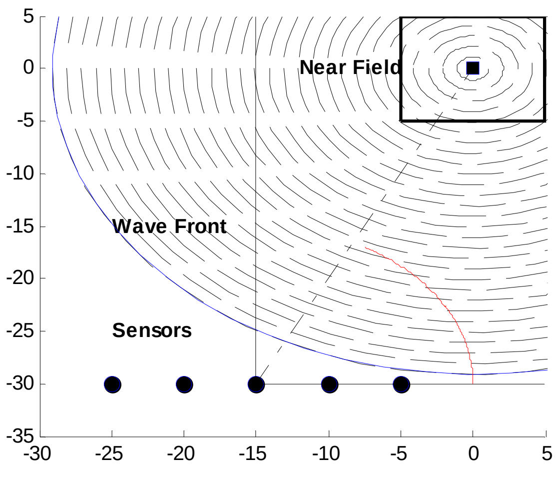

Fig. 1 Propagation model in (x, y) plane

We suppose that we have N sensors to measure the intercepted waves, the measured signals X are proportional to $|\vec{E}|$ after removing the carrier frequencies terms $e^{-j(\omega t)}$ with $\omega = 2\pi f_c$, let us consider that the sensors are isotropic and uniformly placed where the distance between two sensors is $\lambda/2$. During the observation time T, the collected signals at instant t are written by the following equation:

$$X(t) = \sum_{i=1}^{N} \sum_{j=1}^{P} s_j(t) e^{-j\vec{k}_j.\vec{r}_i} + n(t) \quad (3)$$

During the acquisition time we get K samples so $X(t) \in C^{NxK}$, $\vec{r}_i$ is the position of the i^{th} sensor, $s_j(t)$ is the scalar that represents the value of the j^{th} signal at instant t , $n(t)$ is vector of spatially and temporally white noise where it is uncorrelated between sensors and independent of sources. The problem of localization is based on the recovery of the component of the wave vectors $\vec{k}_j$, generally, by means of inter spectral matrix $\Gamma \in C^{NxN}$, its theoretical expression is given by the following equation:

$$\Gamma = \lim_{T\to+\infty} \frac{1}{T}\int_0^T X(t)X^+(t)dt = A\Gamma_s A^+ + \Gamma_n \quad (4)$$

$\Gamma_s \in C^{PxP}$ is the correlation matrix of the signals $s(t)$, $(.)^+$ is the conjugate transpose operator and $A \in C^{NxP}$ is the steering matrix, taking the first sensor as reference, the elements of A are given by:

$$A = \begin{pmatrix} 1 & 1 & \dots & 1 \\ e^{-j\vec{k}_1.\vec{r}_2} & e^{-j\vec{k}_2.\vec{r}_2} & \dots & e^{-j\vec{k}_P.\vec{r}_2} \\ \dots & \dots & \dots & \dots \\ e^{-j\vec{k}_1.\vec{r}_N} & e^{-j\vec{k}_2.\vec{r}_N} & \dots & e^{-j\vec{k}_P.\vec{r}_N} \end{pmatrix} \quad (5)$$

A has Vandermonde structure, $\Gamma_n \in C^{NxN}$ is the correlation matrix of the perturbation noise which expressed in the ideal case as $\Gamma_n = \sigma^2 I_N$, σ is the standard deviation. The quality of Γ estimation is dependent on the number of snapshots, its approximation is given by:

$$\Gamma = K^{-1}\sum_{t=1}^{K} X(t)X^+(t)$$

Note: Due to the models of the parameters $s(t)$ and $n(t)$, the received signals are represented by random matrix $X(t)$ whose joint probability density function is written:

$$p\big(X(t)\big) = \frac{1}{\pi^{NK}\det\{\Gamma\}} e^{(-X^+(t)\Gamma^{-1}X(t))} \quad (6)$$

The spectral decomposition of the matrix Γ is written as a sum of two orthogonal subspaces, called signal and noise subspaces as the following:

$$\Gamma = \sum_{i=1}^{P} \lambda_i u_i u_i^+ + \sum_{j=P+1}^{N} \lambda_j u_j u_j^+$$

where $\lambda_j \cong \sigma^2$ is the noise power which is degenerate $N-P$ times. The largest eigenvalues represent the signal subspace, in descending order, the spectrum of Γ is represented by:

$$\sigma_\Gamma = |\lambda_1 \geq \dots \geq \lambda_P > \lambda_{P+1} \approx \dots \approx \lambda_N >.$$

To detect the column of the matrix A, we search for noise subspace. In compact form, the projectors into the signal and noise subspaces can be written by this equation $U_s U_s^+ + U_n U_n^+ = I_N$, where they form complete base.

The objective herein, is to search for an approximation of the projector into the noise subspace $P_n = I_N - U_s U_s^+$.

III. Generalized Maximum Entropy Operator

The maximum entropy localization technique is based on searching for vector which minimizes the output power of the array as the following:

$$Min\{a^+\Gamma a\} \text{ Subject to } a^+ e_i = 1 \quad (7)$$

where e_i is the i^{th} column of the identity matrix I_N, if we use the Lagrange multiplier we get the function $L(a,\lambda)$ defined by:

$$L(a,\lambda) = a^+\Gamma a - \lambda(1 - a^+ e_i) \quad (8)$$

Searching for the minimum yields to $\frac{L(a,\lambda)}{da} = 0$, since the inter spectral matrix is self adjoint operator, $L(a,\lambda)$ reaches a minimum when we have:

$$2\Gamma a - \lambda e_i = 0 \quad (9)$$

The Lagrange multiplier has the following solution:

$$\lambda = 2/e_i^T \Gamma e_i \quad (10)$$

using the constraint, the optimized vector is obtained as the following:

$$a = \frac{2\Gamma^{-1}e_i}{2e_i^T\Gamma e_i} = \frac{\Gamma^{-1}e_i}{\alpha} \quad (11)$$

where α is constant, the final operator is rank one operator defined as:

$$P_i = aa^+ = \Gamma^{-1}e_i e_i^T \Gamma^{-1} \quad (12)$$

the spatial spectrum is therefore given by the following two dimensional function :

$$f(\theta,\varphi) = \big(a^+(\theta,\varphi)\, P_i\, a((\theta,\varphi)\big)^{-1} \quad (13)$$

The operator P_i is of rank one and is not a projector, its spectrum contains one non zero eigenvalue β:

$$\sigma_{P_i} = |0,0,0,\dots,\beta > \quad (14)$$

For two dimensional localization problem, the operator P_i may not be efficient if there are multiple sources, and this situation can be the same for any index $i = \{1,..N\}$. For this case, we propose a generalization of the MEM principle where each dyadic P_i is transformed into projector.

Following the definition of the spectrum of projector, the spectrum of P_i must contain binaries values, since we have the sum of the eigenvalues as:

$$\sum_{i=1}^{N} \lambda_i = Tr\{P_i\} = \beta \quad (15)$$

The dyadic projector is obtained by $P_i = P_i/Tr\{P_i\}$, now the operator is pure projector such that $\forall\, m \in N, P_i^{\,m} = P_i$ and $\sigma_{P_i} = |0,0,0,\dots,1>$ where the Van Neumann entropy is zero:

$$S(P_i) = -Tr(P_i log(P_i)) = 0 \quad (16)$$

To get full rank operator, we sum over all possible projectors which leads to the following result:

$$Q = \sum_{i=1}^{N} P_i /\ Tr\{P_i\} \quad (17)$$

The final matrix Q is full rank operator characterized by the following spectrum:

$$\sigma_Q = |\lambda_1 \geq \cdots \geq \lambda_{N-P} > \lambda_{N-P+1} \approx \cdots \approx \lambda_N > \quad (18)$$

The new spatial function is computed with new operator Q using (13) which is has more resolution power of detecting the signal subspace $A(\theta,\varphi)$ [7], to prove this remark we present in the next section some numerical results.

IV. Simulation Results

We run some computer simulation to test the performance of the proposed operator, we consider a circular array consisting of $N = 15$ isotropic and identical sensors, 200 samples are collected from three non correlated narrowband sources with positions $(\theta_1 = 15°, \varphi_1 = 50°)$, $(\theta_2 = 40°, \varphi_2 = 44°)$ and $(\theta_3 = 57°, \varphi_3 = 60°)$ with $\lambda = 0.3\,m$. The distance between two consecutive sensors is $\lambda/2$.

The sources have the same power ($\approx 1\,watt$) and the signal to noise ratio is set to $snr = 20log10\left(\frac{1}{0.56}\right) = 5dB$ where the noise is spatially and temporally uncorrelated and the sources are sampled from $s(t) \sim CN(0, I_3)$ which is a model to represent the cosmic sources [5]. We compare the performance analysis of MEM and generalized MEM using Monte Carlo simulation of 1000 trials, the following two graphs represent the average of the realizations for both functions.

In Fig. 2, we remark that the MEM presents some erroneous results due to rank one operator while the generalized MEM gives precise localization because the operator has full rank.

If we compare the spectrum of our operator Q with that of Minimum Variance Distortionless Response (MVDR) [8], we find that there is difference in the magnitude of the eigenvalues, in the third test, we present the sorted eigenvalues of Q and Γ^{-1}, we realize that three first signal eigenvalues of signal subspace are the almost null for both operators while the other eigenvalues belonging to the noise subspace are different in theirs magnitudes.

In the second part of the simulation, we change the configuration of the array, we consider in this case a uniform linear array with the same number of sensors as in the first experiment, the localization problem is focused on azimuth angles only $\theta = [15°, 40°, 57°]$. In Fig. 5, the two functions represent an average of 1000 trials, The magnitudes of the obtained peaks using MEM function are higher than those obtained by the generalized MEM, but for smaller values of the spectra, the functions represent slight differences that can be seen using logarithm scale (dB).

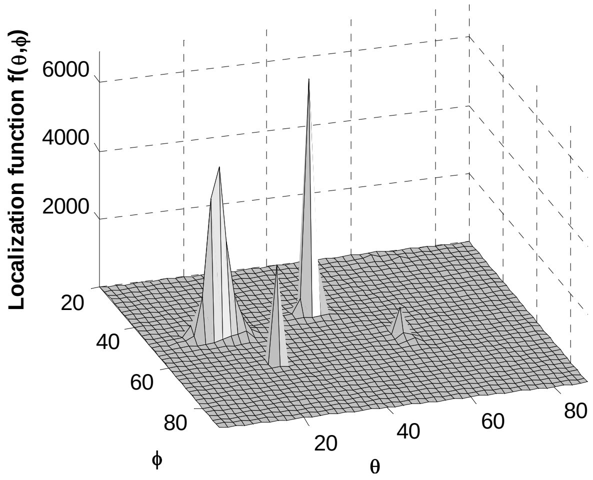

Fig. 2 MEM localization function

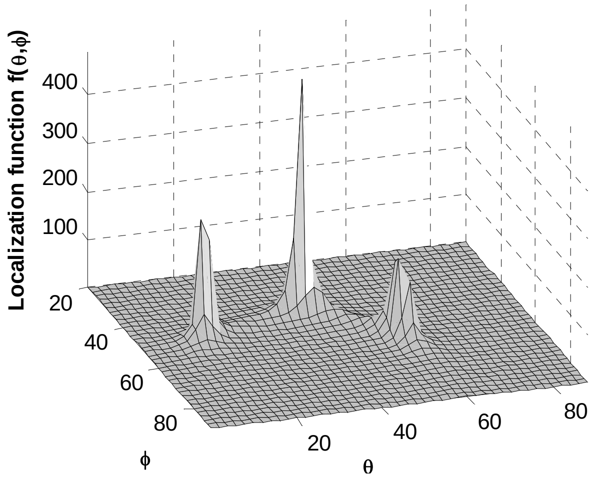

Fig. 3 Generalized MEM localization function

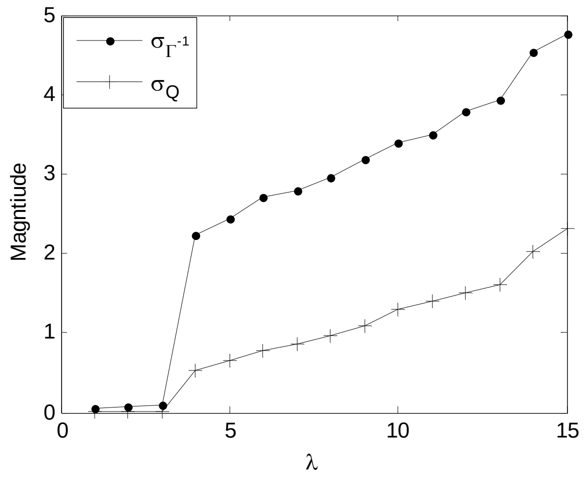

Fig. 4 Eigenvalues of MVDR and proposed operator

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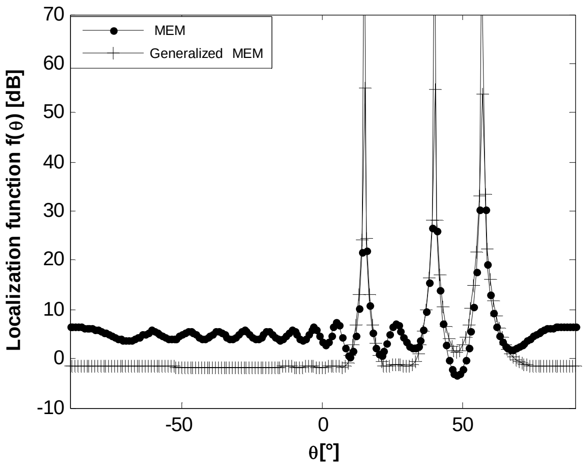

Fig. 5 One dimensional localization, $SNR = 5dB$

V. CONCLUSION

Maximum entropy principle for narrowband DoA problem consists of dyadic operator constructed from any column of the inverse inter spectral matrix when the array is uniform, the technique is robust when the signals of the sources are random which suits the application of DoA in radio astronomy to localize cosmic sources. However if the localization is two dimensional, MEM may give erroneous result, to overcome this issue, we have proposed generalized MEM operator which a sum of all projectors calculated from inverse inter spectral matrix, this new formulation has more resolution power in two dimensional case which is confirmed by simulation results.

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