BasWilCalc – Basket Willow (*Salix viminalis*) Biomass Yield Calculator

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**Abstract**—The aim of the paper was to elaborate a novel calculator BasWilCalc, that allows to estimate the actual amount of biomass on the basket willow plantations. The proposed method is based on the results of field experiment conducted during years 2011-2013 on basket willow plantation in the south-western part of Poland. As input data the results of destructive measurements of the diameter, length and weight of willow stems and non-destructive biometric measurements of diameter in the middle of stems and their length during the growing season performed at weekly intervals were used. Performed analysis enabled to develop the algorithm which, due to the fact that energy plantations are of known and constant planting structure, allows to estimate the actual amount of willow basket biomass on the plantation with a given probability and accuracy specified by the model, based on the number of stems measured and the age of the plantation.

**Keywords**—Basket willow (*Salix viminalis*) biomass, biometric measurements, yield, biomass calculator.

I. INTRODUCTION

**B**IOMASS, being one of the renewable sources of energy, has been of interest to scientists and environmental protection experts. Its use on the global, continental, and national scale has been the topic of a lot of studies for instance in [1]–[8]. Those studies vary in respect of their objectives, approach, and results.

The cooperation of planters of energy crops with the purchasers of biomass in the areas where it is grown makes it necessary to gather information on its current yield on the plantation during the whole period of vegetation. Consequently, for many years now a lot of research centres have been developing mathematical models to estimate the biomass yield from energy crops. Generally, they can be divided into empirical and mechanistic models. The empirical models make use of the data resulting from direct measurements. Their objective is to find relations between the yield of the crops and the selected meteorological as well as soil factors and agrotechnical practices. The mechanistic models, on the other hand, consist in connecting the physiological and morphological features which determine the plant growth. A comprehensive review of the energy crops biomass yield estimation models is presented by Sreeram Nair et al in [9]. For this purpose the models dedicated directly to energy crops are used as well as the adapted existing models of plant yield. Most of those models require many good quality and difficult to get input data regarding for instance the phenological phases, the dynamics of leaf area growth, selected meteorological data and the data on applied agrotechnical practices. Such information is often difficult to get.

This is the reason why the authors of this paper, dealing in their earlier research with modelling the yield size from different cultivated plants (e.g. barley or potatoes) – [10], [11], put forward an innovative method of estimating the current biomass yield on a plantation developed for basket willow (*Salix viminalis*). The method is based only on the results of simple biometric measurements which can be conducted by the planters themselves, practically without any financial expenditure or major difficulties for the plantation. It provided the basis for the development of the plantation actual biomass yield calculator. The estimation can be made both during the vegetation and after it is finished. It is calculated with a set probability, and the size of biomass is determined with the accuracy set by the model, depending on the number of measured stalks.

II. EMPIRICAL DATA

The proposed method was developed on the basis of field studies conducted in 2011-2013 on the basket willow plantation founded in 2010 at the Agro and Hydrometeorology Observatory at Wroclaw University of Environmental and Life Sciences. The facility is located in the south-west part of Poland in Lower Silesia.

In the field experiment basket willow (*Salix viminalis*) is grown on the field of the area of 4.9m × 22.8m in 7 rows in 0.7m intervals; the distance between plants in a row is 0.4m, which gives potentially 392 plants. Their cultivation is conducted with the extensive method. The biomass yield growth model was developed on the results of the measurements taken on the plants in the vegetation periods in 2011-2013 (2nd – 4th year of the plantation). In the analysed years the vegetation began with a varied number of plants, respectively 364 in 2011, 347 in 2012, and 348 in 2013. The measurements regarded all plants with the exception of those growing the outer rows of the field which were considered buffer rows.

Two kinds of measurements are conducted in the ongoing experiment. The first kind destructive measurements, consisting in periodic trimming of randomly selected single shoots (those which were not included in the measurements of the other kind) of the length in excess of 0.5m. The selection

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III. Model

The input data used in building the model included the results of the described in the earlier chapter destructive measurements of the diameter, length, and mass of the willow stalks as well as the biometric measurements of the diameter in the middle of the shoot and the length of the stalks in the vegetation period at weekly intervals. It was assumed that the relation between the stalk mass and its volume does not change its character in the vegetation period. In the paper the two-dimensional gamma distribution [12], [13] was used to estimate mass and the volume index of the basket willow bush stalk (first type of measurements).

\[
q(x, y) = \exp \left( -\frac{p_2x + p_1y}{p_{12}} \right) \frac{x^{y-1}y^{q-1}}{p_{12}^y \Gamma(q)} \Gamma(z) \quad (1)
\]

where \( c = \frac{p_1 p_2 p_{12}}{p_{12}} \), \( q(z) = \sum_{k=0}^{\infty} \frac{\Gamma(z+k)}{\Gamma(z)} \) and \( p_1, p_2, p_{12}, q \) are estimated parameters. Distribution (1) was used to determine the conditional variability of the mass of the whole basket willow bush [14]. The condition is the volume index of one stalk. It was estimated with the use of the first type of measurements as well as the number of stalks in the bush. As the observed characteristic biometric features of the stalks significantly changed in the following years of the plantation, it became necessary to assume that the two-dimensional distribution of mass and volume index of the plant stalk is a mixture of the probability distributions (1):

\[
h(x, y) = \alpha q(x, y) + (1 - \alpha) q_{\alpha}(x, y) \quad (2)
\]

where \( \alpha \) is the estimated mixture parameter. Additionally, it was assumed that the distribution of the stalk number \( W \) in the plant is consistent with the Pascal distribution (negative binomial):

\[
F_W(k) = k = 0, 1, 2, \ldots
\]

where: \( p, v \) are parameters, and \( k + 1 = 1, 2, \ldots \) means the number of stalks in the plant.

On the basis of the conditional distribution \( h(x|y) \) which resulted from distribution (2) the conditional distribution of the mass of a single willow bush was determined. Relevant goodness-of-fit tests demonstrated that this distribution and the Pascal distribution correctly estimates the conditional distribution of the basket willow mass for the two-year-old, the three-year-old, and the four-year-old plantation.

Using the above defined mass distributions and the stalk volume index, the conditional distribution of mass \( M \) of the willow bush was determined with the fixed stalk volume \( y \). That result was generalised to determine the average mass of the willow bush on condition of the measurement of the biometric features of several to about twenty stalks.

IV. Model estimation

Two steps were used to estimate the unknown model parameters.

1) By the application of expectation-maximization algorithm and a hybrid approach [13]–[15] between

**Fig. 1 View of the field in the second and third year of the plantation**

of stalks was conducted in two stages. First, the number of the bush on the field was drawn with replacement, next the number of its shoots was calculated and then a stalk was randomly selected without replacement. It consisted in measuring the length \( h_i \) of \( i \)-th cut shoot, its diameter \( d_i \) in the middle of its length, and then its weighing to determine the mass \( x_i \) of a single shoot. Every time the samples which were taken at the same intervals were of approximately the same number. For each shoot a volume index \( y_i = \frac{1}{3} \pi d_i^2 h_i \) was determined. In this paper instead of shoot volume the volume index is used because the measured mass included both the stalk mass and its leafage. During the whole vegetation period a total of 192 shoots in 2011, 166 shoots in 2012 and 175 shoots in 2013 were cut and measured in the way described above.

The other kind of measurements (biometric non-destructive measurements) were conducted each time on the same 5 willow bushes at weekly intervals. The biometric measurements were started when the plants on the field reached the height of 0.5 m. Each time the number of shoots on the bush was calculated and their length was as well as diameter was measured in the middle of 3 selected (each time the same) and marked shoots.
maximum likelihood and a method of moments the
unknown parameters of distributions (2) were estimated.
2) To estimate the unknown parameters of Pascal
distribution (3) the maximum likelihood method was
used.

In subsequent years (2011-2013) the number of basket
willow stalks showed, as expected, (Fig. 3) the high variability.
Also the positive trend was noticeable. That means, the total
mass of plantation is strongly dependent on the year of
plantation.

V. RESULTS AND CONCLUSIONS

The conducted analyses allowed for the development of
an algorithm which, due to the fact that the plantations of
energy crops have known and uniform planting structure,
allows for the estimation of the current biomass yield on
the plantation with a set probability and accuracy determined
by the model, depending on the number of measured stalks
and the age of the plantation. Fig. 4 presents an example of

There were no reasons to reject the goodness-of-fit
hypothesis on the 0.05 level of significance in each case.
Fig. 2 presents the fitting of both marginal distributions of
(2). The variability of yearly observations and a very good
fitting of compound data were noticed.

such estimations with the measurement of any one, five, and
ten stalks randomly selected from a three-year-old plantation.
The 80% most probable mass areas of the whole bush were
determined.
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REFERENCES