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The Pseudotensors do not Represent

Gravitational Energy

[[1]](#footnote-1)

*Abstract*— It is shown that the energy-momentum pseudotensors of the gravitation field, which were proposed by Einstein or Landau and Lifshitz, give a positive contribution to the expression of the total mass-energy for a gravitation field plus the matter located in it in the case of a quasi-static isolated system. However, as is well known, mass-energy of the gravitation field is negative. So, the pseudotensors incorrectly represent the gravitational field at least in this case.

*Keywords*—conservation law, covariant integration, gravitation field, isolated system.

# Introduction. Energy of the Gravitation Field is Negative

W

hen an electron is attracted to a proton from infinity by the force of the electric field, the energy of the electric field is converted into the kinetic energy, and a part of the electric field is eliminated. As a result, in harmony with the mass-energy conservation law, we have the excitation energy of the atom instead of the energy of the part of the electric field. This means that the total mass-energy of the system is conserved and the mass-energy of the matter is increased due to the disappearance of the field. Then 13.6 eV are radiated, and we get the neutral atom without an external electric field.

Now consider compression of a dilute dust cloud under the action of own gravitational attraction forces. If we adhere to the concept of gravitational field, the cloud is surrounded by its gravitational field. In the compression process, the kinetic energy appears. And this kinetic energy will be converted into the thermal energy when the compression is stopped by the pressure forces. As a result, the mass-energy of the matter is increased in the same way as it is increased at the electron-proton attraction. But in the case of gravitational attraction, the gravitational field of the dust is not eliminated. Instead, the gravitational field strengthened and extends to the volume that has become free from the cloud. So, if we trust that the mass-energy of the system “cloud + its gravitational field” is conserved in the process of compression, we have to ascribe a negative mass-energy to the gravitational field.

Apperdix A in [1] demonstrates the negativity of gravitational energy by the example of a thin sperical shell of mass.

The standard method to calculate the mass-energy of matter, , implies the use of the energy-momentum tensor. The standard method to calculate the mass-energy of gravitational field, , implies the use of the Einstein pseudotensor [2]-[4] or Landau-Lifshitz pseudotensor [5] . The standard method to calculate the total mass-energy of a system “matter + its gravitational field”, , implies the use of the sum . The contribution of the pseudotensor to the mass of the system must be negative, , in order to compensate for the growth of the mass of the matter under compression if we trust that .

# Calculation of the Mass-Energy of Matter

An infinitesimal space element  contans the 4-momentum element of the matter

; (1)

here  is the energy-momentum *tensor density*, and  is the determinant of the metric tensor (see e.g. § 96 in [5]). But a calculation of the total 4-momentum is not possible in general because we cannot sum up vectors (1), which components are defined in different points of a curved space-time. However, in the case of a static body, the infinitesimal 4-momentums (1) are parallel to each other and have only time components  in a static coordinate system. So, in this case it is possible to determine the total mass-energy of a body  by integrating of modules of 4-momentums (1), , over space in this coordinate system:

. (2)

Note, that  is the mass-energy *volume* density (see (95.10) in [3] or § 100 in [5]).

Let us use (2) to calculate the mass-energy of a sphere of perfect fluid. The space-time of such a body is discribed by the Schwarzschild's internal and external solutions of the Einstein equation. The internal solution [6] (see also § 96 in [3]) depends on two parameters,  and :



, (3)

. (4)

Here ,  is the coordinate of the surface where the external Schwarzschild spaceitime is attached, and  is the curvature radius of the inner space determined by the fluid volume density

. (5)

The external solution,

 (6)

depends on the single (Schwarzschild) parameter , which describes external gravitational field. It is seen that at smooth matching

. (7)

In view of the Birkhoff's theorem, the parameter  remains constant under the compression of the sphere.

Now the mass-energy of the sphere can be calculated by using (2):

 (8)

Using (5) and  yields

 (9)

Using the series expansions

,  yields

.(10)

So, . This excess of the matter mass  over the Schwarzschild parameter  was named the (positive) gravitational mass defect (§ 100 in [5]). The point is the parameter  is the *total* mass, i.e. mass of matter *and* of its gravitational field, and this total mass  does not change when compressing of the sphere, according to the Birkhoff''s theorem. This mean that the energy of the gravitation field must be equal  if we wish the total mass, , to be conserved.

# Calculation of the Mass-Energy of the Gravitation Field

Now we must evaluate the integral

, (11)

which is analogical to (2). Here  is the component of the Einstein or Landau-Lifshitz *pseudotensor density*. The Einstein pseudotensor density is [2]-[4]



,

, (12)

The expression (12) is awkward. Luckily, we can use a result of a calculation that was performed by Tolman. For a quasi-static isolated system with respect to quasi-Galilean coordinates, Tolman integrated the sum of *the time components* of the mass-energy matter element  and of the mass-energy field element 

. (13)

The integration gives (see (91.1), (92.4), (97.5) in [3] or [8])

 (14)

where  is pressure.

This integral quantity is not the mass of the system and even is not a component of a vector because it is obtained by adding up the components  belonging to different points where the coordinate bases differ from each other. And there is no coordinate basis to which the quantity (14) could belong as a component. We cannot write  because it is senseless. Nevertheless, this result shows that the integral

 (15)

is *positive* because pressure  is positive. And this means a very important fact: the contribution of the Einstein pseudotensor to the mass of the system, (11), is also positive because :

. (16)

The Landau-Lifshitz pseudotensor gives the same result; it is stated by formula (105.23) in [5].

So, the pseudotensors give positive value for the mass-energy of the gravitational field. The conservation law turns out to be broken.

, , so, . (17)

This means that the pseudotensors do not represent gravitational mass-energy, which is negative. (If we adhere to the concept of gravitational field and trust that the mass-energy of the system is conserved.)

# Why the Invalidity of the Mass-Energy was not Noticed?

Equation (14), , creates an illusion that the pseudotensor  ensures conservation of the sum of the matter mass  and the field mass . But, in reality, the integral (14) does not represent masses at all. Equation (14) is an inadmissible integral of covariant components of the infinitesimal vectors. The total mass  is larger than the quantity (14) because :



. (18)

#  Conclusion

We have to admit that we do not know how to describe the energy of a stationary gravitational field in the frame of the General Relativity. The pseudotensors are not suitable for this.

And here we do not touch the problem of the energy of gravitational waves.

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