Abstract—Drying is a phenomenon that accompanies the hardening of hydraulic materials. This study is concerned with the modelling of drying shrinkage of the hydraulic materials and the prediction of the rate of spontaneous deformations of hydraulic materials during hardening. The model developed takes consideration of the main factors affecting drying shrinkage. There was agreement between drying shrinkage predicted by the developed model and experimental results. In last we show that developed model describe the evolution of the drying shrinkage of high performances concretes correctly.

Keywords—Drying, hydraulic concretes, shrinkage, modeling, prediction.

I. INTRODUCTION

The drying can, if it is not prevented, lead to significant spontaneous dimensional variations, which the cracking is one of events. In this context, cracking promotes the transport of aggressive agents in the material, which can affect the durability of concrete structures.

Drying shrinkage develops over a long period almost 30 years although most occurred during the first three years. Drying shrinkage stabilizes when the material is water balance with the external environment.

The drying shrinkage of cementitious materials is due to the formation of capillary tensions in the pores of the material, which has the consequences of bringing the solid walls of each other [1], [2].

Knowledge of the shrinkage characteristics of concrete is a necessary starting point in the design of structures for crack control. Such knowledge will enable the designer to estimate the probable shrinkage movement in reinforced or prestressed concrete and the appropriate steps can be taken in design to accommodate this movement.

In various fields of science, we often have to explain and / or analyze phenomena whose behavior knows that from experimental measurements. For this reason, it is interesting to synthesize a mathematical model whose behavior is similar to the real phenomenon. In some cases, knowledge of the model parameters and the experimental conditions of phenomena can propose a mathematical model called deterministic model. However, in most cases, it is unclear the precise mechanism of this phenomenon. We can then develop a statistical model which we estimate the parameters from sample measurements. This manner of making has given good results with a study realized by [3].

II. MODELING

In our case, the evolution of drying shrinkage is describe by curves that start with an exponential form and then bend to reach in the end, an asymptotic limit [4], [5]. In mathematics, this type of curve corresponds to the curve of the probability distribution function \( F(t, t_0) \) who gets herself through direct integration with respect to time \( t \) of the probability density function \( f(t, t_0) \).

In the case study, the probability density function \( f(t-t_0) \) is the following mathematical form:

\[
f(t-t_0) = \frac{c}{t-t_0} \times \left( \frac{t-t_0}{t-t_0} \right)^{c-1} \times \exp \left\{ - \frac{(t-t_0)}{(t-t_0)} \right\}
\]

Such as \( t-t_0 > 0 \), \( t_0 \): time of loading, \( f(t, t_0) \) is a probability density function.

Knowing that \( F(t,t_0) \) is the probability distribution function. It is obtained by direct integration with respect to time \( t \) of the probability density function \( f(t, t_0) \) as:

\[
F(t,t_0) = \int f(t, t_0) dt
\]

The plot of \( F(t, t_0) \) begins with an exponential form then inflected to reach in the end, an asymptotic threshold (Fig. 1). Resolution of this equation gives:

\[
F(t,t_0) = -e^{-b(t-t_0)} \bigg|_0^t = 1 - e^{-b(t-t_0)}
\]

To take into account the evolution of the probability distribution function \( F(t, t_0) \) to achieve asymptotic threshold, we multiply (3) by a non-zero positive number \( a \) which gives the form final following:
\[ F(t, t_0) = a \times (1 - e^{-b(t-t_0)}) \]  

(4)

III. ESTIMATION OF MODEL PARAMETERS

For the identification of the model parameters, we used the experimental results. The following relations give the expressions used:

\[
\begin{align*}
    a &= \beta_1 + \beta_2 (HR) + \beta_3 \left( \frac{V/S}{HR} \right) \\
    b &= \beta_4 + \beta_5 (V/S) + \beta_6 \left( \frac{HR}{HR} \right) \\
    c &= \beta_7 + \beta_8 \left( \frac{V/S}{HR} \right)
\end{align*}
\]  

(5)

with: V/S: volume ratio surface (V/S) (mm); RH: relative humidity in%

IV. VALIDATION OF THE MODEL

The model was validated by comparison with experimental results.

The results are summarized in Figs. 2-7.

We observe a perfect concordance between predictions of the model developed and the experimental results.
V. APPLICATION TO THE HIGH PERFORMANCE CONCRETE

Figs. 8-13 give the confrontation between values of drying shrinkage gotten experimentally on the high performance concrete [8], [9] and these predicted by the developed model.

It clearly appears that the developed model permits to predict correctly the drying shrinkage of high performance concrete.

We observe a perfect concordance between predictions of the model developed and the experimental results.

VI. CONCLUSION

The developed model is adapted well to describe the evolution of the drying shrinkage of hydraulic concrete. It has been justified by confrontation to the experimentally results.

The developed model is simple to use and present the advantage to only contain a number limited of parameters.

Results showed that the developed model gave good results with the high performance concrete. However, the developed model can be applied at any type of concrete.
REFERENCES


