Unified Power Flow Controller Placement to Improve Damping of Power Oscillations

M. Salehi, A. A. Motie Birjandi, F. Namdari

Abstract—Weak damping of low frequency oscillations is a frequent phenomenon in electrical power systems. These frequencies can be damped by power system stabilizers. Unified power flow controller (UPFC), as one of the most important FACTS devices, can be applied to increase the damping of power system oscillations and the more effect of this controller on increasing the damping of oscillations depends on its proper placement in power systems. In this paper, a technique based on controllability is proposed to select proper location of UPFC and the best input control signal in order to enhance damping of power oscillations. The effectiveness of the proposed technique is demonstrated in IEEE 9 bus power system.

Keywords—Unified power flow controller (UPFC), controllability, small signal analysis, eigenvalues.

I. INTRODUCTION

DAMPING electromechanical oscillations among connected synchronous generators is essential for secure system functioning. In various conditions, for instance, in heavy loading of power systems, slight signal oscillations appear. Power system stabilizers (PSS) are widely used to damp local and intra-area frequencies [1]. Moreover, FACTS controllers along with complementary controllers with voltage control and power flow are used effectively to damp power system frequencies. The degree of their influence on damping frequencies depends on their proper placement [2]-[6]. Unified power flow controller (UPFC) is used for the simultaneous control of active and reactive power by series voltage injection on line. In addition, in order to stabilize bus voltage for a specific and controllable value on the bus of the beginning of the line, shunt reactive current is injected [7]. This controller can increase the damping of slight signal frequencies in power systems by being located in a proper place. Different methods have been proposed for the placement of FACTS controllers, in which static criteria such as power transfer enhancement, loss minimization, etc. and no dynamic criteria are considered for the proper places of FACTS [8], [9]. In other studies, adjusting FACTS parameter has been proposed for enhancing intra-area frequencies with weak damping [10]-[15]. However, in these methods, no criterion has been proposed for the placement of FACTS controller. Applying controllability and observability criteria is one of the tools proposed for the placement of FACTS devices [16], [17]. In [18], a method for UPFC’s proper place was determined by means of frequency response.

In this study, a technique is proposed for the allocation of UPFC to improve damping of power oscillation and stability. Using controllability index, input control parameter and suitable branch are specified for installing UPFC. The effect of this method is shown by simulation on a 9-bus, 3-machine system.

II. CONTROLLABILITY INDEX

A dynamic system is generally represented as a space as:

\[ \dot{x}_1 = f(x_1, x_2, u) \quad x_1 \in R^n , \quad x_2 \in R^m \]  
(1)

\[ 0 = g(x_1, x_2, u) \quad u \in R^m \]  
(2)

where (1) is a differential equation for dynamic equipments; \( x_1 \) consists of power angle, rotor speed, etc. The second equation is power flow equation: \( x_2 \) consists of domain and phase of node voltage and branch current, etc. and \( u \) is input control vector. By combining (1) and (2) and linearization about the equilibrium point, the following can be written:

\[ \Delta \dot{X} = A \Delta X + B \Delta U \]  
(3)

Assuming that \( u \) is right eigen vector and \( v \) is left eigen vector of matrix \( A \) and in order to omit the mutual conjugation between state variables, a new state variable \( \Delta z \) is located as \( \Delta X = u \Delta Z \) in (3) and this equation results in [9]:

\[ \Delta \dot{Z} = AZ + B' \Delta U \]  
(4)

Matrix \( B' \) which is a combination of the left eigenvector matrix and input matrix is defined as a mode controllability matrix and its row vector \( b_{ki} = v_i^T B_j \) is defined as mode controllability vector \( k \). Domain \( b_{ki} \) shows the degree of input variable controllability \( u_i \) to mode \( k \). The input which has the maximum value \( v_i^T B_j \) is considered the most proper parameter for controlling eigen value \( k \). To control mode \( k \)'s activity, input signals are selected and compared with the amplitude of \( b_{ki} \).

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UPFC around an equilibrium point can be represented in terms of the following state space:

\[
\Delta \dot{X} = A\Delta X + B\Delta U
\]

(5) 

\[0 = C\Delta X + D\Delta U\]

(6)

where:

\[
\Delta X = [\Delta \delta_i \Delta \omega_i \Delta E'_{q_i} \Delta E'_{d_i} \Delta V_f \Delta R_f] 
\]

\[i = 1,2,...,m\]

(7)

\[
\Delta U = [\Delta I_d \Delta I_q \Delta \theta \Delta \delta V_A]
\]

\[k = m + 1,2,...,n\]

(8)

where \(\delta\) is the rotor angle of machine, \(\omega\) is rotor speed, and \(E'_{q}\) and \(E'_{d}\) are induced voltages on axes \(q\) and \(d\), respectively. Also, \(I_q\) and \(I_d\) are stator current components on axes \(d\) and \(q\), respectively, \(E'_{f_d}\) is external excitation voltage, and \(V_{d0}\) is bus complex voltage. Elements of matrices \(A\) and \(B\) correspond to the partial differential dynamic equations of generator and excitations are related to \(X\) and \(U\). Elements of matrices \(C\) and \(D\) correspond to the partial algebraic differential equations of \(X\) and \(U\) [19].

A. UPFC model

UPFC is a combination of two converters of VSC voltage source which have a common capacitor on the DC side. In Fig. 1, a schematic diagram of UPFC is shown. Control diagram block of series and parallel converters is shown in Fig. 2. Shunt converter output can be applied as the voltage source in the power system to bus \(k\) (with the adjustable amplitude and phase \((V_{sh}, \delta_{sh})\)) by a controller to check voltage amplitude at this bus at a specific amount and keep DC voltage fixed. Series converter also injects a voltage with adjustable amplitude and angle \((V_{sc}, \delta_{sc})\) through the transmission line by a controller to monitor active and reactive power of line. \(V_p(t)\) and \(V_{prf}\) are bus \(k\) voltage's phasors and its reference, \(V_{dc}\) and \(V_{dcref}\) are DC voltage and its reference, \(V_{m}\) is bus \(m\)'s voltage phasor, \(I_p(t)\) and \(I_{prf}\) are phasors of series and parallel converter currents, respectively, and \(Q_{ref}\) and \(P_{ref}\) are active and reactive reference power, respectively. Output signals of UPFC \((V_{se}, V_{sh}, \delta_{sc}, \text{and} \delta_{sh})\) can be the control input signals of power system. Therefore, dynamic model of UPFC can be derived as shown in Fig. 3. \(Z_{se}\) and \(Z_{sh}\) are the impedance transformers of series and shunt converters, respectively.

B. Controllability index for UPFC location

It is assumed that UPFC is located between two buses \(k\) and \(m\) as demonstrated in Fig. 3; active and reactive power equations of two buses are as follows: Active power at bus \(k\):

\[
P_k = V_k^2 G_{km} + V_k V_m G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m) + V_k V_{se} G_{km} \cos(\theta_k - \delta_{sc}) + B_{km} \sin(\theta_k - \delta_{sc}) + V_k V_{sh} G_{sh} \cos(\theta_k - \delta_{sh}) + B_{sh} \sin(\theta_k - \delta_{sh})
\]

(9)

Reactive power in bus \(k\):

\[
Q_k = -V_k^2 B_{km} + V_k V_m G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m) + V_k V_{se} G_{km} \sin(\theta_k - \delta_{sc}) - B_{km} \cos(\theta_k - \delta_{sc}) + V_k V_{sh} G_{sh} \sin(\theta_k - \delta_{sh}) - B_{sh} \cos(\theta_k - \delta_{sh})
\]

(10)

Fig. 1 Schematic diagram of UPFC

Fig. 2 Control diagram block of UPFC: (a) Series converter, (b) Parallel converter

Active power in bus \(m\):

\[
P_m = V_m^2 G_{mm} + V_m V_k G_{mk} \cos(\theta_m - \theta_k) + B_{mk} \sin(\theta_m - \theta_k) + V_m V_{se} G_{mm} \cos(\theta_m - \delta_{sc}) + B_{mk} \sin(\theta_m - \delta_{sc})
\]

(11)

Reactive power in bus \(m\):

\[
Q_m = -V_m^2 B_{mm} - V_m V_k G_{mk} \sin(\theta_m - \theta_k) - B_{mk} \cos(\theta_m - \theta_k) - V_m V_{se} G_{mm} \sin(\theta_m - \delta_{sc}) - B_{mk} \cos(\theta_m - \delta_{sc})
\]

(12)

In the above equations, admittance between buses \(k\) and \(m\) is represented as \(Y_{km} = G_{km} + jB_{km}\). By linearizing (9)-(12), algebraic differential equations of network (6) are changed as:

\[
0 = C\Delta X + D\Delta U + F\Delta U_{upfc}
\]

(13)

where \(\Delta U_{upfc} = [\Delta \delta_{se} \Delta \delta_{sh}]\), elements of matrix \(F\) correspond to partial algebraic differential equations to \(\Delta U_{upfc}\), and \(D'\) is the modified matrix \(D\) because of placing UPFC in the power system. By solving (5) and (13):

\[
\Delta \dot{X} = \left(A - BD'^{-1}C\right)\Delta X + \left(-BD'^{-1}F\right)\Delta U_{upfc}
\]

(14)

Equation (14) can be written as:
Controllability of UPFC on line 1 for $\Delta U_{\text{upfc}}$ to mode $k$ is defined as:

$$b_{ki} = v_k^T F_1$$  \hspace{1cm} (16)

where $b_{ki}$ is a $1 \times 4$ vector and the size of each of its elements expresses the controllability of $\Delta U_{\text{upfc}}$ on line 1 to mode $k$. This vector is calculated for all the system lines (except the line with the transformer) and, considering large elements of this vector, the line which is more proper for installing UPFC is determined.

**IV. SIMULATION RESULTS**

To assess the effectiveness of the proposed method, we analyze three machine power system of Fig. 4. The system data are given in [19]. Small signal analysis reveals the important information about oscillatory modes. The above system consists of 3 generators, all of which are equipped with the excitation system. Each generator with its excitation has 7 state variables; so, the rank of state matrix $\mathbf{A}$ is 21. According to the small signal analysis for this system, oscillatory modes with weak damping are determined. Table I shows the corresponding eigen values with electromechanical oscillatory modes to a specific point. More information about oscillatory modes is obtained by participation coefficients and mode shape. Participation coefficient represents state variable's activity in the given mode. Participation coefficient of state variable $k$ to mode $i$ is defined as:

$$p_{ki} = \frac{v_k^T \delta_i}{v_k^T v_k}$$  \hspace{1cm} (17)

where $v_k$ and $v_k$ are right and left eigen vectors to mode $i$, respectively. Among the oscillatory modes, two modes of 1.99 Hz and 1.17 Hz have weak damping and, consequently, these two are considered. Table II shows the participation coefficients of state variables to these two modes. According to the table, it is observed that the state variables of stimulation system do not play an important role in electromechanical oscillatory modes. Therefore, it is possible to neglect these variables and, in most of the studies, the rank four model of generator is used. Large participation coefficients $\delta_{G2}$ and $\omega_{G2}$ (0.371) show generator activity $G_2$ in mode 1.17 Hz and large participation coefficients $\delta_{G3}$ and $\omega_{G3}$ (0.381) represent generator activity $G_3$ in mode 1.99 Hz. $\delta_{G2}$ is determined by observing amplitude and right eigenvector size (mode shape) corresponding to state variable $\delta_{G2}$, in which two above-mentioned oscillatory modes correspond to the rotation of generator $G_2$, unlike that of generator $G_3$. Mode shapes corresponding to two oscillatory modes are shown in Figs. 5 and 6.

<table>
<thead>
<tr>
<th>Eigenvalues $\lambda$</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7194±j12.5502</td>
<td>1.99</td>
<td>0.057</td>
</tr>
<tr>
<td>-0.2872±j7.3645</td>
<td>1.17</td>
<td>0.038</td>
</tr>
<tr>
<td>-5.4605±j8.6669</td>
<td>1.25</td>
<td>0.57</td>
</tr>
<tr>
<td>-5.3775±j8.9330</td>
<td>1.25</td>
<td>0.56</td>
</tr>
<tr>
<td>-5.2370±j8.3930</td>
<td>1.24</td>
<td>0.55</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Participation Coefficients of 9-Bus, 3-Machine System</th>
</tr>
</thead>
<tbody>
<tr>
<td>State variable</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>$\delta_{G2}$</td>
</tr>
<tr>
<td>$\omega_{G2}$</td>
</tr>
<tr>
<td>$E'_{\delta G2}$</td>
</tr>
<tr>
<td>$E'_{\omega G2}$</td>
</tr>
<tr>
<td>$E'_{\delta G3}$</td>
</tr>
<tr>
<td>$V_{BG2}$</td>
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<tr>
<td>$R_{F G2}$</td>
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<td>$\delta_{G3}$</td>
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<td>$E'_{\delta G3}$</td>
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<td>$E'_{\omega G3}$</td>
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<td>$E'_{\delta G3}$</td>
</tr>
<tr>
<td>$V_{BG3}$</td>
</tr>
<tr>
<td>$R_{F G3}$</td>
</tr>
</tbody>
</table>
Controllability indices corresponding to two oscillatory modes with weak damping are calculated for each of the control inputs of UPFC (Δδₑ, ΔVₑ, Δδᵣ, ΔVᵣ) on different lines of system (except lines with the transformer) and are given in Tables III and IV. Table III shows the controllability indices of different lines of UPFC corresponding to mode 1.17 Hz. These indices are expressed as normalized forms. It should be noted that, on line i→j, series convertor of UPFC is connected to bus j and parallel convertor of UPFC is connected to bus i. From Table III, it is observed that the line between two buses 7 and 8 has the maximum amount of controllability index for the place of UPFC than input control signal ΔVₑ. Thus, the most appropriate place for installing UPFC is line 7-8 and ΔVₑ is the best signal for controlling.

Controllability index corresponding to oscillatory mode 1.99 Hz is also computed for the place of UPFC and is given as normalized in Table IV. It can be observed in Table IV that the lines between buses 8 and 9 and buses 7 and 8 have the maximum value of controllability index and, therefore, are the most proper places for installing UPFC. Hence, the line between buses 7 and 8 that has a good controllability index for both oscillatory modes is selected as the most proper place for UPFC. ΔVₑ is the most proper signal for controlling, too.

Since two oscillatory modes are more related to generators G₂ and G₃, Tables III and IV show that buses 7 and 9 which are the closest ones to the generator have the maximum controllability index for UPFC’s place.

To verify the goodness of these results, a three phase fault is applied at bus 8 at t = 1 sec and cleared after 0.1 sec. The original system is restored upon the fault clearance. Figs. 7-10 show the rotor speed of G₂, G₃, rotor angle deviation of G₂ and, active power of line 7–8, respectively.
V. Conclusion

In this paper, a technique based on controllability index was proposed for finding a proper place and input control parameter of UPFC to increase damping oscillations of small signal and improve stability of the system. Controllability index for two critical modes was computed for a three machine system and UPFC was inserted in line with the maximum value of controllability index. Simulation results showed that, by inserting UPFC in a suitable place and input control signal, stability and damping of power oscillations could be quickly and effectively enhanced.

References

