Power Flow and Modal Analysis of a Power System Including Unified Power Flow Controller
Djilani Kobibi Youcef Islam, Hadjeri Samir, Djeahf Mohamed Abdeldjalil

Abstract—The Flexible AC Transmission System (FACTS) technology is a new advanced solution that increases the reliability and provides more flexibility, controllability, and stability of a power system. The Unified Power Flow Controller (UPFC), as the most versatile FACTS device for regulating power flow, is able to control respectively transmission line real power, reactive power, and node voltage. The main purpose of this paper is to analyze the effect of the UPFC on the load flow, the power losses, and the voltage stability using NEPLAN software modules, Newton-Raphson load flow is used for the power flow analysis and the modal analysis is used for the study of the voltage stability. The simulation was carried out on the IEEE 14-bus test system.

Keywords—FACTS, load flow, modal analysis, UPFC, voltage stability.

I. INTRODUCTION

The increasing electric power demand and the restrictions on the power system expansion due to the high cost and environmental issues are the main factors forcing the power systems to operate under increasingly stressed conditions, the ability to maintain voltage stability becomes a growing concern. [1]

The voltage instability of power systems is taking part in increasing the losses in power systems, which is not acceptable, and the events accompanying voltage instability may have disastrous effects, including a resultant low voltage profile in a significant area of the power network, known as the voltage collapse phenomenon [2], [3].

A voltage collapse can be initiated by either a primary fault or an unexpected load demand increase, in combination with insufficient reactive power reserves or transmission capability. [3]

Voltage collapse occurs when increased loading leads to a loss of voltage control in a significant part of a power system. Though voltage instability is a local phenomenon, it may gradually develop a global problem without quick responses. One of the best solutions to avoid voltage instability and to enhance controllability and increase power transfer capability

and reduce losses is the Flexible AC Transmission Systems (FACTS) [4]. FACTS is a static equipment used for controlling the power transmission system [5], [6]. FACTS is defined as "a power electronic based system and other static equipment that provide control of one or more AC transmission system parameters to enhance controllability and increase power transfer capability" [7], [8].

Unified power flow controller (UPFC), being the most versatile FACTS device, can control, concurrently or selectively, the transmission-line impedance, the magnitude of terminal voltage, and the active and reactive power flow in the line and thus, extensively considered for the transmission line compensation [9], [10].

II. UNIFIED POWER FLOW CONTROLLER

The UPFC is one of the FACTS devices, which provides the independent control of the real and reactive power flow, voltage magnitude and enhance the dynamic stability of the system. The UPFC consists of two switching converters like series converter and shunt converter operated from a common DC link. The converters are connected to the power system via coupling transformers. The UPFC structure can be described in Fig. 1 [8].

The series converter provides the main function of the UPFC by injecting a voltage Vse with controllable magnitude and phase angle in series with the line via an insertion transformer. The transmission line current flows through this voltage source resulting in reactive and real power exchange between it and the AC system. The reactive power exchanged at the AC terminal (i.e. at the terminal of the series insertion transformer) is generated internally by the converter. The real power exchanged at the AC terminal is converted into DC power, which appears at the DC link as a positive or negative real power demand.
The basic function of the shunt converter is to supply or absorb the real power demanded by the series converter at the common DC link to support the real power exchange resulting from the series voltage injection. This DC link power demand of the series converter is converted back to AC by the shunt converter and coupled to the transmission line bus via a shunt-connected transformer. In addition, the shunt converter can also generate or absorb controllable reactive power, if it is desired, and thereby provide independent shunt reactive compensation for the line. It is important to note that whereas there is a closed direct path for the real power resulting from the series voltage injection between the UPFC and the transmission line, the corresponding reactive power exchanged is generated or absorbed locally by the series converter and therefore does not have to be transmitted by the line. Thus, the shunt converter can be operated at a unity power factor or it can be controlled to have a reactive power exchange with the line independent of the reactive power exchanged by the series converter. Obviously, there can be no reactive power flow through the UPFC DC link [11], [12].

III. UPFC LOAD FLOW MODEL

Reference [13] has presented an approach of modeling UPFC device in load flow studies. This approach is based on injected active and reactive power at the terminals of UPFC. This UPFC model will be referred as UPFC injection model [14].

A. Series Connected Voltage Source Converter Model

Suppose a series connected voltage source is located between nodes $i$ and $j$ in a power system. The series voltage source converter can be modeled with an ideal series voltage $V_s$ in series with a reactance $X_s$. In Fig. 2, $V_s$ models an ideal voltage source and represents a fictitious voltage behind the series reactance [15]:

$$V_s = V_s + V_i$$

(1)

The series voltage source is controllable in magnitude and phase, i.e.:

$$V_s = rVe^{j\gamma}$$

(2)

where $0 \leq r \leq r_{\text{max}}$ and $-\pi \leq \gamma \leq \pi$

The equivalent circuit vector diagram is shown in Fig. 3.

![Fig. 3 Vector diagram of the equivalent circuit of VSC](image)

In UPFC, the shunt connected voltage source (Converter 1) is used mainly to provide the active power, which is injected, to the network via the series connected voltage source:

$$P_{\text{CONV1}} = P_{\text{CONV2}}$$

The equality above is valid when the losses are neglected. The apparent power supplied by the series voltage source converter is calculated from:

$$S_{\text{CONV}} = V_s^* V_j$$

(3)

The injection powers $S_s$ and $S_j$ are simplified to:

$$S_s = V_s^* V_j = -j b_r V_s^* V_j$$

(4)

where $b_r = b_r V_j^2 \cos \gamma$ and $b_r = b_r V_j^2 \sin \gamma$

(5)

B. UPFC Model

In UPFC, the shunt connected voltage source (Converter 1) is used mainly to provide the active power, which is injected, to the network via the series connected voltage source:

$$P_{\text{CONV1}} = P_{\text{CONV2}}$$

The equality above is valid when the losses are neglected. The apparent power supplied by the series voltage source converter is calculated from:

$$S_{\text{CONV}} = V_s^* V_j$$

(6)

Active and reactive powers supplied by Converter 2 are distinguished as:

$$P_{\text{CONV2}} = r_h V_j \sin(\theta - \phi + \gamma) - r_h V_j^2 \sin \gamma$$

(7)
The reactive power delivered or absorbed by converter 1 is independently controllable by UPFC and can be modeled as a separate controllable shunt reactive source. We assume that \( Q_{CONV1} = 0 \). Consequently, the UPFC injection model is constructed from the series connected voltage source model (Fig. 5) with the addition of a power equivalent to \( P_{CONV} + jQ_{CONV} \) to node i. Thus, the UPFC injection model is shown in Fig. 6. The model shows that the net active power interchange of UPFC with the power system is zero, as is expected for a lossless UPFC.

![Fig. 5 Injection model for a series connected VSC](image)

**C. Series Connected Voltage Source Converter Model**

The UPFC injection model can easily be incorporated in a load flow program. If a UPFC is located between node i and node j in a power system, the admittance matrix is modified by adding a reactance equivalent to \( X_s \) between node i and node j. The Jacobian matrix is modified by addition of appropriate injection powers. If we consider the linearized load flow model as:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
H & N \\
J & L
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix}
\]

(9)

The Jacobian matrix is modified as given in Table I (The superscript 0 denotes the Jacobian elements without UPFC) [15].

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>MODIFICATION OF THE JACOBIAN MATRIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{0i} = H_{0j} = 0 )</td>
<td>( N_{0i} = N_{0j} = 0 )</td>
</tr>
<tr>
<td>( H_{0i} = H_{0j} )</td>
<td>( N_{0i} = N_{0j} + P_0 )</td>
</tr>
<tr>
<td>( H_{0i} = H_{0j} )</td>
<td>( N_{0i} = N_{0j} + P_0 )</td>
</tr>
<tr>
<td>( J_{0i} = P_{0i} )</td>
<td>( L_{0i} = L_{0j} - 2Q_0 )</td>
</tr>
<tr>
<td>( J_{0i} = P_{0i} )</td>
<td>( L_{0i} = L_{0j} + Q_0 )</td>
</tr>
<tr>
<td>( J_{0i} = P_{0i} )</td>
<td>( L_{0i} = L_{0j} + Q_0 )</td>
</tr>
<tr>
<td>( J_{0j} = P_{0j} )</td>
<td>( L_{0j} = L_{0i} + Q_0 )</td>
</tr>
</tbody>
</table>

The matrix \( J_R \) represents the linearized relationship between the incremental changes in bus voltage (\( AV \)) and bus reactive power injection (\( AQ \)). It’s well known that, the system voltage is affected by both real and reactive power variations. In order to focus the study of the reactive demand and supply problem of the system as well as minimize computational effort by reducing dimensions of the Jacobian matrix \( J \) the real power (\( \Delta P = 0 \)) and angle part from the system in (10) are eliminated.

The eigenvalues and eigenvectors of the reduced order Jacobian matrix \( J_R \) are used for the voltage stability characteristics analysis. Voltage instability can be detected by identifying modes of the eigenvalues matrix \( J_R \). The magnitude of the eigenvalues provides a relative measure of proximity to instability. The eigenvectors on the other hand present information related to the mechanism of loss of voltage stability.

![Fig. 6 UPFC model](image)
Modal analysis of $J_R$ results in:

$$J_R = \lambda \Phi \xi$$

(16)

where $\Phi$ = right eigenvector matrix of $J_R$, $\xi$ = left eigenvector matrix of $J_R$, $\lambda = \text{diagonal eigenvalue matrix of } J_R$.

Equation (16) can be written as:

$$J_R^{-1} = \Phi \lambda^{-1} \xi$$

(17)

In general, it can be said that, a system is voltage stable if the eigenvalues of $J_R$ are all positive. This is different from dynamic systems where eigenvalues with negative real parts are stable. The relationship between system voltage stability and eigenvalues of the $J_R$ matrix is best understood by relating the eigenvalues with the V-Q sensitivities of each bus (which must be positive for stability). $J_R$ can be taken as a symmetric matrix and therefore the eigenvalues of $J_R$ are close to being purely real. If all the eigenvalues are positive, $J_R$ is positive definite and the V-Q sensitivities are also positive, indicating that the system is voltage stable.

The system voltage is considered unstable if at least one of the eigenvalues is negative. A zero eigenvalue of $J_R$ means that the system is on the verge of voltage instability. Furthermore, small eigenvalue of $J_R$ determines the proximity of the system to being voltage unstable [18].

There is no need to evaluate all the eigenvalues of $J_R$ for a large power system because it is known that once the minimum eigenvalues become zero, the system Jacobian matrix becomes singular and voltage instability occurs. Therefore, the eigenvalues of importance are the critical eigenvalues of the reduced Jacobian matrix $JR$. Thus, the smallest eigenvalues of $J_R$ are taken to be the least stable modes of the system. The rest of the eigenvalues are neglected because they are considered to be strong enough modes. Once the minimum eigenvalues and the corresponding left and right eigenvectors have been calculated the participation factor can be used to identify the weakest node or bus in the system.

The appropriate definition and determination as to which node or load bus participates in the selected modes become very important. This necessitates a tool, called the participation factor, for identifying the weakest nodes or load buses that are making significant contribution to the selected modes [19]:

$$\Delta V = \sum_i \Phi_i \xi_i \Delta Q$$

(18)

where $\lambda_i$ is the $i^{th}$ eigenvalue, $\Phi_i$ is the $i^{th}$ column right eigenvector and $\xi_i$ is the $i^{th}$ row left eigenvector of matrix $J_R$. Each eigenvalue $\lambda_i$ and corresponding right and left eigenvectors $\Phi_i$ and $\xi_i$ defined the i mode of the system.

A. Identification of the Weak Load Buses

The minimum eigenvalues, which become close to instability, need to be observed more closely. The relationship between system voltage stability and eigenvalues of the $J_R$ matrix is best understood by relating the eigenvalues with the V-Q sensitivities of each bus (which must be positive for stability). $J_R$ can be taken as a symmetric matrix and therefore the eigenvalues of $J_R$ are close to being purely real. If all the eigenvalues are positive, $J_R$ is positive definite and the V-Q sensitivities.

B. V-P Curves

The V-P curve for a bus is obtained by increasing the total load at the bus or area and performing successive power flow calculations, whilst maintaining a constant power factor. The voltage at the bus is plotted as a function of the total active power load, using conventional power-flow programs until the bifurcation point or 'nose' point is reached at critical loading.

The points above the 'nose' correspond to voltage stable operation. Continuation power-flow programs can be used to obtain solutions below the bifurcation point, but this is not usually necessary [20]. Generating V-P curves via automation of power flow simulations can be implemented relatively easily. As the size of the power network increases however, the processing time required to generate V-P curve increases, due to the increase in processing time to perform a single power flow solution. For large networks, processing time can be relatively large. More importantly, V-P curves at several buses must be generated before a system-wide perspective of voltage stability emerges, since there is no way of identifying beforehand the buses that are critical. For large power networks this can be time-consuming and confusing. V-P curves also give no useful insight into the cause of the voltage stability problem in the network [21].

C. Q-V Curves

Many power system planners still utilize Q-V curves in analyzing performance criteria [5]. Such curves indicate the sensitivity and variation of bus voltages with respect to reactive power injections and absorptions. The Q-V curve at a test bus is generated by placing a variable reactive power source with infinite limits at the bus. Successive power flows are performed for different scheduled values of bus voltage, and the required reactive power injection is measured. Q-V curves can therefore give the reactive power load margin at a bus from a stable operating point to the point of voltage instability. Such curves give no insight however into the causes of the voltage stability, such as participating nodes and branches. As with V-P curves, Q-V curves at several buses need to be drawn to obtain a system-wide perspective of the stability problem and decipher which buses in the system have the smallest stability margins, since there is no way of identifying beforehand which buses are critical. Although computation can be easily implemented on software, processing time can be quite large for practical networks with several hundred buses [22].

V. System Under Study

Simulation studies were done for IEEE 14-bus test system, data of the 14-bus system contains 20 lines, 5 generators are taken from [12], and the test system is shown in Fig. 7.
The simulation was carried out with NEPLAN software, the aim of this work is to illustrate the role of the UPFC in improving the power flow, and reducing the power system losses and enhancing the voltage stability, so the steps being taken are:

1. Load flow solution with and without the UPFC.
2. Voltage stability analysis using modal analysis approach of the test system with and without the UPFC.

VI. SIMULATION RESULTS

A. Case 1

The load flow solution is applied on the IEEE 14-bus test system with the UPFC located between Bus 1 and Bus 5, and the results are compared to the test system without the UPFC, to see its effect on the power flow in the steady state conditions, the results are shown in Table II.

We can see from the results that the total active power losses get decreased from 13.59 MW to 13.43 MW and the total reactive power losses get decreased from 27.43 MVAR to 0.89 MVAR, by installing the UPFC in line 2 between Bus 1 and Bus 5.

From Table II, it can be observed that the active power flow in the lines changes by incorporating the UPFC, the lines 1, 3, 4, 5 were relieved by increasing the power transmitted in line 2, 6, 7, which represent an improvement of the load flow.

B. Case 2

The voltage stability analysis was performed on the 14-bus test system without the UPFC and with the UPFC located between Bus 1 and Bus 5; the results are shown in Figs. 8-10.

From Fig. 8 (a), we can see that the lowest system eigenvalues are above zero, confirming that the system is stable and the bifurcation point has not yet been reached. The minimum or critical eigenvalue is 2.079. This critical value is increased as shown in Fig. 8 (b) from 2.079 to 2.688 by incorporating the UPFC in the test system, so the UPFC increases the stability margin.

### Table II

<table>
<thead>
<tr>
<th>Line</th>
<th>Without UPFC (P)</th>
<th>Without UPFC (Q)</th>
<th>With UPFC (P)</th>
<th>With UPFC (Q)</th>
</tr>
</thead>
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<tr>
<td>Line 1</td>
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<td>-20,464</td>
<td>148,979</td>
<td>-18,538</td>
</tr>
<tr>
<td>Line 2</td>
<td>75,457</td>
<td>5,584</td>
<td>83,458</td>
<td>12,284</td>
</tr>
<tr>
<td>Line 3</td>
<td>73,477</td>
<td>3,537</td>
<td>71,661</td>
<td>3,717</td>
</tr>
<tr>
<td>Line 4</td>
<td>55,931</td>
<td>1,795</td>
<td>53,332</td>
<td>-3,698</td>
</tr>
<tr>
<td>Line 5</td>
<td>41,72</td>
<td>3,345</td>
<td>38,416</td>
<td>-5,712</td>
</tr>
<tr>
<td>Line 6</td>
<td>23,46</td>
<td>-9,407</td>
<td>25,174</td>
<td>-3,828</td>
</tr>
<tr>
<td>Line 7</td>
<td>60,116</td>
<td>-8,89</td>
<td>64,666</td>
<td>5,115</td>
</tr>
<tr>
<td>Line 8</td>
<td>27,152</td>
<td>-5,922</td>
<td>27,359</td>
<td>-3,659</td>
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<td>Line 9</td>
<td>15,485</td>
<td>2,928</td>
<td>15,622</td>
<td>3,99</td>
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<td>Line 10</td>
<td>45,768</td>
<td>10,878</td>
<td>45,373</td>
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<td>Line 11</td>
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<td>6,599</td>
<td>8,016</td>
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<td>18,169</td>
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<td>0</td>
<td>24</td>
<td>0</td>
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</tr>
<tr>
<td>Line 15</td>
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<td>15,709</td>
<td>27,359</td>
<td>16,466</td>
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<tr>
<td>Line 16</td>
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<td>-6,681</td>
<td>4,628</td>
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<td>0,465</td>
<td>8,853</td>
<td>1,154</td>
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<td>6,611</td>
<td>4,415</td>
<td>5,511</td>
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<td>Line 19</td>
<td>1,869</td>
<td>1,379</td>
<td>1,81</td>
<td>1,24</td>
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<tr>
<td>Line 20</td>
<td>6,406</td>
<td>4,935</td>
<td>6,23</td>
<td>4,226</td>
</tr>
<tr>
<td>Total</td>
<td>13,597</td>
<td>27,434</td>
<td>13,438</td>
<td>0,895</td>
</tr>
</tbody>
</table>

### Fig. 8

(a) Eigenvalues of the test system (a) without the UPFC and (b) with the UPFC

### Fig. 9

(a) Bus participation factor of the critical Eigenvalue of the test system (a) without the UPFC and (b) with the UPFC
Bus participation factors to the critical eigenvalue are generated to predict the critical buses in the system. The buses with highest participation factors were expected to be the most critical buses or buses closest to instability. These include bus 7, 9, 10, 11 and 14 as shown in Fig. 9. Moreover, the Q-V curve in Fig. 10 of the reactive stability margins shows that the UPFC offer the reactive power support needed to maintain stability. The curves clearly show that the reactive power margin of the power system without the UPFC is much bigger than the reactive power margin of the power system with the UPFC installed, this means that the power system with the UPFC requires less reactive power support to maintain stability.

VII. CONCLUSIONS
In this work, the effect of the UPFC on the losses and the voltage stability has been studied using load flow and voltage stability analysis of NEPLAN software modules, the load flow module, and the voltage stability module, which provides several approaches for static voltage stability analysis of power systems such as V-Q curves and Q-V eigenvalue analysis (modal analysis).

The results show that the UPFC can provide an adjusted distribution of the power flow among the transmission lines, minimize the system losses and enhance the voltage stability.

REFERENCES

