Effect of Subsequent Drying and Wetting on the Small Strain Shear Modulus of Unsaturated Soils

A. Khosravi, S. Ghadirian, J. S. McCartney

Abstract—Evaluation of the seismic-induced settlement of an unsaturated soil layer depends on several variables, among which the small strain shear modulus, \( G_{\text{max}} \), and soil’s state of stress have been demonstrated to be of particular significance. Recent interpretation of trends in \( G_{\text{max}} \) revealed considerable effects of the degree of saturation and hydraulic hysteresis on the shear stiffness of soils in unsaturated states. Accordingly, the soil layer is expected to experience different settlement behaviors depending on the soil saturation and seasonal weathering conditions. In this study, a semi-empirical formulation was adapted to extend an existing \( G_{\text{max}} \) model to infer hysteretic effects along different paths of the SWRC including scanning curves. The suitability of the proposed approach is validated against experimental results from a suction-controlled resonant column test and from data reported in literature. The model was observed to follow the experimental data along different paths of the SWRC, and showed a slight hysteresis in shear modulus along the scanning curves.

Keywords—Hydraulic hysteresis, Scanning path, Small strain shear modulus, Unsaturated soil.

I. INTRODUCTION

Diff erent methods have been proposed to analyze the behavior of soils under cyclic loading and their seismically-induced settlements such as equivalent linear [19] and non-linear [9], [11], [14], [18] analyses. In these methods, the analysis is performed using some relevant terms affecting the settlement behavior such as damping ratio (constant or function of cyclic shear strain), effective stress, pore water pressure, small strain shear modulus, \( G_{\text{max}} \) (constant or function of effective stress), and shear modulus reduction function (\( G/G_{\text{max}} \) as a function of cyclic shear strain), mostly developed for soils in dry and saturated states. However, it has been recognized for some time now that the seismic response of soil at a particular site is dramatically affected by the degree of saturation of soil, \( S_r \), and its unsaturated stress state variables [1], [5].

Reference [1] performed a series of physical model centrifuge tests to simulate ground motions and studied the settlement behavior of an unsaturated layer of sand during seismic loading. The seismic load in this study was provided using an electrohydraulic shaking table mounted on the platform of centrifuge, and the Steady-State Infiltration Method [15] was used to provide uniform target suction through the height of the sand layer. Results of their study revealed the significant effect of degree of saturation on the soil settlement behavior during seismic loading. The soil settlement-degree of saturation curve showed an initial decrease in settlement during drying to a critical point at a \( S_r \) value of approximately 0.7, followed by an increasing trend. It is interesting to note that the trend for the soil settlement was observed to be consistent with the shape of the \( G_{\text{max}}-S_r \) curve during the drying path.

![Fig. 1 Matric suction and settlement variation with \( S_r \)](image)

From the results of the centrifuge tests, [1] proposed and verified an empirical methodology to predict the earthquake induced settlement of a free field unsaturated sand layer. The new methodology incorporated recent advances in the definition of the effective stress and stiffness in unsaturated soils, and soil-water retention curve parameters into existing approaches originally developed for saturated soils. In particular, the small strain shear modulus was defined using an effective stress-based approach proposed by [5] as:

\[
G_{\text{soil}} = \frac{A}{0.3 + 0.7e^T} p^m
\]

where, \( A \) and \( n \) are fitting parameters, \( e \) is the initial void ratio and \( p' \) is the mean effective stress which was defined using the concept of suction stress proposed by [13]. Equation (1) was observed to provide a good prediction of the small strain shear modulus for unsaturated sands. However, it only considered the role of matric suction and net stress on \( G_{\text{max}} \) measurements along the drying path of the SWRC.

Later studies by [8], [16], [22] showed the importance of concurrent consideration of \( S_r \) and \( \psi \) in interpreting \( G_{\text{max}} \) measurements during hydraulic hysteresis, especially when testing fine graded materials, and some correlations between

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the small strain shear modulus and degree of saturation were developed from the results of element scale tests. These correlations improved the prediction of \( G_{\text{max}} \) during hydraulic hysteresis. However, they failed to predict the values of \( G_{\text{max}} \) along the scanning curves of the SWRC. Fluctuating field conditions may result in the occurrence of unsaturated/saturated conditions in the soil layers. This may correspond to wetting and drying on scanning curves between the primary drying and wetting paths of the SWRC.

In this paper, a methodology is proposed to capture changes in \( G_{\text{max}} \) magnitude along different paths of the SWRC including drying/scanning/wetting paths. In this regard, the proposed equation for \( G_{\text{max}} \) by [8] is extended by implementing an empirical relationship that predicts \( S_e \) values along the scanning paths of the SWRC based on [17] approach. The suitability of the proposed methodology is assessed through comparison with experimental results from a suction-controlled resonant column test and from data reported in literature.

II. A METHODOLOGY FOR PREDICTING SMALL STRAIN SHEAR MODULUS ALONG THE SCANNING PATHS OF THE SWRC

Reference [8] adapted the \( G_{\text{max}} \) relationship proposed by [3] for saturated soils to consider the role of matric suction and hydraulic hysteresis on \( G_{\text{max}} \) of low plasticity unsaturated soils. This formulation incorporated two terms including plastic changes in volume and changes in degree of saturation to consider the effect of hydro-mechanical coupling in evaluation of \( G_{\text{max}} \) as:

\[
G_{\text{max}} = A p \left[ \frac{P_{eq} - \exp\left( \frac{\Delta \sigma_f}{\lambda - \kappa} \right)}{P_e - \exp\left( \beta \left[S_{\text{sat}} - S_e \right] \right)} \right]^{D' \left( \frac{P_e}{P_a} \right)^n} \tag{2}
\]

where \( A \) and \( n \) are effective stress dependency parameters, \( P_a \) is the atmospheric pressure, \( P_{eq} \) is the initial mean apparent pre-consolidation stress, \( P_e \) is the net stress, \( p' \) is the mean effective stress defined using the suction stress concept as \( p' = P_e - \psi S_e \), \( \Delta \sigma_f \) is a plastic change in void ratio, \( \lambda \) and \( \kappa \) are the slopes of the isotropic compression and unloading curves, \( K' \) and \( K \) are hardening parameters, \( b \) is the double hardening parameter referring to the effects of changes in soil saturation, and \( S_{\text{sat}} \) and \( S_{eq} \) are the values of effective saturation at current and initial conditions, respectively. The value of effective saturation, \( S_e \), was defined using the SWRC model of [20], which allows the prediction of degree of saturation and hydraulic behavior along the main drying and wetting paths of SWRC. However, it fails to predict the degree of saturation along scanning drying and wetting curves.

In this study, an empirical approach proposed by [17] was implemented in (2) to capture the variations of \( S_e \) along different paths of the SWRC. [17] used two parameters, the air saturation, \( S_{\text{air}} \), and the apparent saturation, \( S_{\text{app}} \), to estimate the amount of trapped air during wetting. \( S_{\text{app}} \) was used to consider the air entrainment along the scanning curves and is obtained using the following equation:

\[
S_{\text{app}} = \frac{S_{\text{rev}} - S_{\text{app}} \left( \psi_{\text{rev}} \right)}{S_{\text{rev}} \left( \psi_{\text{app}} \right) - S_{\text{app}} \left( \psi_{\text{rev}} \right)} + S_{\text{air}} \tag{3}
\]

where \( p \) is an index indicating whether a drying or wetting scanning path is considered. \( S_{\text{rev}} \) is the effective saturation of the main path. \( S_{\text{rev}} \left( \psi_{\text{rev}} \right) \) and \( S_{\text{rev}} \left( \psi_{\text{app}} \right) \) are the values of effective saturation of the most recent and previous reversal points, calculated using the van Genuchten equation, and \( S_{\text{app}} \) and \( S_{\text{air}} \) are the apparent saturations of the most recent and previous reversal points, respectively. Fig. 2 shows a hypothetical SWRC and different paths of drying, wetting and scanning to clarify this designation. For a path like 1-2 which lies on the main drying curve, the most recent reversal point would be point 1 with the values of \( (S_e, \psi_e) = (1,0) \), and the previous reversal point has the values of \( (S_{e,i}, \psi_{e,i}) = (0, \infty) \). Path 2-3 is a scanning wetting path with the values of \( (S_{e,i}, \psi_{e,i}) \) corresponding to point 2 and \( (S_{e,i}, \psi_{e,i}) \) corresponding to the previous point 1. Path 3-4 is a scanning drying path with the most recent and previous reversal points of 3 and 2, respectively. The same manner can be followed through any other path to define the reversal points in calculations.

Fig. 2 Theoretical scanning loops of a hypothetical apparent saturation curve

Now, with the value of apparent saturation (3), the air saturation, \( S_a \) at any intermediate point along the scanning wetting paths is obtained by linearly interpolating air saturations between the end points of primary wetting curves and using the maximum trapped saturation as follows [12]:

\[
S_a = S_{\text{rev}} \left( \frac{S_{\text{app}} - S_{\text{rev}}}{1 - S_{\text{rev}}} \right) \tag{4}
\]

where, \( S_{\text{app}} \) is the apparent saturation, \( S_{\text{rev}} \) is the effective saturation at the reversal point from the main drying curve to a primary scanning wetting path (Fig. 3), and \( S_{\text{rev}} \) is the effective residual saturation of trapped air along the primary scanning path given by the following equations [17]:

![Image](https://via.placeholder.com/150)
\[ S_{\text{res}} = \frac{1 - S_{\text{ev}}}{1 + R \left( 1 - S_{\text{ev}} \right)} \]  

(5)

in which

\[ R = \frac{1}{S_{\text{max,res}}} - 1 \]  

(6)

With the values of apparent saturation (3) and air saturation, the effective saturation of water is obtained as:

\[ S_e = S_{\text{app}} - S_{a} \]  

(7)

Fig. 3 Different parameters involved in air entrapment determination

By implementing (7) into the model proposed by [8], \( G_{\text{max}} \) values along main drying and wetting curves as well as the scanning paths of the SWRC can be obtained. Fig. 4 shows the results of a parametric evaluation to highlight the functional behavior of the extended \( G_{\text{max}} \) model along the scanning loops of the SWRC during hydraulic hysteresis. In this regard, a hypothetical soil specimen subjected to an initial value of mean net stress of \( p_{\text{w}}=100 \) is used and the effect of two hardening parameters, \( b \) in Fig. 4 (a) and \( K' \) in Fig. 4 (b) are discussed.

The presented trends in \( G_{\text{max}} \) imply that an increase in \( K' \) leads to a greater impact of hydraulic hysteresis on \( G_{\text{max}} \) values while imposing greater values of \( b \) results in steeper rate of change in \( G_{\text{max}} \) along the scanning path. Results also indicate that under small changes in matric suction along the scanning curves of the SWRC, the hysteresis effects are negligible and a unique form of SWRC can be assumed for the material.

III. EXPERIMENTAL VALIDATION

The methodology presented in this study for the \( G_{\text{max}} \) values along different paths of the SWRC was validated against a set of \( G_{\text{max}} \) values obtained from a series of resonant column tests on a specimen of silt, obtained from Bonny dam near the Colorado-Kansas Border. The tests were performed using a fixed-free resonant column test device modified with a feedback-controlled flow pump and the axis translation technique to reach different equilibrium values of \( \psi \) and \( S_e \). The soil has a specific gravity of 2.6, plasticity index of 4 and is classified as ML according to the Unified Soil Classification System (USCS). The soil specimen has a diameter of 36mm, height of 72mm, statically compacted at an initial water content of 14% to reach a dry density of 15.09 kN/m³.

In this experimental study, after placement of the soil specimen within the test setup and achieving the full saturation using the backpressure technique, the soil specimen was initially dried to a suction value of 31 kPa along the main drying path of the SWRC under a mean net stress of 70 kPa. After that, the direction of the flow pump was reversed and the applied suction was decreased in stages to a value of 21 kPa to produce the first scanning curve of the test. At each point of suction equilibrium, a resonant column test was performed and the value of \( G_{\text{max}} \) of the soil specimens was measured. The specimen was then re-dried and a similar procedure was followed at suction values of 41 and 61 kPa for simulating three full cycles of drying and wetting. The testing device and procedure for this device are described in detail by [6], [7].

The SWRCs of the tested specimen are shown in Fig. 5 (a) and the variations of \( G_{\text{max}} \) measured after reaching matric suction equilibrium are presented in Fig. 5 (b). Fig. 5 also shows the van Genuchten [20] SWRC model, fitted to the experimental data over the range of suction imposed to the specimen (Fig. 5 (a)). The SWRC model parameters were obtained by fitting the best-fitted curve to the experimental results along different path of the SWRC. The Results of shear modulus at small strain presented in this figure revealed a slight hysteresis in the \( G_{\text{max}} \) measurements along the scanning curves of the SWRC, with a larger loop along the second loop.
where more significant changes in $S$, happened with changing
matric suction. However, in both cases, the value of $G_{\text{max}}$ was
recovered once the main drying path was reached. Fig. 5 (b) also
shows comparisons between the experimental $G_{\text{max}}$ data
and the model for different paths as a function of $\psi$. The
model parameters are presented in Table I. As listed in this
table, the parameters required to predict the evolution of $G_{\text{max}}$
along different paths of the SWRC include the hardening
parameters ($K$, $K'$, $b$), the parameters describe the slope of
virgin compression and unloading curves ($\kappa$, $\lambda$), and the
effective stress dependency parameters ($A$, $n$).

![Experimental data vs Model](image)

Fig. 5 Comparison of the data obtained from the resonant column
results with the proposed methodology: (a) SWRC (b) $G_{\text{max}}$

### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.41</td>
</tr>
<tr>
<td>$n$</td>
<td>0.68</td>
</tr>
<tr>
<td>$K$</td>
<td>0.05</td>
</tr>
<tr>
<td>$K'$</td>
<td>0.9</td>
</tr>
<tr>
<td>$b$</td>
<td>1.06</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.00002</td>
</tr>
<tr>
<td>$p_{c0}$</td>
<td>70 kPa</td>
</tr>
<tr>
<td>$p_{c0}$</td>
<td>720 kPa</td>
</tr>
</tbody>
</table>

In the current study, the values of $p_{c0}$ and $b$ were obtained
from the results of a series of isotropic compression tests on
specimens subjected to different suction values [10], $K$ was
defined using empirical guidance from [2], $A$ and $n$ were
defined by fitting a curve to the values of $G_{\text{max}}$ measured under
saturated conditions (zero suction) and $K'$ was obtained from
the $G_{\text{max}}$ measurements at different equilibrium suction values
along the drying path using least squares minimization. Plastic
changes due to suction change were also considered zero in

(2). Similar values were used by [10] for the specimens with
similar characteristics and dry densities. With the values
presented in Table I, the values of $G_{\text{max}}$ predicted using the
proposed methodology are in good qualitative and quantitative
agreement with the experimental data. The greatest
discrepancy between the experimental data and the model
occurs during wetting for the suction lower than 20 kPa.

### IV. LITERATURE DATA ANALYSIS

In order to further examine the application of the new
model for small stain shear modulus of unsaturated soils along
the scanning paths of the SWRC, the model was fit to $G_{\text{max}}$
values reported in the literature. Figs. 6 and 7 present
experimental SWRCs and $G_{\text{max}}$ data for SP-SC [4], and MH
[21], respectively. Characteristics of each soil are presented in
Table II.

### Table II

<table>
<thead>
<tr>
<th>Literature data</th>
<th>Soil type</th>
<th>$G_{s}$</th>
<th>$w_{1}$ (%)</th>
<th>$w_{r}$ (%)</th>
<th>PI</th>
<th>Soil classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4]</td>
<td>Silty Sand</td>
<td>2.7</td>
<td>25.5</td>
<td>32.5</td>
<td>10</td>
<td>SP-SC</td>
</tr>
<tr>
<td>[21]</td>
<td>Po Silt</td>
<td>2.76</td>
<td>50.4</td>
<td>-</td>
<td>17.9</td>
<td>Inorganic Silt</td>
</tr>
</tbody>
</table>

#### A. $G_{\text{max}}$ Data for SP-SC [4]

In this study, the values of $G_{\text{max}}$ were measured on a
specimen prepared at initial void ratio of 0.4 and dry density of
18.9 kN/m$^3$. The tests were performed using the bender
element technique under an initial mean net stress of 50 kPa
along two successive wetting and drying cycles (Fig. 6). The
SWRC measurements and their corresponding $G_{\text{max}}$ data are
illustrated by the scatter in Figs. 6 (a) and (b), respectively.
The experimental results in this figure are presented with the
data points, while the lines show the predictions.

### Table III

<table>
<thead>
<tr>
<th>Literature data</th>
<th>$A$</th>
<th>$n$</th>
<th>$K$</th>
<th>$K'$</th>
<th>$b$</th>
<th>$\lambda$</th>
<th>$\kappa$</th>
<th>$p_{c0}$ (kPa)</th>
<th>$p_{c0}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4]</td>
<td>3.6</td>
<td>1.1</td>
<td>0.55</td>
<td>0.18</td>
<td>2.8</td>
<td>-</td>
<td>0.0018</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>[21]</td>
<td>3.6</td>
<td>1.1</td>
<td>0.55</td>
<td>0.18</td>
<td>2.8</td>
<td>-</td>
<td>0.0018</td>
<td>50</td>
<td>200</td>
</tr>
</tbody>
</table>

In this study, the van Genuchten parameters, $\alpha$ and $N$, along
the main drying path of the SWRC were obtained using the
SWRC data reported in literature as shown in Fig. 6 (a). Other
parameters including the values of $\alpha$ and $N$ for the main
wetting path were obtained by fitting the best-fitted curve to
the experimental scanning curves. The initial value of $p_{c0}$
were not reported for the tested soil in this studies. So this
variable was assumed and the parameters $A$, $n$, $K$, $K'$ and $b$
were found by fitting the model to the $G_{\text{max}}$ data for
unsaturated conditions. Plastic changes due to hydraulic
loading (suction change) were also ignored. With these parameters, the model was compared against the trends in $G_{\text{max}}$ with $\psi$ along different paths of the SWRC (Fig. 6 (b)). As presented in this figure, for the particular fitting values presented in Table III, a good consistency between the predicted $G_{\text{max}}$ values with those measured experimentally is observed.

![Graph showing effective saturation vs. matric suction](image1)

![Graph showing small strain shear modulus vs. matric suction](image2)

**Fig. 6 Comparison of the data obtained from [4] with the proposed methodology (a) SWRC (b) $G_{\text{max}}$**

**B. $G_{\text{max}}$ Data for MH [21]**

The SWRC measurements and the $G_{\text{max}}$ values obtained from a series of triaxial and resonant column (RC) tests performed by [21] are presented in Figs. 7 (a) and (b), respectively. The results were performed on specimens of a silt used for the construction of embankments on the Po river in Italy, subjected to a mean net stress of 200 kPa. The RC test includes an initial compression stage at a suction of 400 kPa, followed by subsequent wetting and drying cycles between the suction values of 400 and 100 kPa [21]. Due to lack of data on the soil degree of saturation and compression curve, the model parameters were treated as fitting parameters and were defined by fitting the model to the data reported in this study (Table III).

A comparison between the predicted values of $G_{\text{max}}$ and those measured experimentally indicates that the model provides reasonable prediction of the variations in $G_{\text{max}}$ during the first loop of scanning. Results indicated a greater discrepancy between the data and the model, especially along the second path of the scanning curves. Better experimental SWRC measurements would probably result in better prediction of the $S_e$ values along the scanning curves and their corresponding $G_{\text{max}}$ measurements.

![Graph showing effective saturation vs. matric suction](image3)

![Graph showing small strain shear modulus vs. matric suction](image4)

**Fig. 7 Comparison of the data obtained from [21] with the proposed methodology (a) SWRC (b) $G_{\text{max}}$**

**V. CONCLUSION**

A new methodology was presented in this study to infer the effects of the soil’s degree of saturation and hydraulic hysteresis on the seismic response of unsaturated soil. The proposed methodology incorporates an empirical approach to define the characteristic relationships between the degree of saturation and matric suction to simulate the variations of $G_{\text{max}}$ along the main drying and wetting paths of the SWRC as well as the scanning curves.

The validity of the model was investigated through comparison with experimental results from an independent experimental program, using a modified resonant column test device. For a mean net stress of 70 kPa, the small-strain shear modulus of statically-compacted Bonny silt was found to vary from 36 to 55 MPa for matric suctions ranging from 1 to 70 kPa (degrees of saturation from 1.00 to 0.48). The shear modulus of the soil was found to be greater upon re-wetting of the soil, a behavior which is attributed to the hardening caused by soil drying. The results also showed a slight hysteresis in the $G_{\text{max}}$ measurements along the scanning curves of the SWRC. Using the model parameters defined through definition of the SWRC and compression curves, and through parameter fitting to the soil, the model used for the hysteretic SWRC was found to integrate well with the constitutive relationships to represent the change in $G_{\text{max}}$ along different paths of the SWRC.

For further examination of the application of the new methodology for the small stain shear modulus of unsaturated
soils, the model was fit to $G_{max}$ values reported in the literature along the main drying and scanning paths of the SWRC. The proposed methodology was found to fit well the trends in $G_{max}$ values. Future testing on soils of different types and densities is recommended to investigate if the proposed methodology can be more broadly used to characterize the $G_{max}$ behavior of unsaturated soils during hydro-mechanical loading.

REFERENCES


