Distributed Coordination of Connected and Automated Vehicles at Multiple Interconnected Intersections

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Abstract—In connected vehicle systems where wireless communication is available among the involved vehicles and intersection controllers, it is possible to design an intersection coordination strategy that leads the connected and automated vehicles (CAVs) travel through the road intersections without the conventional traffic light control. In this paper, we present a distributed coordination strategy for the CAVs at multiple interconnected intersections that aims at improving system fuel efficiency and system mobility. We present a distributed control solution where in the higher level, the intersection controllers calculate the road desired average velocity and optimally assign reference velocities of each vehicle. In the lower level, every vehicle is considered to use model predictive control (MPC) to track their reference velocity obtained from the higher level controller. The proposed method has been implemented on a simulation-based case with two-interconnected intersection network. Additionally, the effects of mixed vehicle types on the coordination strategy has been explored. Simulation results indicate the improvement on vehicle fuel efficiency and traffic mobility of the proposed method.

Keywords—Connected vehicles, automated vehicles, intersection coordination systems, multiple interconnected intersections, model predictive control.

I. INTRODUCTION

The development of wireless communication and vehicle automation technologies have rapidly lead to the emergence of connected and automated vehicles (CAVs). The CAV technologies can help to address the issues in current transportation systems. Safety, for example, is one of the biggest challenges. 5.6 million of crashes happened in 2013, which ended up with 30,057 fatalities [1]. Although the figures are decreasing over the decade [2], they are still very disappointing. However, combined vehicle communication technologies (e.g., Vehicle-to-Vehicle (V2V), Vehicle-to-Infrastructure (V2I)) can address about 80% of vehicle targeted crashes by providing driver warnings or advisories according to U.S. Department of Transportation (DOT) [3].

In urban transportation system, intersections are one of the most important parts. Vehicles at the intersections approach to each other at high relative velocities. Although the intersections are only a small portion of the traffic network, about 30% of rural crashes and 50% of urban crashes happen at intersections [4]. The intersections are currently controlled by traffic lights and stop signs. These control mechanisms will inevitably generate vehicles’ stop-and-go driving patterns at the intersections, which is very inefficient. Meanwhile, the infrastructures are also under upgrade to enable wireless communication capabilities. Expected by American Association of State Highway and Transportation Officials (AASHTO), up to 80% of intersections will be V2I-enabled by 2040 [5]. Under these circumstances, the research on designing innovative control strategies for CAVs at intelligent intersections becomes an important topic.

Recently numerous researches have focused on the coordination strategies that lead CAVs cross the intersections safely without current intersection control mechanisms, but relying on the cooperation and commination among the involved CAVs and intersection controllers only. Dresner and Stone [6] proposed a multi-agent system approach where the intersection manager coordinates the time-space reservation on the intersection area based on the request from the vehicle agents. This work was later extended to enable platoon reservation [7]. Some other researchers formulated the problem using optimization-based approaches. Lee and Park [8] proposed an algorithm to limit the number of vehicles inside the intersection at each time instance by minimizing different vehicle overlapped trajectory in the intersection area. In such a way collision avoidance can be guaranteed. Jin et al. [9] formulated the optimal control problem to get vehicles’ arrival time at the intersection. And given the optimal arrival time, each vehicle decided its own trajectory. Some other researchers [10], [11] used Model Predictive Control (MPC) to achieve more than one objectives in the cost function. Kamal et al. [10] proposed a centralized MPC method to track the desired velocity, minimize acceleration rate and minimize the risk of collision at the same time. The aforementioned centralized method requires one controller make decisions for all the vehicles involved. On the contrary, [11] proposed a decentralized MPC method, where each vehicle plans its own trajectory. In order to achieve online real-time optimization, [12] developed a closed-form formulation for fuel economic control of the vehicles travelling over merging roads.

Although a lot of research effort has been spent on the coordination of CAVs at isolated intersections, the study of multiple intersections are still performed on macroscopic level with the focus on balancing traffic density over the traffic network [13], [14], maximizing traffic flow [15], and dynamic route selection [16]. Individual vehicle trajectory planning is not the focused in the aforementioned research [13]-[16].
Urban traffic usually consists of multiple intersections interconnected with each other. What happens in one single intersection will influence the behavior of the whole network, so the problem of multiple interconnected intersections becomes more complex.

The coordination strategy is divided into two levels: higher level and lower level. In the higher level, each intersection has an intersection controller which calculates the road desired average velocity based on the traffic density information at the neighborhood intersections and optimally assigns reference velocity to each vehicle with the objective of minimizing velocity deviation from the road average velocity and collision avoidance at the intersection area. In the lower level, each vehicle’s local controller uses MPC to track the reference velocity and maintain minimum headway distance and time to its preceding vehicle.

A. Higher Level

The Greenshield model [17] shows that the average road velocity is a linear function of the road traffic density (red dot dash line in Fig. 2). However, the Greenshield model considers all conventional vehicles without CAV technologies. We modified the original Greenshield model based on the assumption that the current CAV control strategies, such as platooning and coordination adapted curse control (CACC) could make all the vehicles on the same road travel at high velocity when there are only a few vehicles. Due to safety consideration, when the number of vehicles exceeds a threshold, the average velocity will decrease slowly. Fig. 2 (black line) shows the modified relationship between average velocity and traffic density. It can be expressed as:

\[
V = f_\lambda(\rho) = \begin{cases} 
V_f & \rho \leq \rho_f \\
 k_f (\rho - \rho_f) + V_f & \rho > \rho_f 
\end{cases}
\]  

(1)

In (1), \( V \) is the road average velocity and \( f_\lambda(\rho) \) is the mapping from road density \( \rho \) to road average velocity. When the traffic density \( \rho \) on the road is less or equal to the free flow density threshold \( \rho_f \), the road average velocity is considered to be constant and equal to \( V_f \), which can be the speed limit for the road. Otherwise, the velocity would drop linearly with a slope \( k_f \) as the density increases.

We assume the CAVs will travel as the modified traffic flow model, if there is no coordination strategy involved. Under this assumption, when a vehicle enters a road with higher traffic density from the current road with lower density, it will suffer from significant velocity reduction, which is fuel inefficient and uncomfortable for the driver. Consensus algorithm [13], [14] is used in this study to balance the traffic density over the intersection network and minimize the average velocity deviation from road to road. The new road desired average velocity \( V^*_{CR}(k) \) is calculated similar to [13], [14] in (2):

\[
V^*_{CR}(k) = f_\lambda\left(\rho_{CR}(k)\right) + \lambda \left( f_\lambda\left(\rho_{ER}(k)\right) - f_\lambda\left(\rho_{CR}(k)\right) \right)
\]  

(2)

In (2), the road velocity considers current traffic density \( \rho_{CR} \) on its current road \( CR \) and the density difference compared with the road \( ER \), where \( ER \) is the road the vehicles in \( CR \) will finally enter. In (2), \( k \) indicates the time step and \( \lambda \) is a positive constant factor which indicates the density difference convergence rate.

Ideally, all the vehicles on the same road should move at the road desired average velocity calculated from (2). However, there would be a high probability of collision at the intersection area between the vehicles from different
directions. We formulate an optimal control problem to assign individual vehicle reference velocity with the constraints of collision avoidance at the intersection. The problem is formulated as:

$$ J = \min_{u_i} \left( \sum_p w_p \left( v^r_m (k) - \bar{v}^m_p (k) \right)^2 + \sum_q w_q \left( v^r_n (k) - \bar{v}^n_q (k) \right)^2 \right) $$

where

$$ \bar{v}^m_p (k) \geq \tau_0 $$

$$ 0 \leq \bar{v}^m_p (k) \leq \bar{v}_f $$

$$ a_{ib} \leq \bar{v}^m_p (k) - \bar{v}^m_p (k-1) \leq a_{ob} $$

$$ \bar{v}^n_q = \frac{S_p}{\bar{v}^n_q} $$

$m$ and $n$ are the indices of two roads leading to the intersection from different directions and $p$ and $q$ are the vehicle indices in road $m$ and $n$ respectively. $v^r_m$ and $v^r_n$ represents the road desired velocity obtained from (2) of road $m$ and $n$ respectively. In (3), $\bar{v}^m_p$ and $\bar{v}^n_q$ indicate the individual vehicle reference velocities and they are also the decision variables of the optimization problem. The constants $w_p$ and $w_q$ are the weighting factors representing the vehicle types, which will be discussed later in the paper. The cost function is subjected to the constraints from (3b) to (3e). $\tau^m_p$ and $\tau^q_n$ in (3b) are the estimated time of arrival (ETA) to the intersection between roads $m$ and $n$ for the vehicles $p$ and $q$ respectively. The ETA of vehicle $p$ can be expressed as (3e) where $S^m_p$ denotes the distance of vehicle $p$ to the intersection. Equation (3b) is used to avoid collision at the intersection by guaranteeing there is a time interval of $\tau_0$ between the vehicles coming from different directions and approaching the intersection. The constraint in (3c) is making sure that the vehicles reference velocity is within the speed limit range. In (3d), $a_{ob}$ and $a_{ib}$ represent the maximum and minimum velocity change at each time step while $\bar{v}^m_p (k-1)$ indicates the vehicle actual velocity in last time step. The same constraints from (3c) to (3e) are also applied to vehicles $q$ on road $n$.

In this section, at higher level, individual vehicle reference velocity is assigned with the objective of speeding up traffic density balancing (2) and avoiding intersection vehicle collision (3). The next step would be for the vehicle local controller to track its reference velocity while avoiding rear end collision with its preceding vehicle at the lower level.

**B. Lower Level**

MPC can deal with constrained problems and it allows the current time step to be optimized with the consideration of future time steps. The optimal control problem is solved over a finite horizon, but only implements on the current time step [18]. MPC is also proved to be very suitable for tracking problem [19].

In previous works [20], [21], MPC is utilized for fuel economy control of connected vehicles passing multiple intersections where the intersections are controlled by fixed phase timing traffic lights. In this paper, we extended [20], [21] to the case where the traffic lights are replaced with our higher level intersection controllers.

The longitudinal dynamics of any vehicle index of $i$ is given by [22]:

$$ \dot{x}_i = f_i(x_i, u_i) $$

$$ f_i(x_i, u_i) = \left[ -\frac{1}{2M_i} c_D \rho A_i' v_i^2 - \mu g \tan \theta + u_i \right] $$

where $x_i \in \mathbb{R}^{4}$, $u_i \in \mathbb{R}^{4}$ and $n_i = 2$, $n_w = 1$ in our case. In (4), $x_i = [x_i, v_i]$, where $x_i$ is the position of vehicle $i$ and $v_i$ is its velocity. The control input $u_i$ is the traction or braking force per unit mass of a vehicle at any time instance. $M_i$, $c_D$, $\rho$, $A_i'$, $\mu$, $g$ and $\theta$ denote the mass of the vehicle, drag coefficient, air density, frontal area of the vehicle, rolling friction coefficient, gravitational constant and road gradient respectively.

Once the reference velocity $\bar{v}_i$ for any vehicle $i$ is calculated from Section II-A and (3a), the tracking problem is solved as a receding horizon problem. For each vehicle $i$ and a time horizon $T$, the following cost function is solved at each time step $k$ :

$$ J_i = \arg \min_{u_i} \left( \sum_{i=1}^{k-1} \left[ w_1 (v_i (t) - \bar{v}_i (k))^2 + w_2 (v_i (t))^2 + w_3 u_i (t)^2 \right] \left( 5a \right) \right) $$

$$ v_{ib} (t) \leq v_i (t) \leq v_{wb} (t), \quad \forall t $$

$$ u_{ib} (t) \leq u_i (t) \leq u_{wb} (t), \quad \forall t $$

$$ R_i (t) = S_o + t u_{ib} (t) \left( s_i (t) - s_j (t) \right) $$

In the cost function (5a), the first term is used to track the reference velocity $\bar{v}_i (k)$, the second term minimizes the deviation from a desired distance between vehicle $i$ and its preceding vehicle $j$, and the last term minimizes the control effort. In (5a), $w_1$ and $w_3$ are constant weightings, while $w_2 (t)$, is chosen as a function of the relative distance, $(s_i (t) - s_j (t))$, so it increases as the relative distance decreases and vice versa. The choice of $w_2 (t)$ is similar to [22]. $v_{ib} (t)$
and $v_a(t)$ in (5b) indicate the speed limits of the road while $v_c(t)$ and $v_d(t)$ in (5c) denote the vehicle’s traction and deceleration limits. The problem in (5) also needs to be solved considering the constraints of the system dynamics in (4). $S_0$ and $t_{hd}$ in (5d) are predefined critical distance and headway time respectively.

C. Fuel Consumption Evaluation

The rate of fuel consumption for the conventional vehicles is evaluated by the polynomial metamodel proposed in [22]:

$$j_{fuel}(t) = j_{cruise}(t) + j_{accel}(t)$$

(5e)

$$j_{cruise}(t) = b_0 + b_1 \cdot v(t) + b_2 \cdot v(t)^2 + b_3 \cdot v(t)^3$$

(5f)

$$j_{accel}(t) = \dot{a}_i \left( r_1 + \dot{r}_1 \cdot v(t) + r_2 \cdot \dot{v}(t)^2 \right)$$

(5g)

$$\dot{a}_i = -\frac{1}{2M_i} C_D \rho_a \cdot \frac{A}{\theta} v_i - g \cdot g + u_i$$

(5h)

$j_{cruise}(t)$ and $j_{accel}(t)$ denote the fuel consumed by a vehicle travelling at constant velocity and the additional fuel consumption while the vehicle accelerating respectively. The vehicle and environmental related parameters are taken from [22]: $M_i = 1200$ kg, $A_i = 2.5$ m$^2$, $C_D = 0.32$, $\rho_a = 1.184$ kg/m$^3$ and $\theta = 0$. The polynomial coefficients are equal to: $b_0 = 0.1569$, $b_1 = 2.45 \times 10^{-3}$, $b_2 = -7.415 \times 10^{-4}$, $b_3 = 5.975 \times 10^{-5}$, $r_0 = 0.07224$, $r_1 = 9.681 \times 10^{-2}$ and $r_2 = 1.075 \times 10^{-3}$. If $v(t) = 0$ or $u_i < 0$, it means the vehicle is idling. The fuel consumption can be set to be constant as: $j_{fuel}(t) = 0.1$.

III. SIMULATION RESULTS

In this section, the performance of the proposed methodology is evaluated by a simulation-based case. In this case, we consider 52 vehicles initially within the intersection network. New vehicles will come into the control region at designed times. The effects of mixed vehicle types on the coordination strategy has also been discussed.

A. Simulation-Based Case Study

The intersection network and initial setup is shown in Fig. 3. The simulation scenario contains two interconnected intersections. There are 7 one-way roads in total and the length of each road is 250 m. Initially, there are 7 vehicles on each road except on road 4, which contains 10 vehicles initially. The red arrows in Fig. 3 indicate the vehicle moving directions. At first we assume all the vehicles are the same type, which means $w_p = w_g = 1$. There is a 10 m minimum distance between the two adjacent vehicles and 100 m minimum distance between the intersection and the vehicle closest to the intersection on upstream road in the initial set up, which gives the vehicles enough space to adjust their velocities to avoid collisions at intersection. In every 10 seconds, we consider new vehicles entering the roads 2, 5 and 7. We assume the vehicles do not overtake and turn at the intersections in this simulation scenario.

In the traffic flow model for this case, the free flow density threshold considered is $\rho_f = 7$, speed limit $v_f = 10$ m/s, and the slope $k_x = -0.25$. The average road velocity is set to be constant and equal to the speed limit for road 3, road 6 and road 7. The simulation runs for 150 seconds with a prediction horizon $T = 5$ seconds and a time step of $\Delta t = 0.5$ seconds.

The initial velocity of each vehicle $i$ is set to be $v_{0i} = 8$ m/s. In the simulation, we consider the vehicles and the intersection area are points. The vehicle length and the area of intersections are taken into account by the parameters of: time interval $\tau_0 = 1$ s, critical distance $S_0 = 10$ m and headway time $t_{hd} = 1.5$ s.

Fig. 3 Schematic of the initial simulation setup

Fig. 4 All initial vehicle trajectories on x-direction lanes

Fig. 4 shows the trajectories of all 24 vehicles initially on x-direction roads during the simulation. Fig. 5 and Fig. 6 shows the initial 14 vehicle trajectories on y-direction roads of intersection 1 and 2 respectively. The results show no intersection among any of the trajectories, which means there is no rear end collision. Figs. 7 and 8 show the relative distance of the vehicles coming from different directions to the intersection 1 and 2 respectively. It should be noted that the collision between vehicles from different directions (blue and red lines respectively) can only happen at the crossroad area.
As it is shown in Figs. 7 and 8, collision at the intersections are avoided when using our proposed method.

To demonstrate the advantages of our proposed method, we designed two baseline methods. The first one is called no I2I (Infrastructure-to-Infrastructure), which means one intersection does not have the traffic information of its neighborhood intersection. In this method, the road desired velocities ($v^*_m$ and $v^*_n$ in \(3a\)) are not calculated from \(2\), but directly from the modified traffic flow model \(1\). The vehicles will travel as the traffic flow without the consensus algorithm involved. The other baseline method is called no optimization. In this method, the I2I communication is still working. However, the cost function in \(3a\) is set to be constant and the constraints remain the same. So the intersection controller can still avoid vehicle collisions at intersections, but the assigned reference velocity is not based on the minimization of the deviation from the road average velocity.

We evaluate these approaches on two aspects: fuel efficiency (miles per gallon (mpg)) and mobility. For fuel efficiency, we compared the average, maximum and minimum fuel efficiency of all the initial vehicles. The mobility is measured as the total time taken by the initial vehicles to leave the given control region. Since the vehicles initially on road 3, road 6 and road 7 are tracking constant reference velocity and leave the intersection network quickly for all the methods, it is meaningless to take them into account while evaluating the fuel efficiency. The results without the aforementioned vehicles are tabulated in the Table I. The simulation results show fuel economy and mobility improvement over the baseline methods. It should be noted that even the performance the baseline methods are better than the conventional traffic light control due to the elimination of the unnecessary stop-and-go driving patterns.

### TABLE I

<table>
<thead>
<tr>
<th>Approach</th>
<th>Proposed</th>
<th>No I2I</th>
<th>No Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Fuel Efficiency (mpg)</td>
<td>48.22</td>
<td>47.35</td>
<td>47.43</td>
</tr>
<tr>
<td>Maximum Fuel Efficiency (mpg)</td>
<td>50.82</td>
<td>50.71</td>
<td>49.60</td>
</tr>
<tr>
<td>Minimum Fuel Efficiency (mpg)</td>
<td>42.53</td>
<td>41.03</td>
<td>43.41</td>
</tr>
<tr>
<td>All Vehicle Total Travel Time (s)</td>
<td>2054</td>
<td>2064.5</td>
<td>2096</td>
</tr>
</tbody>
</table>

### B. Effects of Mixed Vehicle Types

To study the effects of involving HEVs to our coordination strategy, the other scenario has been explored. In this case, we only consider one intersection and two vehicles approaching the intersection from x-direction and y-direction respectively. The initial velocity and distance to the intersection are the same for both of the vehicles, such that the ETA of the two vehicles to the intersection is the same and they need to adjust their velocities to avoid collision at the intersection. To simplify the simulation, we ignore the reference velocity tracking discussed in Section II B and assume the vehicles will travel at the reference velocity exactly at each time step.

We first set \(w_p = w_q = 1\), indicating the two vehicles are the same type. Then we set \(w_p = 0.8\) and \(w_q = 1\), which means the vehicle from x-direction is a HEV and the vehicle from y-direction is a conventional vehicle. We set the weighting factor of HEV is less than the conventional vehicle, because we want to encourage the velocity changes of HEV and maintain less velocity deviation of conventional vehicle. Since the HEV could recuperate from braking and accelerate with the power form battery, these weighting factors set up would achieve optimal overall fuel efficiency. Fig. 9 shows the simulation results. It can be noticed that in the same vehicle...
type simulation, where the weights are same, the controller will increase the velocity of one vehicle and decrease the other one randomly. In the other case, when different weights are used, the velocity of conventional vehicle remains the same during the simulation and the HEV changes its velocity to avoid collision. The vehicle passing sequence are different for the two cases as shown in Fig. 9. We believe the overall fuel efficiency can be improved by adding appropriate weighting factors representing different vehicle types. However, it is very hard to evaluate the fuel efficiency of the HEV in this case because the HEV only travels for a short distance and time to cross the intersection. Further research effort may worth to expend on this area.

![Fig. 8](image1.png)

**Fig. 8** Relative distance to the intersection of vehicles from road 4 and road 5 of intersection 2

![Fig. 9](image2.png)

**Fig. 9** Effects of vehicle type on intersection passing sequence: (a) two vehicles with the same weighting factor (b) two vehicles with different weighting factors

IV. CONCLUSION AND FUTURE WORK

In this paper, a distributed control strategy of CAVs at multiple interconnected intersections is presented. In the higher level, each intersection is considered to have an intersection controller in charge of sharing the neighborhood intersection traffic density information through I2I. The intersection controllers calculate the road average velocity to speed up the traffic density balance process over the traffic network. In the meanwhile, the intersection controllers assign the reference velocity for each vehicle within the control region based on the objective of minimizing the velocity deviation from the road average velocity and avoiding collision at the intersections. In the lower level, each vehicle is considered to have local controller that sends its position and velocity information to the higher level controller and utilizes MPC to track the reference velocity calculated from the higher level controller.

The proposed method has been implemented on a simulation-based case. The successful implementation indicates our proposed method has the capability to lead the CAVs pass the intersection network safely without conventional traffic light control. Compared with the two baseline methods (no I2I and no Optimization), the proposed method shows improvement of vehicle fuel efficiency and system mobility. A scenario with mixed vehicle type has also been explored. The simulation results show that with the effect of vehicle types, the proposed method can generate a different vehicle passing sequence with the objective of optimal overall fuel efficiency.

Future research directions can be implementing the method to more complex scenarios to demonstrate the scalability and feasibility. Some other future research directions include deeply exploring the effect of mixed vehicle type driving and communication delay or loss.

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