A Sustainable Design Model by Integrated Evaluation of Closed-loop Design and Supply Chain Using a Mathematical Model

Yuan-Jye Tseng, Yi-Shiuan Chen

Abstract—The paper presented a sustainable design model for integrated evaluation of the design and supply chain of a product for the sustainable objectives. To design a product, there can be alternative ways to assign the detailed specifications to fulfill the same design objectives. In the design alternative cases, different material and manufacturing processes with various supply chain activities may be required for the production. Therefore, it is required to evaluate the different design cases based on the sustainable objectives. In this research, a closed-loop design model is developed by integrating the forward design model and reverse design model. From the supply chain point of view, the decisions in the forward design model are connected with the forward supply chain. The decisions in the reverse design model are connected with the reverse supply chain considering the sustainable objectives. The purpose of this research is to develop a mathematical model for analyzing the design cases by integrated evaluating the criteria in the closed-loop design and the closed-loop supply chain. The decision variables are built to represent the design cases of the forward design and reverse design. The cost parameters in a forward design include the costs of material and manufacturing processes. The cost parameters in a reverse design include the costs of recycling, disassembly, reusing, remanufacturing, and disposing. The mathematical model is formulated to minimize the total cost under the design constraints. In practical applications, the decisions of the mathematical model can be used for selecting a design case for the purpose of sustainable design of a product. An example product is demonstrated in the paper. The test result shows that the sustainable design model is useful for integrated evaluation of the design and the supply chain to achieve the sustainable objectives.

Keywords—Closed-loop design, closed-loop supply chain, design evaluation, mathematical model, supply chain management, sustainable design model.

I. INTRODUCTION

To design a product with given design concept and product requirement, a conceptual design model can be constructed. Subsequently, in the detailed design stage, the product specifications can be modeled to fulfill the design objectives. With the given design objectives, there can be alternative ways to design the detailed specifications of the components and product. For example, the components can be designed with different geometric shapes and dimensions. Alternatively, different materials and manufacturing processes can be utilized for processing the components. In this way, the different design cases can be modeled to achieve the same design objectives and can be used to produce the same product. Therefore, in the design evaluation stage, it is required to analyze and evaluate the design cases to select a good design case to be used as the final design.

In the design cases, if the detailed specifications are different, different manufacturing processes and operations may be required for the production. The manufacturing processes and operations will affect the supply chain activities. Therefore, it is necessary to evaluate the different design cases based on the supply chain objectives. In a traditional supply chain management system, the supply chain evaluation is made mainly based on the criteria of time and cost. In contrast, in the concept of green supply chain, the criteria of energy usage and environmental impact are the main factors to be considered in order to achieve the sustainable objectives.

To link design and supply chain, the forward design model and reverse design model are developed in this research. The forward design model is the traditional design model with the common design criteria and objectives. In addition to the original product design objectives, the design decisions in the forward design model need to consider the criteria from the manufacturing point of view. In the design evaluation stage, the criteria and objectives of material and manufacturing processes need to be considered for selecting a good design case. From the supply chain point of view, the forward design model is linked with the forward supply chain. The forward supply chain can be considered as the typical supply chain in which the main evaluation criteria are the manufacturing-related cost items. The typical objective in a forward supply chain is to produce the product with a minimized cost. In this research, the design of a product that considers the forward supply chain can be described as a forward design model.

In this research, the reverse design model is developed to connect with the reverse supply chain for the sustainable objectives. In the reverse design model, the design decisions are made based on the criteria and objectives for sustainable purposes, for example, recycling, disassembly, reusing, remanufacturing, and disposal. For a design evaluation in the reverse design model, it is necessary to consider how the product can be recycled, disassembled, reused, remanufactured, or disposed when the product is designed and produced. In this paper, the design of a product that considers the reverse supply chain is described as a reverse design model. The framework is illustrated in Fig. 1.
As shown in Fig. 1, the concept of a sustainable design model can be achieved by integrating the forward design and reverse design models to form a closed-loop design model. From the supply chain point of view, the decisions and criteria in the forward design model are connected with the forward supply chain considering material and manufacturing processes. The design decisions and criteria in the reverse design model are connected with the reverse supply chain considering recycling, disassembly, reusing, remanufacturing, and disposing. The forward supply chain and reverse supply chain can be integrated to form a closed-loop supply chain to achieve the goal of a sustainable design model.

In order to analyze the design cases, a mathematical programming model is presented for integrated evaluation of the cost parameters in the closed-loop design model. The decision variables are formulated to represent the design cases in the forward design and reverse design. A design case represents a set of detailed specifications including the geometric shapes and dimensions. The cost parameters of a forward supply chain include the costs of material and manufacturing processes. The cost parameters of a reverse supply chain include the costs of recycling, disassembly, reusing, remanufacturing, and disposing. The mathematical model is formulated to minimize the total cost under the constraints of the relationships of the design and the supply chain activities.

In practical applications, the decisions of the mathematical model can be used for decision-making to select the good or the best design case to achieve the sustainable design objectives. The mathematical model has been implemented and tested with example products. It shows that the model is useful for integrated evaluation of forward design and reverse design to form a closed-loop design model. The objectives of the sustainable design can be achieved by integrated evaluating the criteria in the closed-loop design and the closed-loop supply chain models.

This paper is organized as follows. In Chapter I, an introduction is presented. Chapter II presents a literature review. Chapter III describes the mathematical model. In Chapter IV, the application of the model is demonstrated and discussed. Finally, a conclusion is presented in Chapter V.

II. LITERATURE REVIEW

In previous researches, the problems of supplier selection in supply chains have been presented and modeled with various approaches and methods. In the research of [1], a closed-loop design model for sustainable manufacturing by integrating forward design and reverse design is presented. The fuzzy analytical network process model was presented in [1] to evaluate the criteria in forward design, reverse design, and supply chain. In the research of [2], a product development for green logistics model by integrated evaluation of design and manufacturing and green supply chain has been presented. The literature review of [2] investigated the related literature in closed-loop supply chain research. The research in [3]-[5] presented the methods for supplier evaluation and selection. In the research in [6], a literature review of supply chain performance measurement was presented. The models of forward and reverse logics and green supply chains have been presented in [7] and [8]. In the papers of [9] and [10], the problems of close-loop supply chain were discussed and modeled. In the research of [11]-[12], the models of close-loop supply chain were developed with solution methods. The applications of close-loop supply chain models have been presented in [13]-[15].

As discussed in the research of [1] and [2], many of the previous papers presented models for investigating green supply chains and closed-loop supply chains. Many models and solution methods for solving the supplier selection problems have been developed. Several papers presented models and optimization methods for integrating forward and reverse supply chains. However, the issue that the product design can affect the supply chain has rarely been discussed.

Based on review of the above research [1]-[15], the traditional design activities are restricted to the forward design portion. It is observed that the traditional design decisions are suitable to be connected with the forward supply chain. It requires a reverse design model for making the design decisions to connect with the reverse supply chain. If the reverse design model can be built, the closed-loop design model can be utilized to evaluate the closed-loop supply chain in the design evaluation stage. The link of a product design and the closed-loop supply chain needs to be further investigated.

Based on the discussion, in order to make a more complete evaluation of design decisions, in this research, the model of closed-loop design is developed by integrating the forward design and reverse models. With the new concept, a sustainable design model is developed for evaluating the criteria in the closed-loop design and closed-loop supply chain. The mathematical model is used for evaluation of design cases to achieve the sustainable objectives.

III. RESEARCH METHODS AND MATHEMATICAL MODELS

The purpose of the research is to develop a mathematical model for integrated evaluation of the closed-loop design and closed-loop supply chain for the sustainable design objectives. The goal is to find the suitable design case and the selection of material and manufacturing processes. In the developed model,
the related cost parameters are formulated. The total cost is the sum of the defined cost items. The decision variables are modeled as 0-1 integer variables representing whether a design case is selected. A design case represents a set of detailed design specifications including geometric shapes and dimensions. A design case \( i \) is selected if \( i = 1, 2, ..., I \) represents a forward design case. A design case \( l \) is selected if \( l = 1, 2, ..., L \) represents a reverse design case. The model is described as follows.

### A. Notations

1) Notations of Indices

The notations are listed as follows:

- \( i \): a forward design case (\( i = 1, 2, ..., I \))
- \( j \): a type of material (\( j = 1, 2, ..., J \))
- \( k \): a manufacturing process (\( k = 1, 2, ..., K \))
- \( l \): a reverse design case (\( l = 1, 2, ..., L \))

2) Parameters

The parameters are defined as follows:

- \( C \): the total cost
- \( C_{\text{forward}} \): the cost of a forward design connected with the forward supply chain
- \( C_{\text{reverse}} \): the cost of a reverse design connected with the reverse supply chain
- \( C^m_{ijk} \): cost of material \( j \) using manufacturing process \( k \) in forward design case \( i \)
- \( C^p_{ijkl} \): cost of manufacturing process \( k \) using material \( j \) in forward design case \( i \)
- \( C^{r}_{ijkl} \): cost of recycle in reverse design case \( l \) using material \( j \) and manufacturing process \( k \) with forward design case \( i \)
- \( C^{r}_{ijkl} \): cost of reuse in reverse design case \( l \) using material \( j \) and manufacturing process \( k \) with forward design case \( i \)
- \( C^{r}_{ijkl} \): cost of remanufacturing in reverse design case \( l \) using material \( j \) and manufacturing process \( k \) with forward design case \( i \)
- \( C^{d}_{ijkl} \): cost of disassembly in reverse design case \( l \) using material \( j \) and manufacturing process \( k \) with forward design case \( i \)
- \( C^{d}_{ijkl} \): cost of disposal in reverse design case \( l \) using material \( j \) and manufacturing process \( k \) with forward design case \( i \)

3) Variables

The decision variables are defined as follows:

- \( X_{ijkl} \): 0-1 integer variable representing whether forward design case \( i \) selected with the use of material \( j \) and manufacturing process \( k \), where a value of 1 represents that the forward design case decisions are selected.
- \( Y_{ijkl} \): 0-1 integer variable representing whether reverse design case \( l \) is selected with the corresponding forward design case \( i \) with the use of material \( j \) and manufacturing process \( k \), where a value of 1 represents that the design case decisions of forward design case \( i \) and reverse design case \( l \) are selected.

### B. The Mathematical Model

Min \( \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} \left[ C_{\text{forward}} \times X_{ijkl} \right] + \left[ C_{\text{reverse}} \times Y_{ijkl} \right] \)  

(1)

\[ C_{\text{forward}} = \left[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \left( X_{ijkl} \times C^m_{ijk} \right) \right] + \left[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \left( X_{ijkl} \times C^p_{ijkl} \right) \right] \]

(2)

\[ C_{\text{reverse}} = \left[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} \left( Y_{ijkl} \times C^m_{ijkl} \right) \right] + \left[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} \left( Y_{ijkl} \times C^p_{ijkl} \right) \right] \]

(3)

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} X_{ijkl} = 1 \]

(4)

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} Y_{ijkl} = 1 \]

(5)

\[ Y_{ijkl} \leq X_{ijkl} \quad \forall \ i, j, k, l \]

(6)

\[ X_{ijkl} = \begin{cases} 1 & \forall \ i, j, k, l \\ 0 & \text{otherwise} \end{cases} \]

(7)

\[ Y_{ijkl} = \begin{cases} 1 & \forall \ i, j, k, l \\ 0 & \text{otherwise} \end{cases} \]

(8)

### C. Explanation of the Mathematical Model

1) Objective Function

In (1), the objective is to minimize the total cost of the cost parameters in the forward design and reverse design. The goal is to achieve a sustainable design by making decisions of the design cases, materials, and manufacturing processes. The decision variables are modeled as 0-1 integer variables. The decision variable \( X_{ijkl} \) represents the decision in a forward design case and indicating whether material \( j \) and manufacturing process \( k \) is selected for forward design case \( i \). The decision variable \( Y_{ijkl} \) represents the decision in a reverse design case indicating whether material \( j \) and manufacturing process \( k \) is selected for forward design case \( i \) with a corresponding reverse design case \( l \). There are two main cost parameters in a forward design: material cost and manufacturing process cost. There are five main cost parameters in a reverse design: reverse, reusing, remanufacturing, disassembly, and disposing. The total cost is the sum of all the cost parameters in the forward design and reverse design. The objective is to minimize the total cost to achieve the goal of a sustainable design. The cost parameters are described as follows.

2) Forward Design Cost

The forward design cost parameters include material cost and manufacturing process cost.

i. Material Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} X_{ijkl} \times C^m_{ijkl} \]

Material cost can be obtained by multiplying cost parameter of material \( C^m_{ijkl} \) with decision variable \( X_{ijkl} \) indicating whether material \( j \) is selected in design case \( i \).
ii. Manufacturing Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} \times C_{ijkl}^p \]

Manufacturing process cost can be obtained by multiplying cost parameter of manufacturing process \( C_{ijkl}^p \) with decision variable \( X_{ijkl} \) indicating whether manufacturing process \( k \) is selected in design case \( i \).

3) Reverse Design Cost

The reverse design cost parameters include recycle cost, reuse cost, remanufacturing cost, disassembly cost, and disposal cost.

i. Recycle Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} \times C_{ijkl}^r \]

Recycle cost can be obtained by multiplying cost parameter of recycle \( C_{ijkl}^r \) with decision variable \( Y_{ijkl} \) indicating the required recycle cost if the forward design case \( i \) and the corresponding reverse design \( l \) are selected.

ii. Reuse Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} \times C_{ijkl}^u \]

Reuse cost can be obtained by multiplying cost parameter of reuse \( C_{ijkl}^u \) with decision variable \( Y_{ijkl} \) indicating the required reuse cost if the forward design case \( i \) and the corresponding reverse design \( l \) are selected.

iii. Remanufacturing Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} \times C_{ijkl}^f \]

Remanufacturing cost can be obtained by multiplying cost parameter of remanufacturing \( C_{ijkl}^f \) with decision variable \( Y_{ijkl} \) indicating the required remanufacturing cost if the forward design case \( i \) and the corresponding reverse design \( l \) are selected.

iv. Disassembly Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} \times C_{ijkl}^d \]

Disassembly cost can be obtained by multiplying cost parameter of disassembly \( C_{ijkl}^d \) with decision variable \( Y_{ijkl} \) indicating the required disassembly cost if the forward design case \( i \) and the corresponding reverse design \( l \) are selected.

v. Disposal Cost

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} \times C_{ijkl}^g \]

Disposal cost can be obtained by multiplying cost parameter of disposal \( C_{ijkl}^g \) with decision variable \( Y_{ijkl} \) indicating the required disposal cost if the forward design case \( i \) and the corresponding reverse design \( l \) are selected.

4) Assignment Constraints

i. First assignment constraint represents the design constrain of a forward design describing that at least one material and one manufacturing process is selected for a forward design case \( i \):

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} X_{ijkl} = 1 \]

ii. Second assignment constraint represents the design constraint of a reverse design describing that at least one material and one manufacturing process is selected for a reverse design case \( l \) when a corresponding reverse design case \( l \) is selected:

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} Y_{ijkl} = 1 \]

5) Variable Constraints

i. \( X_{ijkl} \) : 0-1 integer variable indicating whether material \( j \) and manufacturing process \( k \) is selected for the forward design case \( i \).

ii. \( Y_{ijkl} \) : 0-1 integer variable indicating whether material \( j \) and manufacturing process \( k \) is selected for the forward design case \( i \) with a corresponding selection of the reverse design case \( l \).

The above mathematical model is formulated to minimize the objective of total cost under the design constraints. From a sustainable design point of view, the cost items can be classified into two main categories: forward design cost and reverse design cost. The forward design cost is linked with the material cost and manufacturing process cost in the forward supply chain. The material and manufacturing process related cost items are the traditional cost items evaluated in the typical supply chain management models. The reverse design cost is linked with the cost items in a reverse supply chain. The reverse design cost items include recycle, reuse, remanufacturing, disassembly, and disposal in the reverse supply chain. The total cost is the sum of the above defined cost items. The objective is to decide a design case with corresponding forward design case \( i \) and reverse design case \( l \) by minimizing the total cost under the assignment constraints and variable constraints.
IV. APPLICATION TO EVALUATION OF DESIGN CASES USING THE MODEL

In this research, the models have been implemented and solved using a linear programming software package on a personal computer. In this presentation, a simplified notebook computer is used as an example product for demonstration. As shown in Fig. 2, a simplified notebook computer is illustrated. The information and data of the product are modeled and defined for testing and computation. Based on the developed concept and the presented models, the product can be designed with different design alternative cases. In the different design cases, the selections of material and the manufacturing processes for producing the components and the product can be different. The framework of decision-making is illustrated in Fig. 3.

![Diagram](image1)

**Fig. 2** A notebook computer is used as the example product for demonstration

![Diagram](image2)

**Fig. 3** The framework of decision-making of forward design case and reverse design case

The numerical data of the cost parameters can be modeled in a table format as shown in Table I. A design case represents a set of detailed design specifications including geometric shapes and dimensions. A design case \( i = 1, 2, ..., I \), represents a forward design case. A design case \( l = 1, 2, ..., L \), represents a reverse design case. With the given information and data, the mathematical model can be formulated and solved. For each component of the product, the best forward design case can be determined as indicated in the decision variable \( X_{ijkl} \). The best reverse design case can be determined as indicated in the decision variable \( Y_{ijkl} \). The decision variables are linked with the detailed numerical data. Therefore, the best design case combination of forward design and reverse design can be decided and the total cost can be calculated. With the decision variable and cost parameters in (1) and the cost data in Table I, the optimized total cost can be calculated.

**TABLE I**

<table>
<thead>
<tr>
<th>Forward Design Case</th>
<th>Material j</th>
<th>Process k</th>
<th>Reverse Design Case</th>
<th>( C_{ijkl} )</th>
<th>( C_{ijkl}^r )</th>
<th>( C_{ijkl}^f )</th>
<th>( C_{ijkl}^d )</th>
<th>( C_{ijkl}^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1, ..., I )</td>
<td>( j = 1, ..., J )</td>
<td>( k = 1, ..., K )</td>
<td>( l = 1, ..., L )</td>
<td>( \bar{l} = 1, ..., \bar{L} )</td>
<td>( \bar{i} = 1, ..., \bar{I} )</td>
<td>( \bar{j} = 1, ..., \bar{J} )</td>
<td>( \bar{k} = 1, ..., \bar{K} )</td>
<td></td>
</tr>
</tbody>
</table>

In the research, the methods and models have been implemented and tested with practical products. In the presentation, an example product is demonstrated and discussed.

V. CONCLUSIONS

In this paper, a concept of sustainable design is presented. The sustainable design model can be executed by using a closed-loop design model by integrating forward design and reverse design. A mathematical model for integrated evaluation of forward design and reverse design is presented. In the design stage, there can be different design cases to achieve the same design requirement and objective. To evaluate the design cases, in the presented mathematical model, the decision variables are formulated to represent the design cases in the forward design and reverse design. The costs in the forward design model are connected with the forward supply chain. The costs in the reverse design model are connected with the reverse supply chain. The cost parameters of a forward design model include the costs of material and manufacturing processes. The cost parameters of a reverse design model include the costs of recycling, disassembly, remanufacturing, and disposing. The mathematical model is formulated to minimize the total cost under the constraints of the relationships of the design and the supply chain activities. In practical applications, the decisions of the mathematical model can be used for decision-making to select the good or the best design case to achieve the sustainable design objectives. The minimized total cost can be calculated according to the decision variables and cost parameter. In the research, the models have been tested with practical products and demonstrated with an example in the presentation. It shows that the model is useful for integrated...
evaluation of forward design and reverse design to achieve a sustainable design objective. Future research can be planned to investigate more detailed mathematical model and cost parameters for further evaluation.

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REFERENCES


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