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Frequency Estimation Using Analytic Signal via Wavelet Transform

Sudipta Majumdar, Akansha Singh

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***Abstract*—**Frequency estimation of a sinusoid in white noise using maximum entropy power spectral estimation has been shown to be very sensitive to initial sinusoidal phase. This paper presents use of wavelet transform to find an analytic signal for frequency estimation using maximum entropy method (MEM) and compared the results with frequency estimation using analytic signal by Hilbert transform method and frequency estimation using real data together with MEM. The presented method shows the improved estimation precision and antinoise performance.

***Keywords*—**Frequency estimation, analytic signal, maximum entropy method, wavelet transform.

I. Introduction

FREQUENCY estimation of a sinusoid from noisy signal occurs in many signal processing problems. Frequency estimation from the narrow band signal recorded in noisy environment requires robust and high resolution spectrum estimation techniques. The spectrum estimation methods are generally categorized into two classes: The classical or nonparametric method and parametric method. The nonparametric methods are based on estimating the autocorrelation sequence from the given set of data. The power spectrum is then obtained by Fourier transform of the estimated autocorrelation sequence. The nonparametric methods include the periodogram method, the modified periodogram method, Bartlett's method, Welch's method, Blackman - Tukey method etc.

The parametric methods of the spectrum estimation are based on a parametric model for the data. These methods select the model, estimate the model parameters using the given data and then, estimate the power spectrum by incorporating the estimated parameters in the parametric form of the spectrum. The MEM is equivalent to spectrum estimation using all pole model.

Burg MEM extrapolates the known auto correlation function using the assumption that unknown information is subjected to the maximum entropy. Burg's algorithm has the problem of line splitting and spectral shifting. The spectral line splitting is due to Levinson recursion and selection of one order reflection coefficient in Yule-Walker equation. Frequency estimation using Burg's method is also affected by various factors such as data length, signal to noise ratio and initial phase of signal data.

Sudipta Majumdar is with the Electronics and Communication Engineering Department Delhi Technological University, Shahbad Daulatpur, Main Bawana Road, Delhi, India (e-mail: korsudipta@rediffmail.com).

Akansha Singh was with the Delhi Technological University, Delhi, India (e-mail:akanshasingh5614@gmail.com).

Chen and Stegen [1] showed that frequency estimation of a sinusoid in white noise using maximum entropy power spectrum estimation with Burg's estimate for reflection coefficient method is very sensitive to initial phase of the sinusoid for short data length. Phase dependent frequency estimation is due to interaction between the positive and negative frequency spectra [2]. This dependence of frequency estimation on phase can be reduced by reducing the interaction between positive and negative frequency. Kay [3] showed that the dependence of frequency estimation of a sinusoid (in white noise) on the initial phase of the signal can be reduced significantly using an analytic signal approach, as the analytic signal of a real sinusoid in white noise obtained using Hilbert transform has zero power in negative Nyquist interval. But, it results in nonwhite noise. For the peak of the estimated power spectral density to be an unbiased estimate of frequency, it requires the whiteness of the noise. Since the noise in this analytic signal is nonwhite, Jackson and Tufts [4], [5] showed that it can be made white by reducing the sampling rate by two. They showed that the performance of frequency estimation can be improved by processing a complex valued version of the real value input signal, with the corresponding sampling rate reduced by half.

Kay used Hilbert transform to obtain an analytic signal. We used wavelet transform theorems presented by Gao [6] to obtain an analytic signal.

II. Hilbert Transform

Hilbert transform (HT) has useful applications in signal processing and network theory. It is also used in radar signal processing, seismic signal processing, speech signal processing, communication systems etc. The Hilbert transform $\hat{f}(t)$ of the function $f(t)$ is defined for all t by

$$\hat{f}(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau} d\tau$$

if the integral exists. P denotes the Cauchy principal value of the integral.

The HT of a signal $f \epsilon L^2(R)$ or generally $f \epsilon L^p(R)$, where, $1 < p < \infty$ is defined in the spatial domain as convolution with the Hilbert kernel.

$$\mathcal{H} f = h * f$$

The Hilbert kernel is given by $h(t) = \frac{1}{\pi t}$

An ideal HT is an all pass filter that provides a 90^o phase shift to the signal at its input. The frequency response of the ideal Hilbert transformer is defined as

$$\mathcal{H}(w) = \begin{cases} j; & 0 < w \leq \pi \\ -j; & -\pi < w < 0 \end{cases} \tag{1}$$

The unit sample response of an ideal HT is

$$h_d(n) = \begin{cases} \frac{2}{\pi}\frac{sin^2\frac{(\pi n)}{2}}{n}; & n \neq 0 \\ 0; & \text{n=0} \end{cases} \tag{2}$$

$h_d(n)$ is infinite in duration and noncausal. The slow decay rate and the infinite length of the impulse of HT generates oscillatory behaviour known as Gibbs effect. This oscillatory behaviour of impulse response of HT reduces the estimation precision in digital implementation.

III. Analytic Signal

Complex signal, whose imaginary part is the HT of its real part, is called the analytic signal. An analytic signal has one sided Fourier spectrum.

IV. Wavelet Transform

Wavelet is a time frequency transform. The wavelet transform (WT) employs a set of basis functions which are scaled version of single mother function. The continuous-time wavelet transform (CWT) of $f(t)$ with respect to a wavelet $\Psi(t)$ is defined as

$$W(a,b) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{|a|}}\Psi^*\left(\frac{t-b}{a}\right)dt$$

Thus, the WT is a two variable function where a and b are real and $*$ denotes complex conjugation. $f(t)$ and $\Psi(t)$ belong to $L^2(R)$, the set of square integrable functions, also called the set of energy signals. a is called the scale or dilation variable. b represents the time shift or translation. The normalization factor of $\frac{1}{\sqrt{|a|}}$ insures that the energy remains the same for all a and b.

As the CWT is generated using dilation and translation of a single function $\Psi(t)$, the wavelet is called as mother wavelet. This mother wavelet has to satisfy two conditions known as regularity and admissibility condition. Also, $\Psi(t)$ must be compactly supported in both time and frequency. The CWT is able to localize events both in time and in frequency, which is useful for nonstationary signal analysis. The WT has zoom in zoom out property, as the size of the analysis window can be changed.

A. Analytic Signal Using Wavelet Transform

We used the wavelet based method proposed by Gao [6] to calculate the analytic part of a real valued signal in $L^2(R)$. Gao showed that the analytic part of any arbitrary real valued signal $s(t) \in L^2(R, dt)$ is

$$\frac{1}{C_g}\int_0^{\infty} S(t,a)\frac{da}{a} = s(t) + jH[s(t)] \tag{3}$$

where $S(t,a)$ is defined as

$$S(b,a) = \frac{1}{a}\int_{-\infty}^{\infty} s(t)\bar{g}(\frac{t-b}{a})dt \tag{4}$$

$t, b \in R$, R is the real number set and $a > 0$. $S(b,a)$ is the wavelet transform of $s(t)$ with respect to the analytic wavelet function $g(t)$. $g(t)$ and its Fourier transform $\hat{g}(\omega)$ satisfying $g(t) \in L^1(R,dt)\bigcap L^2(R,dt)$ and $\hat{g}(\omega) \in L^1(R\backslash\{0\}, d\omega/|\omega|) \cap L^2(R\backslash\{0\}, d\omega/|\omega|)$ respectively. The real part $g_R(t)$ of $g(t)$ is even. $g(t)$ is the complex conjugate of $g(t)$ and $C_g = \int_0^{\infty}(\hat{g}_R(\omega)/\omega)d\omega$, with $0 < C_g < \infty$.

V. Maximum Entropy Method

Spectral estimation using the MEM is used to improve the spectral quality based on the principle of maximum entropy. The method is based on choosing the spectrum which corresponds to the most random or the most unpredictable time series whose autocorrelation function agrees with the known values. Burgs proposed the maximum entropy spectral analysis method [2] to enhance the spectrum resolution and to increase the adaptability of spectral estimation algorithm to signal length, signal to noise ratio and initial phase.

The MEM for spectral estimation is based on an explicit extrapolation of a finite length sequence of a known autocorrelation of a random process [8]. This extrapolation has to be performed in such a way that the random process characterized by the extrapolated autocorrelation sequence has maximum entropy. The random process is assumed to be Gaussian; so that, the maximizing entropy becomes mathematically solvable. For a Gaussian random process $x(n)$, with spectrum $P_x(e^{jw})$ and autocorrelation $r_x(k)$ for lags $|k| \leq p$, MEM extrapolates $r_x(k)$ for $|k| > p$. The entropy of the random variable $x(n)$ is expressed by

$$H(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} lnP_x(e^{jw})dw \tag{5}$$

MEM maximizes $H(x)$ by assuming the following condition

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} P_x(e^{jwk})dw = r_x(k); |k| \leq p \tag{6}$$

Representing the extrapolated autocorrelation by $r_e(k)$, the power spectrum of $x(n)$ can be written as

$$P_x(e^{jw}) = \sum_{k=-p}^{p} r_x(k)e^{-jkw} + \sum_{|k|>p} r_e(k)e^{-jkw} \tag{7}$$

For a valid power spectrum, $P_x(e^{jw})$ should be real valued and nonnegative for all w. A maximum entropy extrapolation is equivalent to finding the sequence of the extrapolated autocorrelations that make x_n as white (random) as possible. From the power spectrum point of view, this maximum entropy extrapolation makes the power spectrum as flat as possible. Assuming $x(n)$ to be the Gaussian process with autocorrelation $r_x(k)$ for $|k| \leq p$, the extrapolated autocorrelation $r_e(k)$ that maximizes the entropy, can be obtained by setting

$$\frac{\partial H(x)}{\partial r_e^*(k)} = 0; |k| > p \tag{8}$$

or

$$\frac{\partial H(x)}{\partial r_e^*(k)} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{P_x(e^{jw})}\frac{\partial P_x(e^{jw})}{\partial r_e^*(k)}dw = 0; |k| > p \tag{9}$$

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where,

$$\frac{\partial P_x(e^{jw})}{\partial r_e^*(k)} = e^{jkw} \tag{10}$$

Substituting (10) in (9)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{P_x(e^{jw})}e^{jkw}dw = 0; |k| > p$$

Defining, $Q_x(^{jw}) = \frac{1}{P_x(e^{jw})}$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}Q_x(^{jw})e^{jkw}dw = 0$$

or,

$$q_x(k) = 0; |k| > p$$

which shows that the inverse Fourier transform of $Q_x(^{jw})$, namely, $q_x(k)$ is equal to zero for $|k| > p$. As the Fourier transform of $q_x(k)$ is

$$Q_x(e^{jw}) = \frac{1}{P_x(e^{jw})} = \sum_{-\infty}^{\infty} q_x(k)e^{-jkw} \tag{11}$$

From (7), the MEM estimate of the power spectrum $P_x(e^{jw})$ for a Gaussian process is defined as

$$\hat{P}_{MEM} = \frac{1}{\sum_{-p}^{p} q_x(k)e^{-jkw}} \tag{12}$$

Noting that $q_x(-k) = q_x^*(k)$, and

$$\sum_{k=-p}^{p} q_x(^k)e^{-jkw} = \sum_{k=1}^{p} q_x^*(k)e^{+jkw} + q_0 + \sum_{k=1}^{p} q_x(k)e^{-jkw} \tag{13}$$

$$\hat{P}_{MEM} = \frac{b(0)b^*(0)}{[1+\sum_{k=1}^{p} a_p(k)e^{-jkw}][1+\sum_{k=1}^{p} a_p^*(k)e^{jkw}]} \tag{14}$$

$$\hat{P}_{MEM} = \frac{|b(0)|^2}{|1+\sum_{k=1}^{p} a_p(k)e^{-jkw}|^2} \tag{15}$$

VI. Maximum Entropy Estimation Using Analytic Signal

We used Key's method of maximum entropy using analytic signal. Let

$$X_t^{'} = Asin(\omega_0 t + \phi) + W_t^{'} \tag{16}$$

be the real discrete signal where $W_t^{'}$ is white noise with zero mean and $R_{W'}(k) = E(W_t^{'}W_{t+k}^{'}) = \sigma_{W'}^2\delta(k)$ then, the corresponding analytic signal is

$$Z_t^{'} = X_t^{'} + j\hat{X}_t^{'} \tag{17}$$

where

$$Z_t^{'} = -jAe^{j(w_0 t+\phi)} + W_t + j\hat{W}_t \tag{18}$$

and $\hat{X}_t^{'}$ is the Hilbert transform of $X_t^{'}$. The resultant downsampled signal is

$$Z_t = Z_{2t}^{'} = -jAe^{j(2\omega_0 t+\phi)} + W_{(t)}^c \tag{19}$$

where $E[W_t^c] = 0$ and $R_{W^c}(k) = E[W_t^{c*}W_{t+k}^c]$.

The maximum entropy power spectral estimator has been used to find the real and complex spectrum. They are

$$P_r(w) = \frac{P_{pr}\Delta_0}{|1+a_r^1 e^{-j\omega} + ... + a_{p_r}^r e^{-j\omega p_r}|^2} \tag{20}$$

$$P_c(w) = \frac{P_{pc}\Delta_0}{|1+a_c^1 e^{-j\omega} + ... + a_{p_c}^c e^{-j\omega p_c}|^2} \tag{21}$$

where p_r and p_c are predictor order for the real and complex spectra respectively. The parameter sets $\{a_1^r, ..., a_{pr}^r, P_{pr}\}$ and $\{a_1^c, ..., a_{pc}^c, P_{pc}\}$ are determined from the Burg estimation method together with Levinson recursion [7]. Here $p_c = \frac{1}{2}p_r$. P_{pr} and P_{pc} are prediction error power for the real and complex spectra respectively.

VII. Simulation Results

A MATLAB based simulation is used on real and analytic signal by Hilbert transform method and wavelet transform method to compare the effect of initial phase on frequency estimation. Single real sinusoid in white Gaussian noise is used. The frequency of the sinusoid is 1 Hz, sampled 20 times/second or sampling time $= \Delta_0 = 0.05$ second. 41 samples of real data has been used. The predictor order used is 9 and SNR=10 dB and phase of input sinusoid is varied. The peak of $P_r(\omega)$ or $P_c(\omega)$ gives estimated frequency.

Analytic signal is formed using Hilbert transform and wavelet transform method. The modified Morlet wavelet is defined as

$$g_\tau(t) = e^{imt}e^{(-1/2)}[\sqrt{2}\sigma m/(2\pi\tau)t]^2$$
$$= cos(mt)e^{-(1/2)[\sqrt{2}\sigma m/(2\pi\tau)t]^2} + jsin(mt)e^{-(1/2)[\sqrt{2}\sigma m/(2\pi\tau)t]^2} \tag{22}$$

where m is the angular frequency, τ is the number of cycles of carrier wave in an envelope. σ is a real number related to precision (when $|g(t)| \leq e^{-\sigma^2}, g(t)$ can be approximated as zero). Let $C = \sqrt{2}\sigma m/2\pi\tau$. For numerical computation $m^2/(4C^2)$ is large enough so that the wavelet defined in (19) satisfies Theorem 2 in [6].

For wavelet method, $\sigma = 5$, $\tau = 4$, $m = 28.28$. Phase of the input sinusoid is varied. The dependence of frequency estimation on sinusoid phase for real signal, analytic signal by Hilbert transform and analytic signal by wavelet transform are shown in Fig. 1. Large variations are obtained, when real data are used and it is reduced when analytic signal by Hilbert transform is used. This variation is further reduced, when analytic signal by wavelet method is used. The maximum frequency deviation using real data is 11 percent, using analytic signal by Hilbert transform is 4.4 percent and using wavelet transform is 0.39 percent. For SNR = 5 dB, the error obtained by using real data is 13.78 percent, 6.67 percent by Hilbert transform method and 0.39 percent by wavelet transform method.

VIII. Conclusion

The wavelet method gives the smallest dependence of frequency estimation on the phase of the sinusoid in white noise. This is due to time frequency localization property

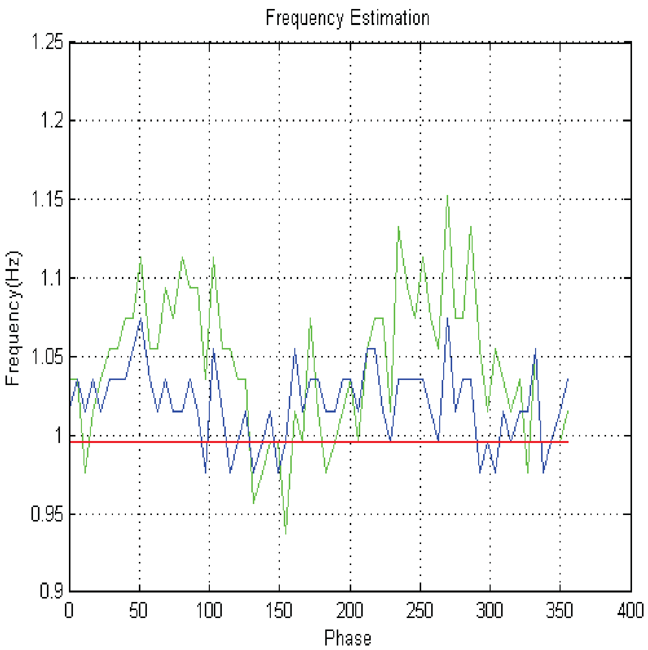

Fig. 1 Frequency estimate versus phase for SNR=10 dB. Green color shows for real data, blue color for analytic signal by Hilbert transform and red color shows for analytic signal using Wavelet transform method

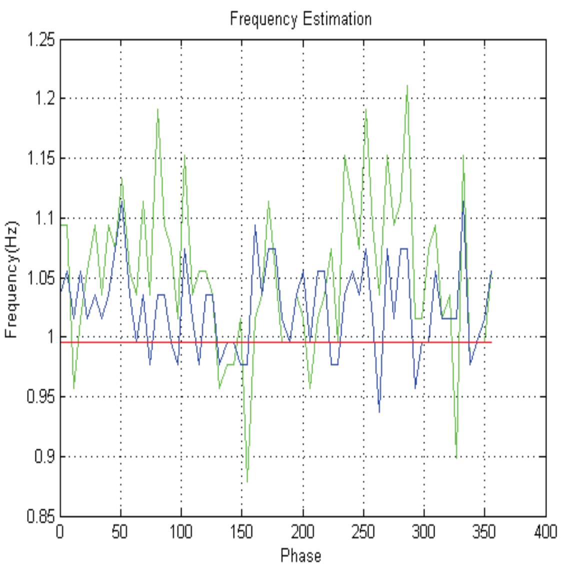

Fig. 2 Frequency estimate versus phase for SNR=5 dB. Green color shows for real data, blue color for analytic signal by Hilbert transform and red color shows for analytic signal using Wavelet transform method

and antinoise performance of the wavelet transform. Also, the oscillatory nature (Gibbs phenomenon) of the impulse response of Hilbert transform reduces the estimation precision in digital implementation.

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Sudipta Majumdar Sudipta Majumdar completed her B. Tech. and M. Tech. from University of Allahabad. She received her Ph.D. degree from Delhi University.Her research interest includes nonlinear analysis and signal processing.

Akansha Singh Akansha Singh completed her M. Tech. from Delhi Technological University.

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