Damage Strain Analysis of Parallel Fiber Eutectic

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Abstract—According to isotropy of parallel fiber eutectic, the no-damage strain field in parallel fiber eutectic is obtained from the flexibility tensor of parallel fiber eutectic. Considering the damage behavior of parallel fiber eutectic, damage variables are introduced to determine the strain field of parallel fiber eutectic. The damage strains in the matrix, interphase, and fiber of parallel fiber eutectic are quantitatively analyzed. Results show that damage strains are not only associated with the fiber volume fraction of parallel fiber eutectic, but also with the damage degree.

Keywords—Parallel fiber eutectic, no-damage strain, damage strain, fiber volume fraction, damage degree.

I.  INTRODUCTION

RECENTLY, using combustion synthesis to fabricate eutectic composite ceramic, the ceramics had high performance, high reliability, and low cost. Directionally solidified eutectics contain a large amount of clean interfaces between two strongly-bonded phases with typical inter-phase spacing in the micron range, and these characteristics result in an improvement of some material properties. For instance, fibrous eutectics present a very high flexural, tensile strength, and high toughness at room and high-temperatures [1]-[3]. By using the laser heated floating zone method to prepare eutectics, Sayir and Farmer [4] found the microstructures and orientation relationship between the phases in the system varied as a function of composition and growth rate, and microstructures varied from lamellar eutectics to fibrous ones as growth rate was increased substantially. Solidification is a surface reaction whose rate depends on the degree of under-cooling that drives it, so high under-cooling degree introduces a high nucleation rate and high solidification rate. At the high cooling rate of the and the high under-cooling degree, the fibrous eutectics are obtained, and within the fibrous eutectics aligned nano/micron fibers are embedded and are nearly perpendicular to the growth orientation of the fibrous eutectic [5]. One of the most significant barriers to the increased use of eutectic composite ceramic is the inability to predict accurately eutectic damage.

To date, the damage research is focused on the long-fiber composites. Under a load, the debonding phenomenon appeared in the interface of long-fiber composites, and then the fiber fracture occurred in the debonding area. There is local strain tensor in parallel fiber eutectic, respectively;

II. NO-DAMAGE STRAIN FIELD IN PARALLEL FIBER EUTECTIC

The parallel fiber eutectic is transverse isotropic. There are five independent elastic constants [10]. The longitudinal elastic module \( E_{ll} = \frac{E_{l}}{(1+h_{111})} \), the transverse elastic modulus \( E_{tt} = \frac{E_{t}}{(1+h_{222})} \), the longitudinal shear modulus \( \mu_{12} = \frac{\mu_{12}}{(1+h_{111})} \), the transverse shear modulus \( \mu_{3} = \frac{\mu_{3}}{(1+h_{222})} \), the Poisson's ratio of longitudinal-transverse \( \nu_{13} = \frac{\nu_{13}}{(1+h_{222})} \). Among them, the \( E_{l} \) and \( E_{t} \) are the elastic modulus and Poisson's ratio of the matrix in parallel fiber eutectic, respectively; \( h_{ijkl} \) is the dimensionless flexibility increment.

Obtained from each flexibility component, the flexibility tensor for no-damage eutectic is as:

\[
S = \begin{bmatrix}
S_{11} & 0 \\
0 & S_{22}
\end{bmatrix}
\]

where,

\[
S_{11} = \begin{bmatrix}
\frac{1}{E_{ll}} & -\frac{\nu_{12}}{E_{ll}} & -\frac{\nu_{13}}{E_{ll}} \\
-\frac{\nu_{12}}{E_{tt}} & \frac{1}{E_{tt}} & -\frac{\nu_{13}}{E_{tt}} \\
-\frac{\nu_{13}}{E_{tt}} & -\frac{\nu_{13}}{E_{tt}} & \frac{1}{E_{tt}}
\end{bmatrix},
S_{22} = \begin{bmatrix}
\frac{1}{\mu_{12}} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{1}{\mu_{13}}
\end{bmatrix},
\]

In the no-damage state, there is no damage in eutectic. The local strain tensor in parallel fiber eutectic can be described as:

\[
\varepsilon' = S\sigma' \]

According to the constitutive relation, the no-damage strain field in parallel fiber eutectic is as:
\[ \{e'_{11} = \frac{1}{E_{11}}\sigma_{11} - \frac{\nu_{12}}{E_{22}}\sigma_{22} - \frac{\nu_{13}}{E_{33}}\sigma_{33} \\
e'_{22} = \frac{1}{E_{22}}\sigma_{22} + \frac{1}{E_{22}}\sigma_{22} - \frac{\nu_{12}}{E_{22}}\sigma_{22} - \frac{\nu_{13}}{E_{33}}\sigma_{33} \\
e'_{33} = \frac{1}{E_{33}}\sigma_{33} - \frac{\nu_{12}}{E_{22}}\sigma_{22} - \frac{\nu_{13}}{E_{33}}\sigma_{33} \} \]

\[ (3) \]

III. DAMAGE STRAIN FIELD IN PARALLEL FIBER EUTECTIC

The damage variable is introduced because of the damage phenomenon in parallel fiber eutectic under loading, as in [11]. The parallel fiber eutectic is transverse isotropic so that one longitudinal damage variable and two transverse damage variables are defined which are correlated with principal stress. Among them, the longitudinal damage variable \( d_l \) represents microcrack in the plane which is vertical to fiber, and the microcrack is correlated with elastic properties along the fiber longitudinal direction in the parallel fiber eutectic. And the transverse damage variable \( d_t \) and \( d_{t2} \) represent microcrack in the plane which is parallel to fiber, and the microcrack is correlated with the elastic properties along the fiber transverse direction in the parallel fiber area of composite eutectic. The shear damage variable \( d_s \) is correlated with shear stress \( \tau_{12} \) and \( \tau_{13} \). Thus, the transverse shear damage is obtained by the coaxial of stress and strain. According to (1), the damage flexibility tensor of parallel fiber eutectic is obtained as:

\[ S' = \begin{pmatrix} S'_{11} & 0 \\ 0 & S'_{22} \end{pmatrix} \]

\[ (4) \]

Among them, \( S'_{11} \) is the flexibility tensor which is correlated with principal stress, and \( S'_{22} \) is the flexibility tensor which is correlated with shear stress. In the isotropic plane, the no-damage elastic constant and damage variable at \( e_3 \) are numerically equal to \( e_3 \). The flexibility tensor which is correlated with principal stress expressed as:

\[ S'_{11} = \begin{pmatrix} \frac{1}{E_{11}} & \frac{-\nu_{12}}{E_{22}} & \frac{-\nu_{13}}{E_{33}} \\ \frac{-\nu_{12}}{E_{22}} & \frac{1}{(1-d_t)E_{22}} & \frac{-\nu_{13}}{E_{33}} \\ \frac{-\nu_{13}}{E_{22}} & \frac{-\nu_{13}}{E_{33}} & \frac{1}{(1-d_t)E_{22}} \end{pmatrix} \]

\[ (5) \]

Reference [12] shows that the stress and strain are coaxial in the isotropic plane, so the transverse shear modulus is expressed as:

\[ \mu'_{23} = \frac{E_{22}(-d_t)}{2[1+\nu_{23}(-d_t)]} \]

\[ (6) \]

Derivation of (3) shows that

\[ \mu'_{23} = \frac{E_{22}(-d_t)}{2[1+\nu_{23}(-d_t)]} \]

\[ (7) \]
When the arc-crack appeared on the two-phase interface, and the maximum debonding area reached half cylinder, based on the equilibrium condition of shear lag analysis [15], the shear stress of half cylinder surface connected with the fiber should be equal to the shear stress of cylinder surface in any ring layer whose radius is $a$

$$\tau_0' = 2\pi a \tau$$  \hspace{1cm} (10)

where $\tau_0'$ is the shear stress along the fibers of half cylinder surface connected to the fiber, $\tau$ is the shear stress of cylinder surface in the ring layer whose radius is $a$. Based on Hooke’s law: $\tau = \mu_m \frac{du}{da}$, in which $u$ is the displacement along the fibers of matrix whose radius of cylinder surface is $a$. $\mu_m$ is the shear modulus of the matrix. $\frac{du}{da} = \frac{\tau_0' a_0}{2 \mu_m a_0}$ can be obtained after the calculation of the simultaneous equations. The integral of the formula is shown in (11):

$$\int u^R du = \frac{\tau_0' a_0}{2 \mu_m a_0} \int \frac{da}{a}$$  \hspace{1cm} (11)

Result calculated from (11) is shown in (12):

$$\tau_0' = \frac{2 \mu_m (u_R - u_0)}{a_0 \ln (R/a_0)}$$  \hspace{1cm} (12)

where $u_R$ is the matrix axial displacement and $u_0$ is the fiber axial displacement. The shear stress of interface at the maximum damage is expressed in (10). When the interface has no damage, (10) can be transformed into $2\pi a_0 \tau_0 = 2\pi a \tau$ where $\tau = \frac{\mu_m (u_R - u_0)}{a_0 \ln (R/a_0)}$ can be obtained by the use of the same method. According to $\tau_{max}' = 2\tau$, and (9), the maximum shear stress is:

$$d_{2,max} = 1/2$$  \hspace{1cm} (13)

The constitutive relations of parallel fiber eutectic can be expressed as shown in (14):

$$\epsilon' = S' \sigma'$$  \hspace{1cm} (14)

Based on the constitutive relations in (14), the damage strain fields in parallel fiber eutectic are shown in (15):

$$\begin{align*}
\epsilon_{11}' &= \frac{1}{E_{11}} \sigma_{11} + \frac{1}{2} \left( \frac{v_{12}}{E_{22}} + \frac{v_{13}}{E_{33}} \right) \sigma_{22} + \frac{v_{12}}{E_{22}} \sigma_{33}, \\
\epsilon_{22}' &= -\frac{v_{11}}{E_{11}} \sigma_{11} + \frac{1}{1 - d_2} \left( \frac{v_{12}}{E_{22}} + \frac{v_{13}}{E_{33}} \right) \sigma_{22} + \frac{v_{13}}{E_{33}} \sigma_{33}, \\
\epsilon_{33}' &= -\frac{v_{11}}{E_{11}} \sigma_{11} + \frac{1}{1 - d_2} \left( \frac{v_{12}}{E_{22}} + \frac{v_{13}}{E_{33}} \right) \sigma_{22} + \frac{v_{12}}{E_{22}} \sigma_{33}
\end{align*}$$  \hspace{1cm} (15)

The above equations show that damage strains are not only associated with the five independent elastic constants of parallel fiber eutectic, but also with the damage variable.

For parallel fiber eutectic, the material constants in the matrix and fiber are $E_m = 402$ GPa, $\nu_m = 0.233$, $E_b = 233$ GPa, $\nu_b = 0.31$. The thickness of the interphase is $\Delta = 1$ nm. $f_2 = \frac{4\Delta}{2\Delta + d_2}$. If a tensile stress $\sigma$ is forced along eutectic axis, the diameter of fiber is set at 125 nm. When $d_2$ is set to 1/3, 2/3 and 1 times of the maximum, the relationship between the strain components in matrix and fiber volume fraction $f_2$ is shown in Figs. 2 and 3.

**Fig. 2** Relationship between the strain in matrix parallel to eutectic axis and fiber fraction

**Fig. 3** Relation between traverse strain in matrix and fiber fraction

According to Fig. 2, the strain of matrices along the eutectic axis increases with the increase of fiber volume fraction. Fig. 3
shows that the traverse strain in matrix decreases with the increase of fiber volume fraction.

The diameter of fiber is set at 125 nm. When \( d_2 \) is set to 1/3, 2/3, and 1 times of the maximum, the relationship between the strain components in interphase and fiber volume fraction \( f_b \) is shown in Figs. 4 and 5.

Figs. 4 and 5 show that the strain components in interphase increase with the increase of fiber volume fraction. The diameter of fiber is set at 125 nm.

When \( d_2 \) is set to 1/3, 2/3, and 1 times of the maximum, the relationship between the strain components in fiber and fiber volume fraction \( f_b \) is shown in Figs. 6 and 7.

Figs. 6 and 7 show that the strain components in fiber increase with the increase of fiber volume fraction.

Comparing Figs. 2 and 7, the maximum linear strain of matrix along the eutectic axis is the main factor of the matrix damage.

IV. CONCLUSION

1) The theoretical analysis indicates that damage strains are not only associated with the five independent elastic constants of parallel fiber eutectic, but also with the damage variable.

2) The damage strain of matrix in eutectic along the eutectic axis increases with the increase of fiber volume fraction, and the traverse strain of matrix decreases with the increase of fiber volume fraction. The greater the change of damage degree, the more intense the change of damage strains of matrix is.

3) The strain components of interphase in eutectic increase with the increase of fiber volume fraction, and the greater the change of damage degree, the more intense the change of damage strains of interphase is.

4) The axial and traverse damage strains of fiber in eutectic increase with the increase of fiber volume fraction, and the greater the change of damage degree, the more intense the change of damage strains of fiber is.

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REFERENCES


