Abstract—In today’s market, striving hard has become necessary for the industries to survive due to the intense competition and globalization. In earlier days, there were few sellers and limited numbers of buyers, so customers were having fewer options to buy the product. But today, the market is highly competitive and volatile. Industries are focusing on robotics, advance manufacturing methods like AJM (Abrasive Jet Machining), EDM (Electric Discharge Machining), ECM (Electrochemical Machining) etc., CAD/CAM, CAE to make quality products and market them in shortest possible time. Leagile manufacturing system is ensuring best available solution at minimum cost to meet the market demand. This paper tries to assimilate the concept of Leagile manufacturing system in today’s scenario and evaluating key factors affecting Leagile manufacturing using digraph technique.

Keywords—Agile manufacturing, digraph, lean manufacturing, leagile manufacturing.

I. INTRODUCTION

LEAN is concerned with elimination of all types of wastes which do not add value to the product. It also means maximizing the value of product by using less of everything i.e. less manpower, fewer resources, less time, less investment etc. It mainly focuses on value stream to make the product at less cost in short span of time. Different authors have defined lean production system [1]-[10]. According to [1], [5], [9], the definition of value stream in lean depends on a customer and cost perspective, rather than organization’s viewpoint, and a lean manufacturing typically has predictable demand, low product variety, longer product life cycles, and cost driven customers. Various advantages of implementing lean system are reduced inventory, reduced lead time, wastage reduction, financial savings, increased productivity, increased market share etc. [5]. Implementation of lean manufacturing depends on type and size of industry. [11]. Leanness means developing a value stream to eliminate all waste, including time and to ensure a lever schedule [7].

Agile manufacturing deals how frequently the system can be reconfigured in order to meet changing needs of customers. According to [3], agile manufacturing is defined as capability of operating profitably in a competitive environment of continually and unpredictably changing customer opportunities. [2] Agile focuses more priority on responsiveness than cost while lean concept mainly focuses on reducing the cost.

Leagility concept has emerged as a profitable strategy nowadays. Reference [8] discusses that postponement can be most important tool of leagility.

II. MATRIX REPRESENTATION OF THE DIGRAPH

Digraph shows relationship among leagility attributes. Leagility Matrix is represented by a binary matrix $a_{ij}$, where $a_{ij}$ represents the relative importance between attributes $i$ and $j$ such that $a_{ij} = 1$, if $i^{th}$ leagility attribute is more important than the $j^{th}$ attribute; $a_{ij} = 0$, otherwise. It is noted that $a_{ii} = 0$ for all $i$, as an attribute cannot have relative importance over itself.

III. KEY FACTORS AFFECTING LEAGILE MANUFACTURING SYSTEM

A. Human Resources
1. Commitment of employee towards work [12]
2. Experience of employee [13], [14]
4. Attitude of employee [5]
5. Teamwork [2]
6. Interpersonal Skills [14]
7. Interest of employees towards R&D activities [5]
8. Resistance to change [7]

B. Production and Automation Engineering Aspects
2. Use of advanced manufacturing methods like AJM, EDM etc. [16], [17]
3. Use of FMS, CNC, DNC [3], [9]
4. Use of robotics and PLC [18], [19]
5. Route sheet or sequence of operation to be followed [20], [21]

C. Quality Tools and Techniques
1. 7 QC Tools [22], [23]
2. Six sigma [4]
3. Acceptance sampling [5]
5. TQM [14]
7. Kaizen [21]
8. Kanban [22]
9. SMED (single minute exchange of dies) [17]
10. Value stream mapping [20]

D. Design Aspects
1. Design of product i.e. simplicity [1]
2. Techniques and methodology used [6], [7]
3. Availability of R&D facilities [10], [13]
4. Supplier and customer involvement in product design [8], [9]

E. Management Aspects [5], [7], [12], [14]
1. Supply Chain Management [15]-[21]
2. Management support towards implementation of strategies [22]
3. Financial expenditure towards training & development [11],
4. Decentralized authority [17]
5. Increment and remuneration policies [12]
6. Employee empowerment [7], [8], [16]

These factors affect Leagile manufacturing system differently. The relationship between these different factors and the amount which individual factor affects the main objective function (Leagile manufacturing system) is equivocal. There are many techniques like graph theory, simulated annealing, grey relational analysis etc. The quantification of inheritance and interactions is not possible by using Delphi, AHP, ANP, SEM and Fuzzy logic etc.

IV. LEAGILITY EVALUATION: GRAPH THEORETIC REPRESENTATION

The effectiveness of the environment depends upon the degree of inheritance of these factors and the amount of interactions present between them, which conventional representations are unable to analyze. These interactions may be direction dependent or independent. Graph representation is used to propose the model of leagile environment. The graph theoretic representation is suitable for visual analysis [6], [7]. The five broad factors, their critical elements and their sub elements identified in the previous section are used to evaluate the extent of the leagile environment for an index known as the Leagile index. Thus

\[
\text{leagile index} = f(\text{critical elements})
\]

The Endeavour is made to co-relate these five critical elements, their quantification based on subcomponents and interdependency of critical elements. Based on the above quantification, it is proposed to find the capacity of an organization in terms of the Leagile environment. This is achieved through a structural approach called graph theoretical approach. It consists of the representation of matrix, digraph and the permanent function. The digraph is the visual representation of characteristics and their interdependencies. The matrix converts the digraph into mathematical form. The permanent function helps to determine index by mathematical model [12].

A. Digraph Representation

![Fig. 1 Schematic representation of Leagile Enablers](image1)

![Fig. 2 Leagility Digraph](image2)

B. Matrix Representation

Since a digraph is a visual representation, it helps in analysis to a limited extent only. To establish an expression for Leagile elements, the digraph is represented in matrix form, which is convenient in computer processing also.

Let us represent a digraph of n factors leading to an nth order symmetric matrix \( A = [T_{ij}] \).

\[
T_{ij} = 1; \text{ if factor } i \text{ is connected to factor } j \\
T_{ij} = 0, \text{ otherwise}
\]

Generally \( T_{ij} \neq T_{ji} \) as Leagile factors are directional and \( T_{ii} = 0 \), as a factor is not interacting with itself. The Leagile matrix is square and non-symmetric and is analogous to the adjacency matrix in graph theory. The Leagile matrix representing the digraph shown in Fig. 2 is written as:

\[
A = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(1)

Off-diagonal elements with value 0 or 1 represent the
interdependency of Leagile elements. The diagonal elements are 0 since the effect/inheritance of Leagile elements is not taken into consideration. To consider this, another Leagile characteristic matrix is defined.

C. Leagility Characteristic Matrix (LCM)

Let us consider an identity matrix I, and T as the variable representing Leagile elements. The characteristic matrix is used to characterize leagile elements. The Leagile characteristic matrix, B, for the digraph shown in Fig. 2, may be expressed as [T I-A], represented in (2)

\[
B = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
T & -1 & -1 & -1 & -1 \\
0 & T & 0 & -1 & -1 \\
-1 & 0 & T & -1 & -1 \\
0 & 0 & 0 & 0 & T
\end{bmatrix}
\] (2)

In the above matrix, the value of all diagonal elements is the same, i.e. all Leagile elements have been assigned the same value which is not true in practice, since all Leagile elements have different values depending on various parameters affecting them. Moreover, interdependencies have been assigned values of 0 and 1 depending on whether it is there or not. To consider this, another matrix, the Leagile variable characteristic matrix is considered.

D. Leagile Variable Characteristic Matrix (VCM-Leagile)

The Leagile variable characteristic matrix takes into consideration the effect of different Leagile factors and their interactions. Fig. 2 is considered for defining VCM-leagile. As stated earlier the \(T_i\)'s and \(T_j\)'s represent nodes and edges, respectively, in the digraph. Consider a matrix C with off-diagonal elements \(T_{ij}\) represents interactions between Leagile factors, i.e. instead of 1 (as in matrix (1)). Another matrix D is taken with diagonal elements \(T_i\), i = 1, 2, 3, 4, 5 where the \(T_i\) represents the effect of various factors, i.e. instead of \(T\) only (as in matrix (2)).

Considering matrices C and D, VCM- leagile is expressed as: E = [D-C]

\[
E = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
T & -T_{12} & -T_{13} & -T_{14} & -T_{15} \\
0 & T_{23} & 0 & -T_{25} & -T_{25} \\
0 & 0 & T_{35} & 0 & -T_{35} \\
-T_{41} & 0 & -T_{43} & T_{45} & 0 \\
0 & 0 & 0 & 0 & T_{5}
\end{bmatrix}
\] (3)

The matrix (3) analyses performance through its determinant. This is a characteristic of the system and represents the Leagile environment of the system, considering the effect of Leagile elements and their interactions. Due to consideration of selective interaction of Leagile factors (as per Fig. 2), some of the diagonal elements in the matrix in (3) are zero.

The determinant of the matrix in (3), i.e. the variable characteristic Leagile multinomial, carries positive and negative signs with some of its coefficients.

E. Leagile Variable Permanent Matrix (VPM-Leagile)

The environment in an organization is LEAGILE-enabled when the effect of all the factors is maximum.

\[
T = D + C
\]

where D and C have the same meaning as stated earlier. Thus, the variable permanent Leagile matrix (VPM- Leagile) corresponding to the five-critical element Leagile digraph (Fig. 2) is given by

\[
\text{VPM - LEAGILE} = T^* = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
T_{1} & T_{12} & T_{13} & T_{14} & T_{15} \\
T_{21} & T_{2} & T_{23} & T_{24} & T_{25} \\
T_{31} & T_{32} & T_{3} & T_{34} & T_{35} \\
T_{41} & T_{42} & T_{43} & T_{4} & T_{45} \\
T_{51} & T_{52} & T_{53} & T_{54} & T_{5}
\end{bmatrix}
\] (4)

The diagonal elements \(T_{1} , T_{2} , T_{3} , T_{4} \) and \(T_{5}\) represent the contribution of the five critical factors in creating the LEAGILE environment and the off-diagonal elements represent interdependencies of each element in the matrix. The contribution can be expressed quantitatively and is explained later in this paper.

F. Permanent Representation

To develop a unique representation, independent of labeling, a permanent function of the matrix VPM-Leagile (Variable Permanent Matrix- Leagile) is proposed for this purpose. The value of permanent function is obtained in a similar manner as that of determinant. A negative sign appears in the calculation of determinant while in the permanent, i.e. the variable permanent function, only positive signs come. These computation processes results in a multinomial (7) whose every term has a physical significance related to the environment. This multinomial representation includes all the information regarding critical elements including human aspects and strategic policies and interactions amongst them.
Quantitative Leagile evaluation of an organization is obtained from VPF-Leagile by substituting numerical values of the $T_i'$s and $T_{ij}'$s which are obtained analytically or by comparing to ideal cases. This single numerical index is the representation of a typical Leagile environment in quantitative terms. The variable permanent Leagile function (VPF-Leagile), being the characteristic of the Leagile environment of an organization is a powerful tool for its analysis. The VPF-Leagile expression corresponds to the five-factor digraph/VPM-Leagile and is given by

$$\text{VPF-Leagile} = \text{per} \star \left( \sum_{i=1}^{5} T_i + \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{k=1}^{5} \sum_{l=1}^{5} \sum_{m=1}^{5} (T_{ij}T_{jk}T_{kl} + T_{ik}T_{jk}T_{lj}) T_i T_j T_k T_m + \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{k=1}^{5} \sum_{l=1}^{5} \sum_{m=1}^{5} (T_{ij}T_{jk}T_{kl}) T_i T_j T_k T_m + \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{k=1}^{5} \sum_{l=1}^{5} \sum_{m=1}^{5} \sum_{n=1}^{5} (T_{ij}T_{jk}T_{kl}T_{mn}) T_i T_j T_k T_m T_n + \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{k=1}^{5} \sum_{l=1}^{5} \sum_{m=1}^{5} \sum_{n=1}^{5} (T_{ij}T_{jk}T_{kl}T_{mn}T_{nl}) T_i T_j T_k T_m T_n T_l + \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{k=1}^{5} \sum_{l=1}^{5} \sum_{m=1}^{5} \sum_{n=1}^{5} \sum_{p=1}^{5} (T_{ij}T_{jk}T_{kl}T_{mn}T_{np}) T_i T_j T_k T_m T_n T_p \right) \tag{7}$$

The permanent of the matrix (i.e. (7)) is a mathematical expression in symbolic form. It ensures an estimate of the Leagile environment in an organization. Equation (7) contains 5! terms. Each term is useful for Leagile experts as each term serves as a test for the effectiveness of the relevant group in Per $T'$ [24].

Equation (7) contains terms arranged in $N + 1$ groups, where $N$ is the number of elements.
- The first term (grouping) represents a set of $N$ unconnected Leagile elements, i.e. $T_{1}', T_{2}'$ ............. $T_{N}'$ [24].
- The second grouping is absent in the absence of self-loops [24].
- Each term of the third grouping represents a set of two-element Leagile loops i.e. $(T_{ij}T_{jk})$ and is the resultant Leagile dependence of characteristics $i$ and $j$ and the Leagile measure of the remaining $N-2$ unconnected elements [24].
- Each term of the fourth grouping represents a set of three-element Leagile loops $(T_{ij}T_{jk}T_{kl})$ or its pair $T_{ik}T_{jk}T_{lj}$ and the Leagile measure of the remaining $N-3$ unconnected elements [24].
- The fifth grouping contains two subgroups. The terms of the first sub group consist of two-element Leagile loops (i.e. $T_{ij}T_{ji}$ and $T_{kl}T_{lk}$) and leagile component ($T_{mn}$). The terms of the second grouping are a product of four-element leagile loops (i.e. $T_{ij}T_{jk}T_{kl}T_{lj}$) or its pair (i.e. $T_{ik}T_{jk}T_{lj}$) and leagile component (i.e. $T_{mn}$) [24].
- The terms of the sixth grouping are also arranged in two sub groupings. The terms of the first subgrouping are a product of a two-element leagile loop (i.e. $T_{ij}T_{ji}$) and a three-element leagile loop (i.e. $T_{kl}T_{lm}T_{mk}$) or its pair (i.e. $T_{lm}T_{ml}T_{km}$). The second sub-grouping consists of a five-component leagile loop (i.e. $T_{ij}T_{jk}T_{kl}T_{lm}T_{ml}$) or its pair ($T_{lm}T_{ml}T_{ik}T_{kj}$) [24].

V. QUANTIFICATION OF $T_i'$S AND $T_{ij}'$S

The quality measure of the elements (i.e. the $T_i'$s) is evaluated considering each $T_i$ as a subsystem and the graph theoretic approach is applied in each system. The various sub factors affecting $T_i$ are identified. Corresponding subsystem digraph is also identified as evaluated in (7). In order to avoid complexity at sub subsystem level, values for inheritance may be taken from Table I.

The dependence between the elements at system level or subsystem level cannot be measured directly. However, values can be assigned through proper interpretation by experts. Table II suggests these qualitative values of interdependencies of elements.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Quantitative measure of Leagility elements</th>
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<td>Exceptional low</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Very low</td>
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</table>

<table>
<thead>
<tr>
<th>S.No</th>
<th>Quantitative measure of dependencies</th>
<th>Assigned Value</th>
</tr>
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<tbody>
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<td>1</td>
<td>Very strong</td>
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</tr>
<tr>
<td>2</td>
<td>Strong</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Medium</td>
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</tr>
<tr>
<td>4</td>
<td>Weak</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Very weak</td>
<td>1</td>
</tr>
</tbody>
</table>

VI. LEAGILITY INDEX

As we discussed, the leagile environment in an organization is a function of five elements and their interdependence.
Leagile = per $T^* \rightarrow$ Permanent value of VPM – leagile

The leagile index for different organizations can be found using the methodology presented in this article and thus various organizations can be compared. It is a versatile tool and the methodology can cater to the ever-changing environment by including new factors that are affecting the change. The methodology discussed earlier is presented in terms of the salient steps for application in industry.

A. Methodology

The graph theoretic approach evaluates the leagile environment of an organization in terms of a single numerical index. This takes into consideration the effect of factors and their interdependencies. The various steps in the approach are presented, which will help organizations to take up the evaluation process of the leagile environment.

1. Identify the various elements affecting the leagile environment. Different industries may have a different set of elements affecting the leagile environment depending on the type of product (variable due to type of technology), size of organization, cultural values depending on geographical location and environmental factors.

2. Broadly group these elements (as five broad factors are framed in this study based on Table I). For the application of this methodology, the elements are written in composite form to avoid mathematical complexity in the further analysis.

3. Logically develop a digraph between the factors depending on their interdependencies [24] (similar to Fig. 2).

4. Identify the sub factors affecting each factor.

5. Develop a variable permanent matrix at the system level (similar to (5) and (6)) based on the digraph developed in step 3.

6. For each factor, develop the digraph among sub factors based on interactions among them.

7. Develop the variable permanent matrix for each factor based on the sub factor digraphs developed in step 6. This is the variable permanent matrix at subsystem level.

8. Find the permanent function at subsystem level using (7). At the subsystem level or sub-subsystem level, the values of interactions and inheritance may be taken from Tables I and II, respectively, to avoid complexity.

9. The permanent value at each subsystem provides inheritance of each factor (i.e. the diagonal element in step 5). The values of off-diagonal elements (i.e. interactions) are to be determined by experts based on Table II.

10. Evaluate the permanent function of the variable permanent matrix at system level (developed in step 5) using (7). This is the value of the leagile index, which mathematically characterizes the leagile environment based on factors/elements and their interactions.

B. Example

An example is considered for the demonstration of the methodology. Certain values are assumed for inheritance and interactions, as the actual values are to be obtained by conducting a field study at every system/subsystem level by experts. The leagile index is determined using numerical values of all factors and their interdependencies, i.e. all values in (6).

A numerical value is to be assigned to factors contributing to the leagile environment i.e. $T_1 , T_2 , T_3 , T_4$ and $T_5$, so as to evaluate a quality measure of each factor. Each factor is identified as a subsystem and a graph theoretical approach is applied in each subsystem. [2] For example, in the first factor $T_1$, i.e. behavioral factor, the employer is considered as a sub-subsystem denoted by $T_e$. The contribution of the human resource to the leagile environment in an organization is a function of eight elements, which are:

1. Commitment of employee towards work
2. Experience of Employee
3. Skills of operator
4. Attitude of Employee
5. Team work
6. Interpersonal Skills
7. Interest of employees towards R&D activities
8. Resistance to change

Based on the interdependencies of these elements a sub-subsystem digraph is developed and, similar to (5), these elements form a variable permanent matrix for the sub-subsystem human resource, which is given as

$$VPM - \text{Leagile}_e =$$

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
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<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
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<tr>
<td>$T_4$</td>
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<tr>
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<td>5</td>
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<td>8</td>
</tr>
</tbody>
</table>

To find the value of the measure of the employer contribution to the leagile environment, the permanent function of (8) is determined. In a similar way, elements contributing to other sub factors are analyzed and corresponding matrices are written. We can use the graph theoretical approach at every subsystem. In order to avoid complexity, a suitable scale may be used to assign values to each element. Normalization may be done, if required, using the scale depicted in Table I.
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Leagile viewpoint if their leagile digraphs are isomorphic. Two organizations are similar from the leagile viewpoint if their VPF-leagile. Two organizations are similar from the leagile viewpoint if their leagile digraphs are isomorphic if they have identical VPF-leagile. This means not only numbers of terms in each grouping/sub-grouping are the same but also the values are the same. Based on this, a composite leagile identification set for an organization is written as:

\[
\left[\frac{J_1^T}{J_2^T}/\frac{J_1^T}{J_2^T}/\frac{J_1^T}{J_2^T}/\frac{J_1^T}{J_2^T}/\frac{J_1^T}{J_2^T}/\frac{J_1^T}{J_2^T}/\ldots\right] \left[\frac{V_1^T}{V_2^T}/\frac{V_1^T}{V_2^T}/\frac{V_1^T}{V_2^T}/\frac{V_1^T}{V_2^T}/\frac{V_1^T}{V_2^T}/\frac{V_1^T}{V_2^T}/\ldots\right]
\]

where \(J_i^T\) represents the total number of terms in the \(i^{th}\) grouping, \(J_j^T\) represents the total number of terms in \(j^{th}\) subgrouping of the \(i^{th}\) grouping. In case there is no sub grouping, the \(J_j\) is the same as \(J_i\); similarly \(V_i^T\) is the value of the \(i^{th}\) grouping; \(V_j^T\) is the numerical value of the \(j^{th}\) subgrouping in the \(i^{th}\) grouping. Numerical values of the \(T_i\)'s and \(T_j\)'s are substituted in the subgrouping or grouping to obtain \(V_j^T\). The subgroupings are arranged in decreasing order of size (i.e. number of characteristics in a loop).

In general, two organizations may not be isomorphic from the leagile viewpoint. Comparison is also carried out on the basis of the coefficient of similarity. If the value of distinct terms in the \(j^{th}\) subgrouping of the \(i^{th}\) grouping of VPF-LEAGILE of two organizations under consideration are denoted by \(V_j^T\) and \(V_j^T\), then criterion 1 of the coefficient of dissimilarity is given as:

\[
C_{d-1}^T = \left(\frac{1}{Y_i} \sum_{i} \sum_{j} \phi_{ij} \right)^{1/2}
\]

where

\[
Y_i = \max \left[ \sum_j T_i \left| V_j^T \right| \text{ and } \sum_j T_i \left| V_j^T \right| \right]
\]

In case of absence of sub-groupings, \(V_j = V_j^T\) and \(V_j^T = V_j^T\). Also,

\[
\phi_{ij} = \left| V_{ij} - V_{ij}^T \right| \text{ when the subgrouping exists}
\]

\[
\phi_{ij} = \left| V_{ij} - V_{ij}^T \right| \text{ when the subgrouping are absent}
\]

Criterion 2 of the coefficient of dissimilarity is given as

\[
C_{d-2}^T = \left(\frac{1}{Y_i} \sum_{i} \sum_{j} \phi_{ij}^2 \right)^{1/2}
\]
where $\phi_j$ is same as described above and

$$Y_2 = \max \sum \sum \left( V_{ij}^T \right)^2 \text{and} \sum \sum \left( V_{ij}^T \right)^2$$

Using (11) and (12), the co-efficient of similarity is given as

$$C_{s-1}^T = 1 - C_{d-1}^T$$
$$C_{s-2}^T = 1 - C_{d-2}^T$$

where $C_{s-1}^T$ and $C_{s-2}^T$ are the coefficient of similarity of two organizations under consideration based on criteria 1 and 2.

A. Example for Comparison of Organizations

Similarity or dissimilarity among organizations in terms of the leagile environment may be estimated using the methodology discussed in the previous section. An example is considered for demonstration of the methodology. Two organizations are considered whose leagile environment is a function of three broad factors identified as organizing, human resource focus and system techniques. These are designated by $T_1$, $T_2$, $T_3$ and $T_1'$, $T_2'$, $T_3'$ for organization 1 and organization 2, respectively. Although the factors identified are the same for both organizations, it is the inheritance and interaction of these factors that will lead to comparison. Based on the methodology discussed in Section VI A, the variable permanent matrix (VPM) for organization 1 may be written as

$$N = \begin{bmatrix}
T_1 & T_2 & T_3 & 1 \\
T_1 & T_2 & T_3 & 2 \\
T_3 & T_2 & T_3 & 3
\end{bmatrix}$$

(13)

Variable permanent function of (13) will lead to the permanent of the matrix per $T_1^*$ using (7).

$$\text{Per } T_1^* = T_1T_2T_3 + T_1T_2T_3 + T_1T_2T_3 + T_1T_2T_3 + T_1T_2T_3 + T_1T_2T_3$$

(14)

The expression for per $T_1^*$ is arranged in $N + 1$ i.e. four groups (the second group is absent as there are no self-loops) and contains $N!$, i.e. six terms.

Similar to (13) and (14), VPM and the permanent function for organization 2 may be written down.

The values of inheritances and interactions, i.e. $T_i$’s and $Tij$’s are to be determined by experts. For the demonstration of the methodology these are taken from Tables I and II as

$T_1 = 9$, $T_2 = 5$, $T_3 = 6$, $T_{12} = 4$, $T_{13} = 4$, $T_{21} = 4$, $T_{23} = 3$, $T_{31} = 5$, $T_{32} = 3$ and $T_{1}' = 8$, $T_{2}' = 5$, $T_{3}' = 6$, $T_{12}' = 3$, $T_{13}' = 4$, $T_{21}' = 6$, $T_{23}' = 5$, $T_{31}' = 4$, $T_{32}' = 3$

Substituting the values, the permanent for organization 1 (14) may be written as

$$\text{Per } T_1^* = 270 + 96 + 100 + 81 + 60 + 48 = 655.$$ 

Based on (10), (14) and the value of per $T_1^*$, the identification set for organization 1 may be written as

$$[1/0/3/2] [270/0/277/108]$$

On similar lines, the permanent for organization 2 may be written as

$$\text{Per } T_2^* = 240 + 108 + 80 + 120 + 60 + 72 = 680.$$ 

The identification set for organization 2 may be written as

$$[1/0/3/2] [240/0/308/132]$$

Based on criterion 1, the coefficient of dissimilarity between the two organizations is 0.315. Thus the coefficient of similarity is 0.685. The comparison procedure helps organizations compare different groups in identification sets, based on which they can analyze and improve the weak link in the leagile environment. Moreover, organizations can be compared or ranked in increasing or decreasing order of value of the coefficient of similarity or dissimilarity. Also organizations may be grouped in a given range of the value of the coefficient and the leagile index may be compared for a given range.

VIII. Conclusion

In this paper, a methodology for evaluation of the leagile index is presented using a digraph and matrix approach. It is a powerful tool for evaluating the degree to which leagile concepts are followed. It identifies five characteristics, which parameterize the leagile environment of an industry. The graph theoretical methodology consists of the leagile digraph, the leagile matrix and the leagile permanent function.

The leagile digraph is the visual representation of the characteristics and their interdependence.

The leagile matrix converts digraph into mathematical form. The leagile permanent function is a mathematical model, which helps to determine the Leagile index. Thus, the approach helps to express leagile in quantitative terms, which has more often been expressed in qualitative terms. The procedure also helps to compare different industries in terms of leagile characteristics and rate them for a particular period of time. The advantages of using graph theory approach are:

1. It permits modeling of interactions/dependences existing between factors.
2. The leagile environment can be represented by graph theoretic, matrix and permanent function models.
3. The environment is quantified by a single numerical index representing its competitiveness and suitability.
4. These models can easily be modified to consider new factors emerging in global trading.
5. The methodology is useful for continuous improvement (Kaizen) as well as for breakthrough improvement.
6. Sensitivity analysis to identify the critical elements is easily carried out.
7. The method permits us to generate alternative environments.
8. This is an effective tool for evaluation, comparison, ranking and selection of an optimum environment.

The application of graph theoretic methodology to the sphere of leagile would add value to the leagile body of knowledge. The interdependency of various factors would help us understand and unveil the complexity of leagile factors. Evaluation and comparison will also lead to identify critical areas that are roadblocks to leagile. A comparison based on this would lead to healthy competition among industries.

REFERENCES