Application of De Novo Programming Approach for Optimizing the Business Process

Z. Babic, I. Veza, A. Balic, M. Crnjac

Abstract—The linear programming model is sometimes difficult to apply in real business situations due to its assumption of proportionality. This paper shows an example of how to use De Novo programming approach instead of linear programming. In the De Novo programming, resources are not fixed like in linear programming but resource quantities depend only on available budget. Budget is a new, important element of the De Novo approach. Two different production situations are presented: increasing costs and quantity discounts of raw materials. The focus of this paper is on advantages of the De Novo approach in the optimization of production plan for production company which produces souvenirs made from famous stone from the island of Brac, one of the greatest islands from Croatia.

Keywords—De Novo Programming, production plan, stone souvenirs, variable prices.

I. INTRODUCTION

De Novo programming, initiated by Zeleny [1], presents a special approach to optimization. Instead of "optimizing a given system", De Novo suggests a way of "designing an optimal system". In the De Novo approach the resources are not limited because the necessary resource quantities can be obtained at certain prices. The resources maximum quantity is limited only by the available budget, which is an important element of the De Novo programming.

Most cases can be handled more effectively by using De Novo than using the standard programming model (see [2]-[4]). Changes in prices, technological coefficients, increasing costs of raw materials, quantity discounts and other similar and real production situations can be easily incorporated into the De Novo model and can provide very satisfactory solutions.

II. DE NOVO PROGRAMMING

The traditional resource allocation problem in economics is modeled via standard linear programming formulation of the single-objective product-mix problem.

In the De Novo formulation, the purpose is to design an optimal system and the following formulation is of interest:

\begin{align}
\text{Max } z &= c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \\
\text{s.t. } a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n &= b_1 \\
a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n &= b_2 \\
&\vdots \\
a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n &= b_m \\
\begin{pmatrix} x_1, & b_1, & b_2, & \ldots, & b_m \end{pmatrix} &\geq 0, \quad j = 1, 2, \ldots, n; \quad i = 1, 2, \ldots, m \\
\end{align}

where, \( b = (b_1, b_2, \ldots, b_m) \) — set of decision variables representing the level of resource \( i \) to be purchased, \( p_i \) — unit price of resource \( i \), \( B \) — total available budget for the given system.

Now, the problem is to allocate the budget so that the resulting portfolio of resources maximizes the value of the product mix (with given unit prices of \( m \) resources, and with given total available budget).

The main difference of the two models lies in the treatment of the resources which become decision variable \( b_i \) in the De Novo formulation.

A. Varying Cost of Raw Materials

A frequent phenomenon arising in real production problems is the varying price of the same resource. Namely, if a company needs additional quantities of raw materials it is possible to buy them from another supplier but at a different (usually higher) price. Let us assume that \( i \) raw material can be purchased at the price \( p_i \), but only for the quantity lower (or equal) than \( Q \). To purchase \( i \) raw material above that quantity, it is necessary to take another supplier whose price is \( p_i' > p_i \).

Then, the relation for the \( i \) raw material is transformed into:

\begin{align}
a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n &= b_i + d_i, \\
\end{align}

with additional constraint \( b_i \leq Q \), where \( d_i \) is the additional quantity of the \( i \) raw material with the unit price \( p_i' \).

Let us now consider such production situation when there are quantity discounts granted for bulk orders of raw materials. Therefore, in addition to the increasing cost effect, we have to introduce this possibility into the model. Let us assume that, for the \( k \) resource \( (b_k) \), the valid price is \( p_k \) as long as the purchased quantity is below \( Q \), and the discounted price \( p_k' \) is valid for the entire quantity if the purchased quantity is higher (or equal to) than \( Q \). Consequently, the assumption is opposite to the one in the previous model, i.e. \( p_k' < p_k \).

The previous formulation is not applicable since the optimization model will prefer using the less expensive material without satisfying the quota (Q). A different model
has to be formulated with a slightly more complicated procedure.

Let \( b_k, p_k \) – the amount and price of \( k \) raw material if it is purchased at less than the quantity discount volume; \( d_k, p'_k \) – the amount and price of \( k \) raw material if it is purchased at the quantity discount.

The new model, in that case, instead of one equation for \( k \) raw material has some more relations, and those are:

\[
a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n = b_k + d_k
\]

and, according to this, the budget constraint is:

\[
p_1y_1 + p_2y_2 + \cdots + p_ky_k + p'_k d_k + \cdots + p'_n y_n \leq B,
\]

where \( M \) is a very large positive number (\( M >> 0 \)), or the upper limit for the procurement of the resource \( k \), and \( Q^* \) is a number which is slightly lower than \( Q \). Variables \( y_1 \) and \( y_2 \) are integer 0-1 variables, for which is:

\[
y_1 + y_2 = 1
\]

In the above model, there are two 0-1 variables \( y_1 \) and \( y_2 \), where only one of them always equals 1, and the other equals zero. Naturally, if the model comprises a number of resources that can be purchased at a discounted price then there are more 0-1 variables.

Since the same raw material has different price variable, the income from end product unit is not constant anymore. Therefore, maximizing the sum of \( c_j x_j \) would not be an accurate measure of net income. Net income (1) should be recalculated as the difference between sales and total cost of materials, where the objective function will include materials at both prices. Consequently, if \( s_j \) is the sales price of \( j \) product, the objective function has the following form:

\[
\text{Max } z = \sum_{j=1}^{n} s_j x_j - \sum_{i=1}^{m} p_i b_i - \sum_{k \in K} p'_k d_k
\]

In that equation set, \( K \) presents the indices of raw materials that have increasing or discounted prices, and \( d_k \) (\( k \in K \)) stays for those materials where additional quantities can be bought only at a higher price (\( p'_k \)), or the quantities of raw materials if we bought them with quantity discounts.

In the budget equation, it is also necessary to introduce costs for additional quantities of raw materials, so that it now takes the following form:

\[
\sum_{i=1}^{m} p_i b_i + \sum_{k \in K} p'_k d_k \leq B
\]

There is no need to specify that \( b_i \) should reach the maximum value of \( Q \) first, before allowing \( d_i \) greater than zero. The optimization model ensures \( b_i \) reaching the maximum value of \( Q \) because of the lower penalty, i.e. lower price \( p_i \).

### III. PROBLEM SETTING

This paper analyses the production planning problem in one production company which produces stone souvenirs made from famous stone from the island of Brac. This company produces 13 different stone souvenirs, and these articles can be seen in Table I. In this table, there are lower bounds for the six-month production and selling prices of all the articles.

Table II presents the list of raw materials that are used in production of these articles. There are 26 different raw materials, and the purchasing prices for every of them are also presented in the table.

The amount of raw materials in one unit of articles (\( a_{ij} \)) is also used in production planning problem, and their values are presented in Tables III and IV.

<table>
<thead>
<tr>
<th>Mark</th>
<th>Article name</th>
<th>Six-month amount production (kn)</th>
<th>Selling prices (kn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRM (A1)</td>
<td>Bracelet - metal</td>
<td>900</td>
<td>55</td>
</tr>
<tr>
<td>BRSP (A2)</td>
<td>Bracelet – semiprecious stone</td>
<td>700</td>
<td>55</td>
</tr>
<tr>
<td>BRR (A3)</td>
<td>Bracelet – rope</td>
<td>6000</td>
<td>15</td>
</tr>
<tr>
<td>CRS (A4)</td>
<td>Crown - small</td>
<td>600</td>
<td>40</td>
</tr>
<tr>
<td>CRB (A5)</td>
<td>Crown - big</td>
<td>100</td>
<td>125</td>
</tr>
<tr>
<td>NECM (A6)</td>
<td>Necklace - metal</td>
<td>120</td>
<td>70</td>
</tr>
<tr>
<td>NECSP (A7)</td>
<td>Necklace - semiprecious stone</td>
<td>120</td>
<td>70</td>
</tr>
<tr>
<td>NECFM (A8)</td>
<td>Necklace full – metal</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>NECFSP (A9)</td>
<td>Necklace full - semiprecious stone</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>EMS (A10)</td>
<td>Earrings metal - small</td>
<td>500</td>
<td>20</td>
</tr>
<tr>
<td>ESPS (A11)</td>
<td>Earrings semiprecious stone - small</td>
<td>500</td>
<td>20</td>
</tr>
<tr>
<td>EMB (A12)</td>
<td>Earrings metal - big</td>
<td>220</td>
<td>30</td>
</tr>
<tr>
<td>ESPB (A13)</td>
<td>Earrings semiprecious stone - big</td>
<td>250</td>
<td>30</td>
</tr>
</tbody>
</table>
According to these data, the production planning problem can be posted as the linear programming model with one or more objective functions. Here, we will consider the production company which produces stone souvenirs where
we have the varying price of the same resource, i.e. increasing costs of the same raw material or quantity discounts for some of raw materials as can be seen in Table II.

TABLE IV
THE RECIPES – PART II
<table>
<thead>
<tr>
<th>OPM-OG</th>
<th>OPP-OG</th>
<th>NMM-NAU</th>
<th>NPM-NAU</th>
<th>NMV-NAU</th>
<th>NPV-NAU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stone 8mm</td>
<td>32</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Stone 10mm</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Stone 12mm</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Skullcap small</td>
<td>16</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Skullcap big</td>
<td>24</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Space big</td>
<td>8</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Space small</td>
<td>24</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bead metal 4mm</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bead metal 8mm</td>
<td>8</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bead metal 10mm</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sajla</td>
<td>0,6</td>
<td>0,6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cable</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Semiprecious stone 4 mm</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Semiprecious stone 8 mm</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Semiprecious stone 10mm</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wire</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rope</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Small medal</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Small cross</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Big medal</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Big cross</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Needle 20mm</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Needle 30mm</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Crochet</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Since raw materials are now of different costs, variable prices of end product are not constant any more. Therefore, maximizing the sum of $c_j x_j$, where $c_j$ is the unit profit for article $A_j$, would not be an accurate measure of profit. Rather, profit equation should be recalculated as sales income less total cost of materials.

If $x_j$ is the production quantity of $i$ stone product, the model which will take this increasing costs and quantity discounts into consideration is as follows:

Objective function (total contribution) which has to be maximized is:

$$\text{Max } z = \sum_{j=1}^{13} a_{ij} x_j - b_i - d_i = 0, \ i = 1, \ldots, 26$$  \hspace{1cm} (9)

In that equation set, $K$ presents the indices of raw materials that have increasing or discounted prices. In our case, that happens for $S_7$, $S_8$, $S_{12}$ and $S_{15}$. Let us consider such situation for our souvenirs production model. The eight and fifteenth raw material ($S_8$ - Bead metal 4 mm and $S_{15}$ - Semiprecious stone 12 mm) can be purchased at a discounted price if the bought quantity is $Q_1 > 2500$ pieces and $Q_2 > 4200$ pieces, and this reduced price is valid for the entire quantity supplied, i.e. $p_1' = 0.02$ kn and $p_2' = 1$ kn. In addition to this, let us assume increasing costs for Small space ($S_7$) and Semiprecious stone 4 mm ($S_{12}$) in this way:

The limit of $S_7$ purchased at a lower price is 30000 pieces, while this limit in $S_{12}$ is 3500 pieces. The purchasing price of the additional quantity of $S_7$ is $p_1' = 0.08$, and of $S_{12}$ $p_{12}' = 0.25$ currency units. Assuming that the budget level is $B = 99430$ kn, and selling prices as in Table I, the constraints in the production model are:

Raw material constraints:

$$b_8 - 24990 y_1 \leq 0, d_8 - 25000 y_2 \geq 0, d_8 - M y_2 \leq 0$$  \hspace{1cm} (11)

$$b_{15} - 4190 y_3 \leq 0, d_{15} - 4200 y_4 \geq 0, d_{15} - M y_4 \leq 0$$  \hspace{1cm} (12)

where $M$ is a very large positive number ($M >> 0$), or the upper limit for the procurement of the specific resource $k$. Variables $y_1, y_2, y_3$ and $y_4$ are integer $0 \leq \text{ integer } 1$, variables, for which $y_1 + y_2 = 1$ and $y_3 + y_4 = 1$ is valid. In the above model there are four 0-1 variables, where due to the upper relations only one of them in each pair always equals 1, and the other equals zero. Naturally, if the model comprises a number of resources
that can be purchased at a discounted price then there are more 0-1 variables. Last is the budget constraint:

\[ \sum_{i=1}^{26} p_i b_i + \sum_{k=K} p_k d_k \leq 99430 \]  

(13)

In addition to that the model has 13 integer variables (units of articles - \( x_j \)). Of course due to the data from Table I, all articles have the lower bounds which are 13 more constraints. There are two more constraints for the raw materials which have increasing costs:

\[ b_7 \leq 30000, \quad b_{12} \leq 3500 \]  

(14)

In addition to that, we have some production lines (capacities) constraints and that is:

\[ x_1 + x_2 \leq 1800 \quad \text{(bracelet production line)} \]  

(15)
\[ x_6 + x_7 \leq 300 \quad \text{(necklace 1 production line)} \]  

(16)
\[ x_8 + x_9 \leq 200 \quad \text{(necklace 2 production line)} \]  

(17)
\[ x_{10} + x_{11} \leq 1000 \quad \text{(earrings 1 production line)} \]  

(18)
\[ x_{12} + x_{13} \leq 500 \quad \text{(earrings 2 production line)} \]  

(19)

\( d_{15} = 4380 \)  
\( b_{15} = 0 \)  
\( y_{15} = 1 \)  
\( y_1 = 1 \)  
\( y_2 = 0 \)  
\( y_3 = 0 \)  
\( y_4 = 1 \)  
\( y_5 = 1 \)  
\( y_6 = 0 \)  
\( y_7 = 0 \)  
\( y_8 = 1 \)  
\( y_9 = 1 \)  
\( y_{10} = 0 \)  
\( y_{11} = 0 \)  
\( y_{12} = 0 \)  
\( y_{13} = 0 \)  
\( y_{14} = 1 \)  
\( y_{15} = 1 \)  
\( y_{16} = 1 \)  
\( y_{17} = 1 \)  
\( y_{18} = 1 \)  
\( y_{19} = 1 \)  
\( y_{20} = 1 \)  
\( y_{21} = 1 \)  
\( y_{22} = 1 \)

\( z^* = 228682.20 \)

IV. CONCLUSION

In this paper, the De Novo programming model in the production plan optimization of the production of stone souvenirs is considered. The efficiency of the proposed model is investigated on the case of a company that produces 13 various souvenirs. The De Novo approach does not limit the resources as most of the necessary resource quantities can be obtained at certain prices. Resources, of course, are actually limited because their maximum quantity is controlled by the budget, which is an important element of De Novo.

The obtained results indicate a high application efficiency of the proposed model by using the De Novo programming in solving the production plan optimization problem in various production companies. Using the De Novo approach, most varied cases can be handled more effectively than by the standard programming models and in this paper increasing costs of raw materials and quantity discounts for some raw materials in souvenirs production are investigated.

The future work on this issue will investigate the possibilities of introducing new objective functions in the model, and solving this production problem as the multi-criteria ones [4]-[6].

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REFERENCES


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