Numerical Solution of Transient Natural Convection in Vertical Heated Rectangular Channel between Two Vertical Parallel MTR-Type Fuel Plates

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Abstract—The aim of this paper is to perform, by mean of the finite volume method, a numerical solution of the transient natural convection in a narrow rectangular channel between two vertical parallel Material Testing Reactor (MTR)-type fuel plates, imposed under a heat flux with a cosine shape to determine the margin of the nuclear core power at which the natural convection cooling mode can ensure a safe core cooling, where the cladding temperature should not reach a specific safety limits (90 °C). For this purpose, a computer program is developed to determine the principal parameters related to the nuclear core safety, such as the temperature distribution in the fuel plate and in the coolant (light water) as a function of the reactor core power. Throughout the obtained results, we noticed that the core power should not reach 400 kW, to ensure a safe passive residual heat removal from the nuclear core. For this purpose, a computer program is developed to determine the distribution of the cladding and the coolant temperatures along the channel as a function of the core power.

Keywords—Buoyancy force, friction force, friction factor, finite volume method, transient natural convection, thermal hydraulic analysis, vertical heated rectangular channel.

I. INTRODUCTION

The uses of the passive cooling mode like the free natural convection in the reactor technologies, are increased in the last decade, which can evacuate the reactor decay heat after a normal or accident shutdown efficiently without the need of any power supply, and this can satisfy some nuclear safety requirements and allowed us to overcome some nuclear safety issues.

For the open pool nuclear research reactor, the upward natural convection induced by the density difference between the core and the pool should be investigated to ensure that the cladding temperature will not reach 90 °C, which is considered like a safety limit, to avoid any undesired effect to the integrity of the fuel plat.

In this last decade, many authors are attracted by the features of the natural convection cooling mode, especially in nuclear field, mainly for their passive residual heat removal from the nuclear core.

Among the many published works, our whole interest goes toward the work done by Jo [1], which is considered like a reference of our study, for the application and the verification of our computer program. In their work, the authors carried out, by the both RELAP5/MOD3 and NATCON codes, a numerical simulation of plate type research reactors during the natural convective cooling mode in a hot spot of a fuel assembly. Several convective heat transfer correlations are implemented into the simulations; then the coolant and cladding temperatures and ONB temperature margin, as a function of core power, are obtained from the simulations with a good agreement between the both used codes.

In the present study, a numerical solution of a transient upward natural convection of light water in a vertical channel between two parallel MTR-type fuel plates is carried out by the finite volume method to ensure a safe passive residual heat removal from the nuclear core. For this purpose, a computer program is developed to determine the distribution of the cladding and the coolant temperatures along the channel as a function of the core power.

II. THE TRANSIENT NATURAL CONVECTION GOVERNING EQUATIONS

For the case of one-dimensional monophasic transient and free fluid flow, the momentum and the energy equations are respectively expressed by the two equations below [2].

\[ \rho \frac{dv}{dt} + \rho v \frac{dv}{dx} = (\dot{\rho}_c - \rho_{pool})g - \frac{(\rho_{in}v_{in})^2}{\rho_p} \]

\[ \rho Ac_p \frac{dT}{dt} + m c_p \frac{dT}{dx} = \frac{\rho_p g \cdot \Delta \cdot \cos \left( \frac{\pi x}{2l_p} \right)}{2} \]

And for calculating the temperature distribution in the fuel meat and in the cladding, the heat equation is used.

\[ \rho c_p \frac{dT}{dt} = \frac{d}{dx} \left( k \frac{dT}{dx} \right) \]

The first and the second terms of the right side of (1), represent respectively the buoyancy and the friction forces, where \( \dot{\rho}_c \), \( \rho_c \), \( \rho_{in} \) and \( \rho_{pool} \) (kg/m\(^3\)) are respectively the mean, the local, the inlet and the Pool coolant density. \( v \) and \( v_{in} \) (m/s) are respectively the local and the inlet coolant velocity, \( f \) is the friction factor, \( P_h(m) \) is the channel heated perimeter, \( \lambda_p(m) \) is the extrapolated length. \( T(C) \), \( C_p(J/kg \cdot s) \) and \( q_{max}(W/m^2) \) are the coolant temperature, the coolant specific heat, and the surface heat flux generated in the hot channel.

A. The Friction Factor Correlations

To calculate the friction factor in the rectangular heated channel more accurately two corrections are introduced, the first one is given by the factor (ξ) as follows [3]:

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\[ f = \xi f_d \]  

where for water \( \xi = \frac{1}{1.186} \) and \( \mu_w, \mu_b \) are respectively the fluid dynamics viscosity for the temperature of the wall and the bulk temperature. The Darcy friction factor \( f_d \) is calculated according to the flow regime of the coolant where the three following cases are considered.

**B. Laminar Fluid Flow**

For laminar flow, the correlation used is valid only for Reynolds number less than 2000, and \( K_R \) represents the Reynolds correction for the non-circular channel [3].

\[ f_d = \frac{64}{Re K_R} \]  

where,

\[ K_R = \frac{2}{3} + \frac{11}{24} \alpha (1 - \alpha) \]

\[ \alpha = \text{the channel width} \]
\[ \text{the channel length} \]

**C. Transient Fluid Flow**

For the case of transient fluid flow where the Reynolds number varies between 2000 and 5000, the friction factor without taking into account the Reynolds correction for the non-circular channel, is evaluated by a linear interpolation as [4]

\[ f_d = f l + \left( \frac{Re - 2000}{3000} \right) (f_l - f_0) \]

\( fl \) is the friction factor for laminar flow for Reynolds number equal to 2000, \( f_0 \) is the friction factor for turbulent flow for Reynolds number equal to 5000.

After the calculation of the friction factor as a function of the core power which depends only on the coolant velocity, when the coolant velocity increases with the core power, the friction factor is decreased as shown in Fig. 1.

**E. The Coolant Velocity**

The coolant velocity along the channel is determined by the discretization of (1), by the finite volume method over a control volume \( \Delta V = A \, dz \, dt \), Then, the following algebraic equation is obtained.

\[ a_p v_p^{n+1} + a_E v_E^n + a_w v_w^n = b \]  

with,

\[ a_p = \frac{\rho A \Delta z}{\Delta t} + a_E + a_w \]

\[ a_E = \max(\rho(\dot{m}_c,0); \quad \alpha = \max(\dot{m}_w,0) \]

\[ b = q'' p_h \Delta z + \frac{\rho c_p \Delta z}{\Delta t} T_p^n \]

where \( \dot{m}_c \) and \( \dot{m}_w \) are the control volume interfaces masse flow rates, which are calculated in this study as all the control volume interfaces parameters by a centered scheme.

**F. The Coolant Pressure**

The pressure distribution along the channel is calculated after a static pressure analysis by the following expression.

\[ P(z) = P_{atm} + \rho g (H - z) - \frac{1}{2} \rho \, \frac{\partial^2 z}{\partial t^2} \]

**III. THE TEMPERATURE IN THE FUEL PLATE**

**A. The Coolant Temperature**

To obtain the temporal and axial coolant temperature distribution along the channel active length, (2) is discretized by the finite volume method over the control volume \( \Delta V = A \, dz \, dt \), then the following algebraic equation is obtained.

\[ a_p T_p^{n+1} + a_w T_w^n + a_E T_E^n + b \]  

with,

\[ a_p = \frac{\rho c_p \Delta z}{\Delta t} + a_w + a_E \]

\[ a_E = \max[\dot{m}_w c_p,0] ; \quad \alpha = \max[-\dot{m}_w c_p,0] \]

\[ b = q'' p_h \Delta z + \frac{\rho c_p \Delta z}{\Delta t} T_p^n \]

**B. The Cladding Temperature**

The transient outer surface clad temperature is obtained throughout a combined operation, between the discretized heat equation (3) over the considered control volume and with the Newton’s law, which is expressed by the simple equation below.

\[ T_{cl}^{n+1}(z) = \frac{\eta_{max}}{h} \cos \left( \frac{\pi z}{2y_p} \right) + T_{c}^{n+1} \]
So finally, the following algebraic equation is given.

\[ a_F T_p^n + a_q T_q^n + a_W T_W^n = b \]  
(12)

with,

\[ a_F = \left(1 - \frac{(k_{eF} + k_{nF})\Delta t}{\rho_{eF} c_{eF} \Delta z^2} \right) \]

\[ a_E = \frac{k_{eE} \Delta t}{\rho_{eE} c_{eE} \Delta z^2}; \quad a_W = \frac{k_{eW} \Delta t}{\rho_{eW} c_{eW} \Delta z^2} \]

\[ b = \frac{T_p^n}{n} + T_e^{n+1} \]

where \( h \left(\text{w/m}^2\cdot\text{C}^0\right) \) is the convective heat transfer coefficient.

To calculate this coefficient, three different correlations of Nusselt number are analyzed.

- **Elnabass correlation** [5]:

\[ \text{Nu} = \frac{1}{24} \left( G_r P_r \right) \left( 1 - e^{-\frac{5}{4} \left( \frac{a_{SF} + a_{SP}}{a_{SF} + a_{SP}} \right)^{0.75}} \right) \]

\[ G_r = \left[ \frac{\beta \mu}{\rho (T_e - T)} \right]^{0.75} \]

\[ P_r = \frac{\beta \mu}{\kappa} \]

- **Kreith & Bohn correlation** [3]:

\[ \text{Nu} = \frac{h_a}{24} \left( 1 - e^{-\frac{35}{R_a}} \right)^{0.75} \]

\[ R_a = \frac{\rho \beta T_0 (T_e - T) \Delta z^2}{\kappa \nu} \]

- **McAdams correlation** [6]:

\[ \left\{ \begin{array}{l} N_u = 0.59 R_a^{0.25} \quad 10^4 < R_a < 10^9 \\ N_u = 0.129 R_a^{0.33} \quad 10^9 < R_a \end{array} \right. \]

It is evident that the three correlations give almost the same values of the convective heat transfer coefficient. In our study, the McAdams correlation has been chosen because it is simpler and used in some validated nuclear codes.

**C. The Fuel Temperature**

Also, by the same way, the transient fuel temperature is carried out by a combined operation with the electrical analogy expression of the clad-fuel exchanged heat flux, which is expressed by the simple equation

\[ T_p^n(z) = T_p^n + q_{max} \cdot \cos \left( \frac{\pi z}{2 L_p} \right) \left( \frac{x}{k_f} + \frac{c_{eF}}{k_{eF}} \right) \]  
(13)

So finally, the following typical algebraic equation is obtained.

\[ a_F T_p^n + a_q T_q^n + a_W T_W^n = b \]  
(14)

where, \( a_F, a_q, a_W, \) and \( b \) are respectively the half fuel and the cladding thicknesses, while \( k_f \left(\text{w/m} \cdot \text{C}^0\right) \) and \( k_{eF} \left(\text{w/m} \cdot \text{C}^0\right) \) are respectively the nuclear fuel and cladding thermal conductivities.

**IV. RESULTS AND DISCUSSIONS**

In this study, our computer program is applied to the same nuclear reactor core data of the work done by Jo [1], which is considered like a reference work of this study, where the main nuclear core and fuel element characteristics data used to perform our calculation are presented in Table I [1]. All the water properties are calculated as a function of pressure and temperature by the polynomial correlations of the work [7].

**Fig. 2** The convective heat transfer coefficient for different correlations as a function of the core power

After we have evaluated the convective heat transfer coefficient by the three correlations as a function of the core power, the obtained results are presented in Fig. 2.

**TABLE I**

<table>
<thead>
<tr>
<th>THE MAIN CORE GEOMETRIC DATA</th>
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<tbody>
<tr>
<td><strong>Pool depth</strong></td>
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<tr>
<td><strong>number of assembly</strong></td>
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<tr>
<td><strong>number of fuel plates</strong></td>
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<tr>
<td><strong>number of channels</strong></td>
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<tr>
<td><strong>Plate thickness</strong></td>
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<tr>
<td><strong>Meat thickness</strong></td>
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<tr>
<td><strong>Meat width</strong></td>
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<td><strong>Clad thickness</strong></td>
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<td><strong>Channel thickness</strong></td>
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<tr>
<td><strong>Channel width</strong></td>
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<tr>
<td><strong>Plate length</strong></td>
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<td><strong>Heated length</strong></td>
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<td><strong>Unheated length</strong></td>
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To carry out the distribution of the interested parameters related to the reactor core safety along the channel, an iterative process is established where it completely depends on the coolant inlet velocity. So, in this iterative process, we increase a guessed inlet velocity with a fixed increment until the difference between the buoyancy and the friction forces satisfies a convergence criterion as shown in Fig. 3.

The buoyancy and the friction forces are respectively calculated by means of the following expressions [1]

\[ F_B = (\rho_c - \rho_{pool}) L g \] (15)

\[ F_F = \frac{(\rho_{in} v_{in})^2}{2} A \left[ \frac{1}{2\rho_{in}} + \sum_{i=1}^{n} f \frac{\Delta S_i}{\rho_p \rho_{in}} + \frac{1}{\rho_{out}} \right] \] (16)

where \( \rho_{in}, \rho_{out}, \) and \( \rho_p (\text{kg/m}^3) \) are respectively the inlet, the outlet, and the local coolant densities. \( L [\text{m}] \) and \( [\text{m}^2] \), are respectively the channel length and cross section. In Figs. 4-7, we display the typical results that we obtained by means of our simple and fast calculation computer program for 200 kW core power.

In Fig. 8, we show the comparison of the coolant and cladding temperatures in the hot channel at 400 kW core power of both results obtained by our computer program and those published in the reference work.
In Figs. 9-11, we present respectively the coolant velocity, hottest cladding, and the outlet coolant temperatures which are compared always with the same reference work, in the hot channel as a function of the core power. The nominal difference between the two calculation tools is more significant for the coolant velocity which can reach 20% for 100 kW and 200 kW core power. But, it is significantly less for the both outlet coolant and cladding temperatures where it does not exceed respectively the 10% and 2%. Finally, in Fig. 12, we present the outlet coolant pressure, where it is evident that it decreases implicitly with the increasing of the coolant velocity by the increasing of the core power.

V. CONCLUSIONS

In this study, we performed a thermal hydraulic analysis by solving numerically a transient natural convection in a heated rectangular channel between two vertical parallel MTR-type fuel plates, for the same core reactor of the reference work. The main goal of this study is to determine the core power margin to employ safely the natural convection cooling mode without reach a critical state, where the cladding temperature must stay below a specific safety limit (90 °C). For this purpose, a computer program is developed to calculate and determine the coolant and cladding temperature distributions in the hot channel of nuclear fuel element, as a function of the
core power. Then, our results are validating throughout a comparison against other published study. The core power should not reach 400 kW, to ensure a safe passive residual heat removal from the nuclear core by the upward natural convection cooling mode.

REFERENCES