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An Improved Total Variation Regularization Method for Denoising Magnetocardiography

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***Abstract*—**The application of magnetocardiography signals to detect cardiac electrical function is a new technology developed in recent years. The magnetocardiography signal is detected with Superconducting Quantum Interference Devices (SQUID) and has considerable advantages over electrocardiography (ECG). It is difficult to extract Magnetocardiography (MCG) signal which is buried in the noise, which is a critical issue to be resolved in cardiac monitoring system and MCG applications. In order to remove the severe background noise, the Total Variation (TV) regularization method is proposed to denoise MCG signal. The approach transforms the denoising problem into a minimization optimization problem and the Majorization-minimization algorithm is applied to iteratively solve the minimization problem. However, traditional TV regularization method tends to cause step effect and lacks constraint adaptability. In this paper, an improved TV regularization method for denoising MCG signal is proposed to improve the denoising precision. The improvement of this method is mainly divided into three parts. First, high-order TV is applied to reduce the step effect, and the corresponding second derivative matrix is used to substitute the first order. Then, the positions of the non-zero elements in the second order derivative matrix are determined based on the peak positions that are detected by the detection window. Finally, adaptive constraint parameters are defined to eliminate noises and preserve signal peak characteristics. Theoretical analysis and experimental results show that this algorithm can effectively improve the output signal-to-noise ratio and has superior performance.

***Keywords*—**Constraint parameters, derivative matrix, magnetocardiography, regular term, total variation.

I. Introduction

DIFFERENT from other physiological signals, the MCG signal is a non-invasive and non-contact technique [1], which provides electrical characteristics of the heart on the other hand relative to ECG signals [2], [3]. The MCG signal is detected with SQUID that has unparalleled sensitivity for measurement of the magnetic fields associated with the electrical activity of the heart [4], [5]. Generally, such measurements are conducted in order to detect small magnetic field signals in the presence of large background noise [6], [7], which makes it difficult to extract useful information at low SNR. So, removing background noise and recovering useful signals are chief objectives. To extract the signals which are buried in the noise, signal preprocessing contains essential and various tools, e.g., wavelet [8]-[10], EMD [11], [12] and filtering methods such as the Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) [13], [14], have been evolved and are used. The wavelet transform method for signal denoising is based on the use of a set of predefined basis functions, in order to decompose the measured signals and remove components corresponding to noise. The main disadvantage of this method is that the selection of wavelet basis seriously affects the denoising results. Empirical Mode Decomposition (EMD) is one of the decomposition methods of signal denoising, and is widely used to decompose a signal into different modes recursively. This method is, however, prone to mode mixing, and limited by sensitivity to noise and sampling [15]. The mode mixing is significantly reduced by a modified noise-assisted data analysis method known as the Ensemble Empirical Mode Decomposition (EEMD) method [15], [16]. However, the decomposition results are unsatisfactory because of the low signal-to-noise ratio. Lately, a new adaptive decomposition method called Variational Mode Decomposition (VMD) has been proposed [17], [18]. However, the research results showed that the decomposition results lack of adaptability. The TV [19], [20] regularization model is proposed at an early stage and was considered to be a signal denoising model that can reasonably maintain signal characteristics. Recently, some scholars have applied TV based approach to the denoising of one-dimensional signals and peak detection [21], [22]. However, the denoising results show that the algorithm may produce step effect and lack of constraint adaptability. So, the denoising performance needs to be improved.

In order to overcome the problems above, we propose an improved TV regularization method whose regular term introduces second derivative matrix with adaptive constraint parameters. The rest of this paper is organized as follows: Section II introduces the data model required for the TV denoising algorithm. In Section III, a TV denoising scheme based on the second derivative matrix is proposed. The application for denoising methods of MCG signal is shown in Section IV. Conclusion is given in Section V.

II. Data Model

For one-dimensional signal $x(n), 1 \le n \le N$, the TV, that is, the sum of the degree of change in the signal is defined as:

$$TV(x) = \sum_{n=1}^{N} \left| x(n+1) - x(n) \right| \tag{1}$$

For ease of expression, (1) can also be written as:

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$TV(x) = \|Dx\|_1$, where $\|\cdot\|_1$ represents the l_1 norm, then the first-order derivative matrix D of size $(N-1)\times N$ is defined as,

$$D = \begin{bmatrix} -1 & 1 & & \cdots & & \\ & -1 & 1 & & \cdots & \\ & & \ddots & & & \cdots \\ & & & & -1 & 1 \end{bmatrix}_{(N-1)\times N} \quad (2)$$

Assume that the measurement signal $y(n)$ is the signal buried in the noise $w(n)$. $x(n)$ is the original signal, and the expression is as follows:

$$y(n) = x(n) + w(n) \quad (3)$$

where $w(n)$ is the noise component. Compared with the other signal denoising methods, the TVregularization method converts the signal recovery into a well-posed problem by introducing certain constraints, and ensures the existence and uniqueness of the signal recovery result, which has the advantage of less noise interference. The method of TV regularization for solving the original clear signal can be transformed into the following problem of minimizing the objective function $F(x)$:

$$F(x) = \frac{1}{2}\|y-x\|_2^2 + \lambda\|Dx\|_1 \quad (4)$$

The former is the fidelity constraint, and the latter is the TV regularization term. λ is the regularization parameter. The method is expected to achieve the best denoising effect by adjusting the regularization parameter to adjust the two parts of the above equation and calculate the minimum value.

Since the l_1 norm is not divisible, the general optimization methods cannot be used to optimize the objective function $F(x)$. The Majorization-minimization algorithm uses the iterative idea to achieve better results in solving the optimization problem [23]-[25], which is essentially an iterative algorithm. The algorithm consists of two steps: first, we need look for an optimization function $G_k(x)$ that is easy to find the optimal solution and is related to the objective function. Then, we can solve the minimum point of the optimization function. It should be noted that the selected optimization function $G_k(x)$ satisfies the following two conditions:

1) $G(x, x^{(k)}) \geq F(x), \forall x$
2) $G(x^{(k)}, x^{(k)}) = F(x^{(k)})$

where $x^{(k)}$ represents the current iteration value of the signal to be sought. We find the optimization function that satisfies the above conditions as follows:

$$G(x, x^{(k)}) = \frac{1}{2}\|y-x\|_2^2 + \lambda\frac{1}{2}x^T D^T \Lambda_k^{-1} Dx + \lambda\frac{1}{2}\|Dx^{(k)}\|_1, \quad \Lambda_k = diag(|Dx^{(k)}|) \quad (5)$$

where $diag(v)$ can generate a diagonal matrix, whose diagonal elements are the elements of v. The next iteration value can be expressed as follows:

$$x^{(k+1)} = \arg\min_x G(x, x^{(k)}) \quad (6)$$

The $k+1$ th estimated value of the signal can be obtained by minimizing $G(x, x^{(k)})$. Matrix inversion lemma is applied to solve the iterative formula:

$$x^{(k+1)} = y - D^T\left(\frac{1}{\lambda}diag(|Dx^{(k)}|) + DD^T\right)^{-1} Dy \quad (7)$$

The above content is the original TV method denoising process. The basic principle of the TV denoising method is to use the regular term as the prior knowledge of the original signal to constrain the structural distribution of the reconstructed signal, thereby obtaining the estimated signal with high signal to noise ratio.

III. Proposed New TV Scheme

TV denoising method can remove unsmooth noises by minimizing the difference between adjacent signals. However, the method has limited denoising effect on smooth noise such as baseline drift noise. In order to further remove the baseline drift noise based on the original denoising result, we constrain the minimum difference of adjacent data while also constraining the difference of the same position in the adjacent period. Based on the quasi-periodicity of the MCG signal, a new regularization matrix $D1$ is defined:

$$D1 = \begin{bmatrix} 1 & -2 & \cdots & 1 & \cdots & & \\ & 1 & -2 & \cdots & 1 & \cdots & \\ & & \ddots & & & & \\ & & & 1 & -2 & \cdots & 1 \end{bmatrix}_{(N-p)\times N} \quad (8)$$

For the i th row of the matrix, the non-zero elements are the i th and $(i+1)$ th and $(i+p_i)$ th elements, and the corresponding values are 1,-2,1, where p_i represents the number of data points between the i th data point and the point at the same position in the next cycle. We can use the QRS peak as a benchmark to find p_i. The specific process is as follows:

1) Detect signal QRS peak position. First, we need determine the window length L of a detection window. In general, L is the number of sampling points in a heartbeat cycle.
2) Determine whether the i th data point of the noise signal

$y(i)$ is a peak. If $y(i)$ satisfies the formula: $y(i)=\max\left\{y(n), i-\frac{L}{2}<n\le i+\frac{L}{2}\right\}$, it is a peak. Then, we continue to detect the next point until all peaks are recorded.

3) Calculate the peak point closest to the i th data point, where p_i is defined as the distance between the peak point and the next peak point.

The new matrix can constrain the correlation of adjacent periods of the signal to effectively remove baseline drift noise.

There are multiple peaks in the MCG signal. Constraining the difference between adjacent data will weaken the peaks. Although the traditional algorithm can retain the signal characteristic information, the details are partially distorted. In order to improve the adaptability of the algorithm, we introduce adaptive constraint parameter θ. The function of this parameter is to reduce the constraint strength in the sharp wave band and increase the constraint strength in the smooth wave band. Based on the above matrix $D1$, each row of the matrix is multiplied by a constraint parameter. We can get a new adaptive constraint matrix $D2$.

$$D2=\left[\theta_1 D1_1, \theta_2 D1_2, \cdots, \theta_{N-p} D1_{N-p}\right]^T \quad (9)$$

where $D1_i$ represents the i th row of the matrix $D1$, which is

$$D2=\begin{bmatrix} \theta_1 & -2\theta_1 & \cdots & \theta_1 & & \cdots & \\ & \theta_2 & -2\theta_2 & \cdots & \theta_2 & \cdots & \\ & & \ddots & & & & \\ & & & \theta_{N-p} & -2\theta_{N-p} & \cdots & \theta_{N-p} \end{bmatrix}_{(N-p)\times N} \quad (10)$$

Multiplying the i th row of the matrix by the sequence x, $D2_i x$ can adaptively constrain the difference between the adjacent data and the adjacent period. The above matrix shows that the greater the value of θ, the stronger the constraint strength, conversely, the smaller the value of θ, the weaker the constraint strength. We expect to obtain the value of the adaptive parameter by priori knowledge of the noise signal. Here, we need to segment the noise signal and then calculate the variance to solve θ. It is obvious that the adaptive parameter θ_i is inversely proportional to the variance that indicates the degree of signal variation at the i point. The adaptive parameter θ_i is defined as:

$$\theta_i = a\frac{\frac{1}{N}\sum_{i=1}^{N}\mathrm{var}\left(y_{i,seg}\right)}{\mathrm{var}\left(y_{i,seg}\right)} \quad (11)$$

where $y_{i,seg}$ represents a small segment of noise signal with a fixed length L_{seg} and is centered on i. a is a constant used to adjust the value of the adaptive parameter. The larger the variance of each segment of the signal, the more severely the changing near the center of the signal. At this time, the adaptive parameters become smaller and the constraint strength becomes smaller, which ensures that the signal characteristics are preserved. We replace the original matrix with the new matrix $D2$ as the regular term. The iterative formula can be derived by Majorization-minimization algorithm:

$$x^{(k+1)} = y - D2^T\left(\frac{1}{\lambda}diag\left(\left|D2x^{(k)}\right|\right)+D2D2^T\right)^{-1}D2y \quad (12)$$

Summarizing this denoising process might be done as follows: first, the detection window is applied to detect the peak position of the noise signal. Then, the parameter values p_i and θ_i are calculated to determine the form of the matrix $D2$. Finally, majorization-minimization algorithm is performed to obtain the iterative formula. The detailed procedures are as follows:

1) Set the regularization parameter λ. Generally, the higher the signal-to-noise ratio, the smaller the value of λ, the range is taken 0~2.
2) Determine the form of the regularization matrix $D1$, firstly use the detection window method to detect the signal peaks, and find the same position of the to-be estimated point in the next cycle based on the adjacent peak value, so as to determine the value of p_i. That is, for the i th row of the matrix, the value of the i th and the $(i+p_i)$ th elements is 1, and the value of the $(i+1)$ th element is -2. Finally, each row of $D1$ is obtained in turn.
3) Calculate the constraint parameters θ_i corresponding to each data point to determine the form of the regularization matrix $D2$.
4) Let $k=0$, initialize the value of x_0.
5) According to the principle of majorization-minimization algorithm, choose an optimization function $G_k(x)$ to satisfy:

$$G\left(x,x^{(k)}\right)\ge F(x), \forall x$$
$$G\left(x^{(k)},x^{(k)}\right)=F\left(x^{(k)}\right)$$

6) Minimize the optimization function to get estimated sequence:

$$x_{k+1}=\arg\min G_k(x)$$

7) Calculate the sequence x_{k+1} using the iterative formula. Iteration is stopped until the minimum condition is satisfied. Otherwise let $k=k+1$, and return to step 5.

IV. Results and Discussion

Three different types of noise have been added to the MCG signal, in order to investigate the effectiveness of denoising by an improved TV denoising method. The types of noise include a low frequency (0.3 Hz) sinusoidal signal for simulating the baseline drift, 50 Hz sinusoidal signal for simulating the interference at power line frequency, and high frequency random noise. Reference [26] shows the denoising results of the MCG signals by the EEMD algorithm and the VMD algorithm. The denoising performance of these two algorithms is discussed below.

The signal is denoised by different algorithms. In Fig. 1, we compare the original signal with the reconstructed signals obtained from EEMD based denoising methods using soft and hard thresholding. The results show that the hard threshold processing can reconstruct the QRS peak waves, but there are obvious errors in the reconstruction of other signal parts.

Based on the bandwidth of the measurement signal and multiple tests, the number of modes decomposed by VMD is assigned to 6. The initial value of quadratic penalty α is assigned to 2000, and the default of the bandwidth τ is 0. In Fig. 2, we compare the original signal with the reconstructed signals obtained from VMD methods with soft and hard thresholding. The results show that the baseline of the reconstructed signal is not uniform with the original signal. To make matters worse, there is serious distortion in the reconstructed signal from the soft thresholding processing.

In order to get more useful research results, the TV denoising method is applied to denoise the MCG signal. The denoising result of the original algorithm is shown in the upper part of Fig. 3. It is seen from the figure that TV denoising method cannot effectively remove baseline drift noise. At the same time, we can see that the signal waveform is not smooth and slightly distorted. The reason is that the algorithm is prone to step effect.

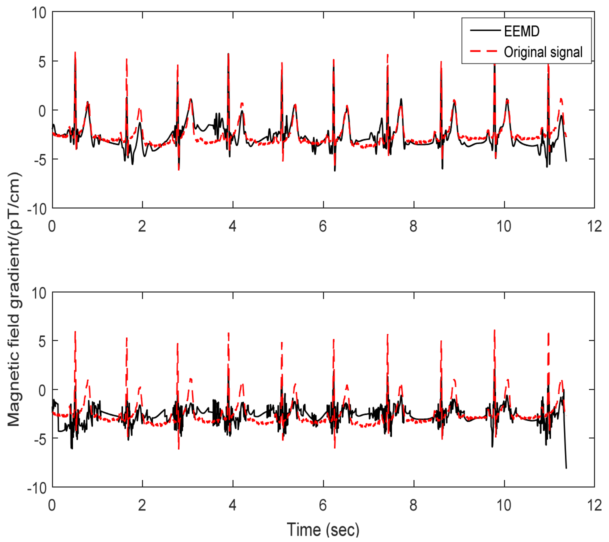

Fig. 1 A comparison of the original signal (dotted line) with the reconstructed signals (solid line) obtained from EEMD based denoising methods with soft and hard thresholding. The panel above is the result of hard thresholding processing and the following panel is the result of soft thresholding processing

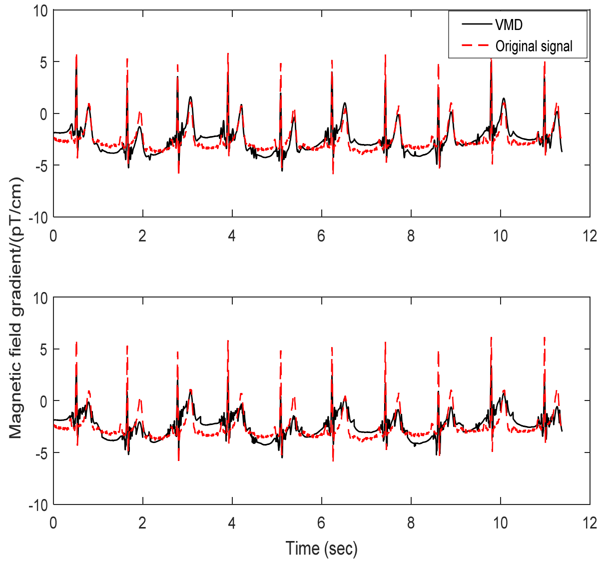

Fig. 2 A comparison of the original signal (dotted line) with the reconstructed signals (solid line) obtained from VMD based denoising methods with soft and hard thresholding is shown in the figure

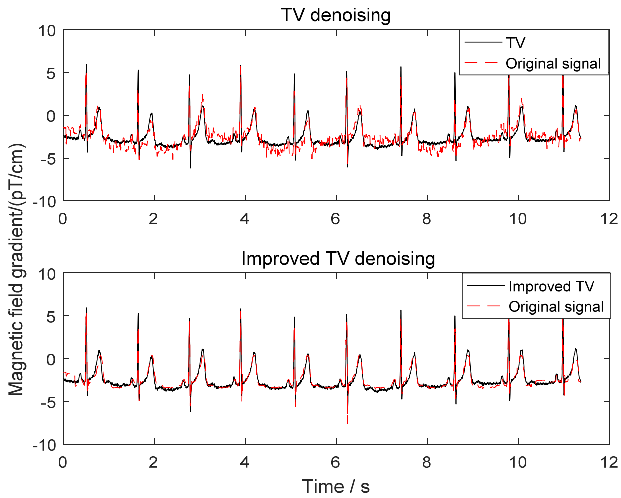

Fig. 3 A comparison of the original signal (dotted line) with the reconstructed signals (solid line) obtained from TV based denoising method and the proposed method is shown in the figure

To get a better result, we use the proposed method to denoise the signal. According to many experiments, we choose the parameter value with better effect as the next simulation initial values. The regularization parameter λ is set in the range of 0 to 2. The length L of the detection window is assigned as 360, which is the number of sampling points in a heartbeat cycle. The length L_{seg} of the data segment that needs to be solved for variance is assigned as 31. The constant a for adjusting the adaptive constraint parameters can be set according to the detailed data features and is the range 1 and 3. The denoising result of the improved TV denoising algorithm is shown in the lower part of Fig. 3. By comparing the reconstructed signal with the original signal, we can find that the fitting degree between the reconstructed signal and the original signal is good, and all three kinds of noise in the signal are effectively removed. It may also be noted that, the signal to noise ratio improvement of the improved TV denoising method is much better than other methods.

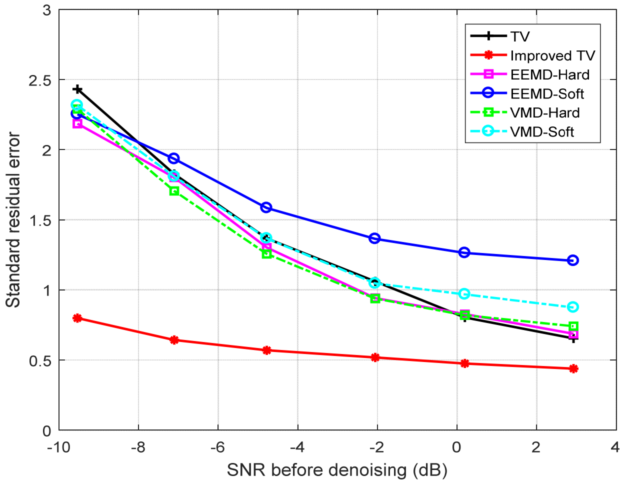

Fig. 4 The root-mean-square deviation (RMSE) of the EEMD, the VMD, and the improved TV methods are revealed. The improved TV method outperforms other methods

In order to better compare the performance of the algorithms, we use the root-mean-square error (the square root of the mean of the sum of squared residuals, RMSE) to characterize the fitting degree of the reconstructed signal and the original signal. In Fig. 4, we compare the root-mean-square deviation (RMSE) of the four methods with the different input signal-to-noise ratio (SNR). In the case of low input SNR, the RMSE of the improved TV denoising method is significantly less than the other three methods, and the method has better denoising performance even with the low input SNR.

In Fig. 5, we compare the performance of the original TV method, the improved TV method, the EEMD method, and the VMD method. For computing SNR, the logarithmic ratio of variance of a signal (from the beginning of P-wave to the end of T-wave for one cardiac cycle) to the variance of noise (from the end of T-wave to the beginning of P-wave, i.e., in the TP interval) has been taken. It is seen from Fig. 5 that the original TV algorithm applied to denoise the MCG signal cannot effectively improve the signal to noise ratio. Signal to noise ratio improvement of other threshold-based denoising algorithms is limited. Obviously, the improved TV method is capable of achieving better SNR when compared with EEMD and VMD methods.

From the above simulation results, we can see that the proposed algorithm has better denoising performance compared with EEMD and VMD methods. Although there are slight disturbances in the QRS spikes and S-T waves obtained by filtering, it does not affect the calculation of heart related parameters. Measurements of magnetic field energy and current density remain accurate. In the case of the low input signal-to-noise ratio used in this paper, the SNR improvement of the proposed algorithm can be above 15 dB. The algorithm filtering results can support feature extraction of MCG and detection of heart disease.

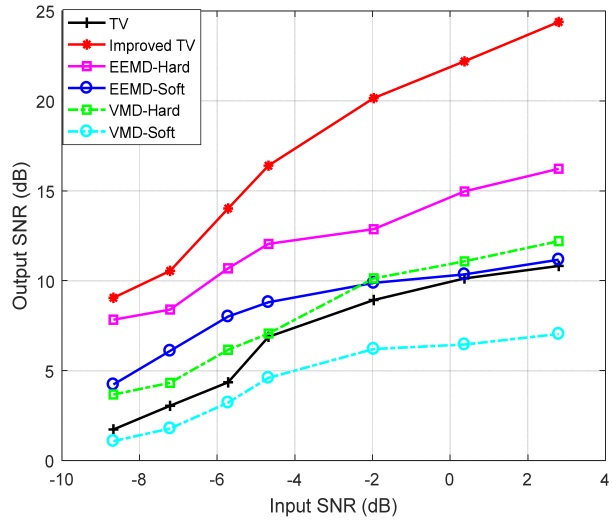

Fig. 5 The variation of the output signal-to-noise ratio (SNR) by EEMD, VMD， TV and the improved TV methods. The improved TV method outperforms other methods.

V. Conclusion

The proposed method in this paper overcomes the unadaptable constraint problem of TV, which improves the availability and precision of denoising of the MCG signal. This method adaptively adjusts the second derivative matrix of regular term by detecting signal peaks and calculating the variance of the segmentation signal. The low-frequency noise is eliminated, according to adjusting the position of the last non-zero element of each row of the second derivative matrix. The variance described above is used to calculate the adaptive constraint parameters. Then, iterative formula is used to solve the denoised sequence. The simulation experiments show the superiority of the improved TV in denoising performance for the MCG signal. The acceleration of the proposed method and the suitable signal preprocessing methods should be considered in future research and applications.

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