Effects of Mixed Convection and Double Dispersion on Semi Infinite Vertical Plate in Presence of Radiation


Abstract—In this paper, the effects of radiation, chemical reaction and double dispersion on mixed convection heat and mass transfer along a semi vertical plate are considered. The plate is embedded in a Newtonian fluid saturated non-Darcy (Forchheimer flow model) porous medium. The Forchheimer extension and first order chemical reaction are considered in the flow equations. The governing sets of partial differential equations are non-dimensionalized and reduced to a set of ordinary differential equations which are then solved numerically by Fourth order Runge-Kutta method. Numerical results for the detail of the velocity, temperature, and concentration profiles as well as heat transfer rates (Nusselt number) and mass transfer rates (Sherwood number) against various parameters are presented in graphs. The obtained results are checked against previously published work for special cases of the problem and are found to be in good agreement.

Keywords—Radiation, Chemical reaction, Double dispersion, Mixed convection, Heat and Mass transfer

I. INTRODUCTION

Many investigations have been made over convection heat transfer from vertical surfaces embedded in porous media. This is due to fact that these flows have many engineering and geo-physical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, groundwater pollution, and underground energy transport. In the study of fluid flow over heated surfaces, the buoyancy forces are, generally, neglected when the flow is horizontal. However, for vertical or inclined surfaces, the buoyancy forces exert strong influence on the flow field. Hence, it is not possible to neglect the effect of buoyancy forces for vertical or inclined surfaces. Nield and Bejan [1] and Pop and Ingham [2] have made comprehensive reviews of the studies of heat transfer in relation to the above applications. Ali et.al [3] studied the natural convection-radiation interaction in boundary layer flow over horizontal surfaces. The effect of radiation on free convection of an optically dense viscous incompressible fluid along a heated inclined flat surface maintained at uniform temperature placed in a saturated porous medium[4]. Yih [5] investigated the radiation effect on the mixed convection flow of an optically dense viscous fluid adjacent to an isothermal cone embedded in a saturated porous medium. Bakier [6] analyzed the effect of radiation on mixed convection from a vertical plate in a saturated medium. Cheng and Minkowycz [7] presented similarity solutions for free convective heat and mass transfer from a vertical plate in a fluid saturated porous medium. Chen and Chen [8] studied free convection from a vertical wall in a non-Newtonian fluid saturated porous medium. Mehta and Rao [9] studied the free convection heat transfer in a porous medium past a vertical flat plate with non-uniform surface heat flux at the wall. Mucoglu and Chen [10] have studied the mixed convection flow over an inclined surface for both the assisting and the opposing buoyancy forces. Free convection flows occur not only due to temperature difference, but also due to concentration difference or the combination of these two. Anjalidevi and Kandasamy [11] studied the effects caused by the chemical-diffusion mechanisms and the inclusion of a general chemical reaction of order $n$ on the combined forced and natural convection flows over a semi-infinite horizontal plate immersed in an ambient fluid. Kandasamy [12] analyzed the effects of chemical reaction heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection. The mixed convection boundary layer flow on an impermeable vertical surface embedded in a saturated porous medium has been treated by R. R. Kairi [13]. Amin, et.al [14] have studied the effects of chemical reaction and double dispersion on non-Darcy free convective heat transfer flows and aimed at analyzing the effects of radiation with chemical reaction over a vertical surface embedded in a porous medium. The effects of double dispersion on natural convection heat and mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium have been investigated by A.J. Chamka. [15]. Murti A.S.N. et.al [16] analyzed the effects of Radiation, chemical reaction and double dispersion on non-Darcy free convective flow. Ali J. Chamkha [17] analyzed heat and mass transfer for a non-Newtonian fluid flow along a surface embedded in a porous medium with uniform wall heat and mass fluxes and heat generation or absorption. Ramachandran et.al [18] have studied the mixed convection flow over vertical and inclined surfaces, theoretically as well as experimentally. Merkin and Mahmood [19] have obtained the similarity solution of the mixed...
convection flow over a vertical plate for the constant heat flux case. Murti et.al [20] obtained heat and mass transfer effects over a vertical plate in a free convective non-Newtonian fluid with double dispersion. The objective of the present paper is to analyze the effects of thermal radiation and first order chemical reaction on a non-Darcy mixed convective Newtonian fluid over a semi infinite vertical plate embedded in a porous medium subject to constant temperature. The equations of continuity, momentum, energy and concentration, which govern the flow field, are solved numerically by Runge-Kutta method of fourth order.

II. FORMULATION OF PROBLEM

Consider the mixed convection in a porous medium saturated with a Newtonian fluid over a semi vertical plate. The x-coordinate is measured along the surface and the y-coordinate normal to it. The wall is maintained at constant temperature and constant concentration. The ambient conditions for temperature and concentration are assumed to be constant as shown in Fig1. The governing equations for steady non-Darcy flow in a saturated porous medium can be written as follows.

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(Continuity Equation)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{K}{\mu} (\frac{\partial p}{\partial x} + \rho g) \]  

(Momentum Equation)

\[ \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D_x \frac{\partial C}{\partial x} + D_y \frac{\partial C}{\partial y} - K_e (C - C_\infty) \]  

(Concentration Equation)

\[ \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = -\frac{K}{\rho C_p} \frac{\partial q}{\partial y} \]  

(Energy Equation)

Energy Equation:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} \]  

(4)

where

\[ \frac{\partial q}{\partial y} = -16\sigma RT_\infty^4 (T_e - T) \]  

(5)

Concentration Equation:

\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial y} \left( D_x \frac{\partial C}{\partial x} + D_y \frac{\partial C}{\partial y} \right) - K_e (C - C_\infty) \]  

(6)

\[ \rho = \rho_s \left[ 1 - \beta (T - T_e) - \beta' (C - C_\infty) \right] \]  

(7)

\[ y \to 0; v = 0, T_u = \text{constant}, C_u = \text{constant} \]  

(8)

Following Telles and Trevisan [21], the quantities of \( \alpha_x \) and \( D_y \) are variables defined as \( \alpha_x = \alpha + \gamma d [v] \) and \( D_y = D + \zeta d [v] \) represent thermal diffusion and solutal dispersion respectively. This model for thermal dispersion has been used extensively by Cheng [7], Plumb [22], Hong and Tien [23]; Murthy and Singh [24], in studies of non-Darcy convective heat transfer in porous media. Introducing the stream function \( \psi \) such that \( u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \) and the similarity space variables are defined as follows;

\[ \psi = f(\eta) \alpha \sqrt{Ra_x}, \eta = Ra_x \frac{y}{x}, \theta(\eta) = \frac{T - T_e}{T_e - T_\infty} \]  

and

\[ \phi(\eta) = \frac{C - C_\infty}{C_u - C_\infty} \]  

Then the equations of motion reduce to

\[ f' + 2F_o Ra_x f' f'' = \varepsilon \left( \theta' + N \phi \right) \]  

(9)

\[ \theta' + \frac{1}{f} \frac{\partial}{\partial \eta} \left[ f' \theta' + f \theta'' \right] + \frac{4R}{3} \left[ 3 \theta' (\theta + c') - 3 \phi' (\theta + c') \right] = 0 \]  

(10)

\[ \phi' + \frac{1}{2} Lef \phi' + \zeta Ra_x \left( f' \phi' + f \phi'' \right) - \varepsilon Sc \lambda \frac{Gc}{Re_x^2} \phi = 0 \]  

(11)

where \( F_o = \frac{c \sqrt{K}}{\nu d} \) collects a set of parameters that depends on the structure of the porous medium and the thermo-physical properties of the fluid saturating it, \( \varepsilon = \frac{Ra_x}{Pe_x} \) is the mixed convection parameter. \( \varepsilon = \frac{4\sigma \theta^3}{kk} \) is the conduction radiation parameter. \( Ra_x = \frac{Kg \beta (T_e - T_\infty) d}{a \nu} \) is the modified pore diameter-dependent Rayleigh number, \( N = \frac{\beta' (C_u - C_\infty)}{\beta (T_e - T_\infty)} \) is the buoyancy ratio parameter. The modified Grashof number Local Reynolds number, Schmidt number, chemical reaction – porous medium parameter, Lewis number, Prandtl numbers are defined respectively in the following way.

![Fig. 1 Schematic diagram](image-url)
The reference velocity by Elbashbeshy [25] is

\[ u_r = \sqrt{g \beta d (T_s - T_u)} \]

By above quantities we now modify the equation (11) as

\[ \phi' + \frac{1}{2} \text{Re} \phi' + \xi \text{Ra}_d \text{Le}(f' \phi' + f'' \phi') - e \chi \phi = 0 \]  

where \( \chi = \frac{Sc \alpha Gc}{\text{Re}_c^2} \) is Chemical reaction- porous medium parameter. The boundary conditions given in the equation (8) change to

\[ f(0) = 0, \theta(0) = \phi(0) = 1, f'(\infty) = 0, \theta'(\infty) = \phi'(\infty) = 0 \]  

The behaviors of velocity and temperature profiles for different values of the non-Darcy parameter \( F_0 \) and mixed convection parameter \( Ra/Pe \), are shown in Figs. 2 and 3, respectively. The following set of parameters was assumed: \( Le=0.5, Ra_d = 0.7, N=-0.1, \gamma = \zeta=0.0, \chi=0.02 \). The buoyancy ratio, \( N \), set less than zero, \( (N<0) \) indicating that the temperature and concentration buoyancy effects are in opposing direction. It is clear from the fig.2 that velocity increases with the increase in the mixed convection parameter. Both in aiding and opposing flows, velocity decreases with increase of non-Darcy parameter. This is due to the fact that the non-linear drag is more pronounced when the velocity is larger. From fig. 3 it is noticed that temperature decreases with the increase of mixed convection parameter. It is also observed that both in aiding and opposing flows temperature increases with the increase in the non-Darcy parameter.

### III. NUMERICAL SIMULATION

The coupled non-linear ordinary differential equations (9), (10) and (112) together with the boundary conditions (13) are solved numerically by the fourth order Runge-Kutta method with the double shooting technique. By giving appropriate hypothetical values for \( f'(0), \theta'(0) \) and \( \phi'(0) \) we get the corresponding boundary conditions for \( f'(\infty), \theta'(\infty) \) and \( \phi'(\infty) \) respectively. In addition, the boundary condition \( \eta \to \infty \) is approximated by \( \eta_{max} = 4 \) which is found to be sufficiently large for the velocity and temperature to approach the relevant free stream properties. The choice \( \eta_{max} \) helps to compare our results with those of earlier works.

### IV. RESULT AND DISCUSSION

In order to get the physical insight, the system of ordinary differential equations 9, 10 and 12 along with the boundary conditions 13, is integrated with shooting technique. The step size \( \Delta \eta=0.05 \) is used, while obtaining the numerical solution with \( \eta_{max}=4 \) and five decimal accuracy as the criterion for convergence. Numerical computations are carried out for parameters \( F_0, Le, Ra_d, \gamma, \zeta, \chi \), for the two cases aiding buoyancy (\( N>0 \)) and opposing buoyancy (\( N<0 \)) and first order of chemical reaction (\( n=1 \)) in both aiding flows (\( Ra_d/Pe,>0 \)) and opposing flows (\( Ra_d/Pe, <0 \)).

Figures 4 and 5 illustrate the effects of Lewis number, \( Le \), and mixed convection parameter \( Ra/Pe \), on velocity and concentration profiles, respectively, with respect to the following set of parameters: \( F_0=0.3, Ra_d = 0.7, N=-0.1, \gamma = \zeta=0.0, \chi=0.02 \). Concentration is more affected than the velocity by increasing the \( Le \) number, which may be attributed to the fact that increase in Lewis number implies that heat dispersion, is more pronounced than mass dispersion which results in larger concentration gradient near the wall. In
opposing flows both temperature and concentration increase with increase in Lewis number ($Le$). Also it is observed that for aiding flows increase of $Le$ velocity increases/decreases in free/mixed convection and concentration decreases. Figures 6,7&8 show the velocity, temperature, and concentration profiles, respectively, with respect to the following set of parameters: $F_0=0.3$, $Ra_d=0.7$, $Le=0.5$, $\gamma=\zeta=0.0$, $\chi=0.02$.

Figures 6, 7, & 8 show that, increase in buoyancy ($N$) reduces/enhances the velocity near the wall in opposing/aiding flow. This is because of changing from opposing buoyancy to an aiding buoyancy mechanism and vice versa. Figures 7 & 8 show that increase in buoyancy enhances/reduces temperature and concentration in opposing/aiding flow.
The behavior of the velocity profiles according to the variation of the parameter $Ra_d$ is shown in fig. 9. It is clear from the figure that the increase in the parameter $Ra_d$ slightly reduces the velocity close to the wall, while it almost has no effect far from it both in opposing and aiding flows. From fig.10, we notice that increase in the chemical reaction parameter, $\chi$, reduces/enhances the concentration in aiding/opposing flows. Figs.11 & 12 illustrate, respectively, the effect of thermal dispersion coefficient ($\gamma$) on the velocity and temperature profiles with the set of parameters: $F_0=0.5$, $Ra_d=0.7$, $N=0.1$, $Le=0.5$, $\zeta=0.0$, $\chi=0.02$. From fig.11, it can be seen that increase in thermal dispersion coefficient $\gamma$ increases/decreases the velocity in aiding/opposing flows. It is observed from Fig.12 that both in aiding and opposing flows temperature enhances with the increase in thermal dispersion coefficient. From figs 13 and 14, we notice that both in aiding and opposing flows velocity and temperature increase slightly with an increase in solutal dispersion coefficient $\zeta$. 

![Fig. 9 Variation of dimensionless velocity $f'$ with similarity space variable $\eta$ for different $Ra_d$, $Ra_x/Pe_x$](image1)

![Fig. 10 Variation of dimensionless concentration $\phi$ with similarity space variable $\eta$ for different $\chi$, $Ra_x/Pe_x$](image2)

![Fig. 11 Variation of dimensionless velocity $f'$ with similarity space variable $\eta$ for different $\gamma$, $Ra_x/Pe_x$, $(Le=0.5, R=1, F_0=0.3, \chi=0.02, Ra_d=0.7, N=0.1, \zeta=0)$](image3)

![Fig. 12 Variation of dimensionless concentration $\theta$ with similarity space variable $\eta$ for different $\gamma$, $Ra_x/Pe_x$, $(Le=0.5, R=1, F_0=0.3, \chi=0.02, Ra_d=0.7, N=0.1, \zeta=0)$](image4)

![Fig. 13 Variation of dimensionless velocity $f'$ with similarity space variable $\eta$ for different $\zeta$, $Ra_x/Pe_x$, $(Le=0.5, R=1, F_0=0.3, \chi=0.02, Ra_d=0.7, N=0.1, \gamma=0)$](image5)
Lewis number is used to characterize fluid flows where there is simultaneous heat and mass transfer by convection and is defined as the ratio of thermal diffusivity to mass diffusivity. The effect of Lewis number on Nusselt number, ratio of convection heat transfer to diffusion heat transfer, and Sherwood number, ratio of convective mass transport to diffusive mass transport, for different $F_0$ and Mixed convection parameter $Ra_d/Pe_e$ are shown in Figs.15 and 16, respectively with the following set of parameters: $Ra_d = 0.7$, $N = -0.1$, $F_0 = 0.3$, $Ra = 0.7$, $N = 0$.

Fig. 14 Variation of dimensionless temperature $\theta$ with similarity space variable $\eta$ for different $\zeta$, $Ra_d/Pe_e$, ($Le = 0.5$, $R = 1$, $F_0 = 0.3$, $\chi = 0.02$, $Ra = 0.7$).

Fig. 15 Effect of Lewis number on Nusselt number for different $F_0$, $Ra_d/Pe_e$, ($N = -0.1$, $R = 1$, $F_0 = 0.3$, $Ra = 0.7$, $\gamma = \zeta = 0$).

It is noticed that increase in buoyancy ratio $N$ enhances both heat and mass transfer rates near the wall. This is due to the increased convection near the wall induced by existence of...
Nusselt number decreases with the increase in $Ra$ and the opposite is true for high values of $Le$. This increase in $Le$ seems to reduce the Sherwood number with small values of $Le < 2$ and the opposite is true for high values of $Le > 2$.

Fig. 19 Effect of Lewis number on Nusselt number for different $Ra_d$, $Ra_d/Pe_\gamma$,
$(N=0.1, R=1, \chi=0.02, F_\gamma=0.3, \zeta=\zeta_0=0)$

Fig. 20 Effect of Lewis number on Sherwood number for different $Ra_d$, $Ra_d/Pe_\gamma$,
$(N=0.1, R=1, \chi=0.02, F_\gamma=0.3, \zeta=\zeta_0=0)$

Effects of Lewis number $N$ and mixed convection parameter $Ra_d/Pe_\gamma$ on Nusselt number and Sherwood number for different $Ra_d$ are shown in figs. 19 and 20, respectively. Fig. 19 shows that in both the cases of aiding and opposing flows Nusselt number decreases with the increase in $Ra_d$. It can be seen from Fig. 20 that mass transfer rate decreases with the increase in $Ra_d$ in aiding flows and in case of opposing flows increase in $Ra_d$ reduces the Sherwood number with small values of $Le < 1$ and this change in $Ra_d$ does not affect mass transfer rates with high values of $Le > 1$. Figures 21 and 22 illustrate, respectively, the effect of $Le$ on Nusselt number and Sherwood number for different mechanical-solutal dispersion coefficient $\zeta$ with the following parameters: $F_\gamma=0.3, N=0.1, Ra_d=0.7, \chi=0.0, \gamma=0.02$. From fig. 21 we notice that there is an insignificant reduction in heat transfer with increase of $\zeta$ in both aiding and opposing flows. Also from fig. 22, it can be noticed that the parameter $\zeta$ enhances/reduces the mass transfer rate with small values of $Le < 1.75/Le > 1.2$ and the opposite is true for high values of $Le > 1.75/Le > 1.2$. This may be explained as follows: for small values of Lewis number the increase of $\zeta$ causes mass dispersion mechanism to be higher and since the concentration at the wall is kept constant this increases concentration gradient near the wall and hence increases/decreases Sherwood number. As $Le$ increases, heat dispersion outweighs mass dispersion and with the increase in $\zeta$ concentration gradient near the wall becomes smaller and this results in decrease/increase in Sherwood number in aiding/opposing flow. Figure 23 indicates that thermal-dispersion parameter enhances/reduces the heat transfer rate in aiding/opposing flow. It is also observed from figure 24 that increasing the chemical reaction parameter $\chi$ enhances/reduces the mass transfer rate in aiding/opposing flow.
Fig. 23 Effect of Lewis number on Nusselt number for different $\gamma$, $Ra_Pe_\gamma$
$(N=0.1, R=1, 1/N=0.3, Ra=0.7, 0=0$)

Fig. 24 Effect of Lewis number on Sherwood number for different $\chi$, $Ra_Pe_\gamma$
$(N=0.1, R=1, 1/N=0.3, Ra=0.7, 0=0$)

REFERENCES


