Rational Structure of Panel with Curved Plywood Ribs

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Abstract—Optimization of rational geometrical and mechanical parameters of panel with curved plywood ribs is considered in this paper. The panel consists of cylindrical plywood ribs manufactured from Finish plywood, upper and bottom plywood flange, stiffness diaphragms. Panel is filled with foam. Minimal ratio of structure self weight and load that could be applied to structure is considered as rationality criteria. Optimization is done, by using classical beam theory without nonlinearities. Optimization of discreet design variables is done by Genetic algorithm.

Keywords—Curved plywood ribs, genetic algorithm, rational parameters of ribbed panel, structure optimization.

I. INTRODUCTION

MINIMAL material consumption, structure weight, new rational large span structures and use of environmentally friendly materials in structures are the main directions of research in today’s structural engineering science. Wood is one of the renewable natural resources and plywood sheets provide a rational use of wood. Traditionally used flat plywood sheets are not rational in many cases because of their slenderness and insufficient load bearing capacity. The load bearing capacity of plywood sheets could be increased significantly by using sheets with curved shape. Therefore, in this paper a new panel structure is proposed, which is based on cylindrical plywood ribs in combination with other plywood elements – top and bottom flange, shear stiffness diaphragms, plywood stiffeners, and inside filled with foams. The structure becomes more efficient if it’s geometrical and mechanical parameters (technologically changeable) are optimized. In this paper optimization of the proposed panel structure is elaborated.

II. STRUCTURE OF PANEL AND PARAMETERS TO BE OPTIMIZED

In this paper way of optimization of geometrical and mechanical parameters of panel with curved plywood ribs is shown. Cross section, longitudinal section and structural analysis scheme of provided structure are showed in fig 1.

The curved plywood shell could be manufactured by using hot pressing, cold pressing, vacuum pressing or making structure with special nonsymmetrical structure, curved with moisture difference [1]-[3]. In this work it is assumed that curved plywood shells are made with symmetrical structure with respect to its mid surface and curved by hot pressing.

The most important geometrical and mechanical parameters of panel are:

- \( t_1 \) - thickness of cylindrical shell ribs,
- \( t_2 \) - thickness of top plywood flange,
- \( t_3 \) - thickness of bottom plywood flange,
- \( t_4 \) - average thickness of plywood stiffener,
- \( t_5 \) - thickness of side element, during optimization it is constant \( t_5 = 6.5 \text{ mm} \),
- \( t_6 \) - thickness of shear stiffness diaphragms, during optimization it is constant \( t_6 = 9 \text{ mm} \),
- \( b \) - width of cylindrical shell,
- \( h \) - total height of panel,
- \( b_1 \) - width of plywood stiffener,
- \( L_1 \) - distance between shear stiffness diaphragms (count of shear stiffness diaphragms is \( n = \frac{L}{L_1} - 1 \)),
- \( \rho_0 \) - density of foam.

The panel thickness \( h \) depends on heat and sound insulation requirements and load bearing capacity requirements [4]. Therefore this parameter is not included in design vector. The design vector parameters are optimized for each discrete values of thickness of panel \( h = 200, 250, 300, 350 \text{ mm} \).

The span of panel depends on building’s overall structure of the building, therefore it is not considered as an optimized parameter. According to classical knowledge of this type of
structure the material consumption is smaller for smaller span [5]. For each value of span \( L = 4000, 6000, 8000, 10000 \) mm are found optimal values of design vector components. 

Other mentioned parameters can be optimized therefore the design vector consists of eight parameters: 

\[
x = \{ y_1, y_2, y_3, y_4, b_1, b_2, n, \rho_p \}
\]

(1)

Each component of design vector takes one value from eight discrete values. These values are shown in table 1.

### TABLE I: PARAMETERS TO BE OPTIMIZED AND ITS DISCRETE VALUES

<table>
<thead>
<tr>
<th>( t_1, t_2, t_3, t_4 ) mm</th>
<th>( b, b_1, b_2 ) mm</th>
<th>( n )</th>
<th>( \rho_p ) kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>400</td>
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<td>9</td>
<td>450</td>
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<td>24</td>
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<tr>
<td>27</td>
<td>750</td>
<td>150</td>
<td>15</td>
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</table>

* Each design variable could be different from each other.

The thickness of plywood elements is taken according to Finnish birch plywood standards. For each thickness of Finnish birch plywood its appropriate characteristic strength and mean stiffness properties are used in the analysis. The minimized objective function is the ratio of structure’s weight- \( W \) and maximum value of load- \( Q \) that could be applied to structure:

\[
f(x) = \frac{W}{Q}
\]

(2)

where \( W \) - total weight of panel, \( Q \)- maximal value of load that is uniformly distributed and satisfies ULS (ultimate limit state) and SLS (serviceability limit state) criteria.

III. DISCREET PARAMETER OPTIMIZATION WITH GENETIC ALGORITHM

A. Overview of optimization algorithms

There are many methods that provide optimization of the structures. The classical methods are based on calculus and use of function derivatives. Serious problems arise in the cases if there are local extremes, discrete design variables, discontinuous objective function or nonconvex design space [6]. These properties of objective function are difficult to identify for large and relatively novel systems. In this case a new structure is provided and there is no preliminary knowledge of its behavior depending of its parameters, therefore use of nontraditional optimization technique is more rationally. There are several nontraditional optimization techniques- Genetic algorithms [7]-[8], Simulated annealing [9], Particle swarm optimization, Ant colony optimization, Fuzzy optimization, Neural-network-based methods. For this type of problem according to [6]-[10] the most convenient is Genetic algorithm, therefore it will be used in further structure optimization.

B. Genetic algorithm (GA)

A general constrained minimization problem can be stated as

\[
\begin{align*}
\text{Minimize} & \ (f(x)) \\
\text{subject to} & \ g_i(x) \leq 0, \ i = 1, m \\
& \ h_j(x) = 0, \ j = 1, p
\end{align*}
\]

(3)

where \( x \) - vector of design variables, \( f(x) \) - objective function, \( g_i(x), h_j(x) \) - constrains that in form of inequalities and equalities.

GA can be used only for unconstrained problems. Therefore problem (3) is converted into an equal unconstrained minimization problem by using concept of penalty functions as

\[
\text{Minimize} (\Phi(x))
\]

\[
\Phi(x) = f(x) + \sum_{i=1}^{m} r_i (G_i(x))^2 + \sum_{j=1}^{p} R_j (h_j(x))^2
\]

(4)

where \( r_i, R_j \) - penalty parameters, its values are constant during optimization, \( G_i(x) \) is defined as

\[
G_i(x) = \begin{cases} 
0, & g_i(x) > 0 \\
0, & g_i(x) \leq 0
\end{cases}
\]

(5)

GA is based on the principles of natural genetics and natural selection (Darwin’s theory of survival of the fittest). The basic operators of natural genetics are reproduction, crossover and mutation. Simplified flowchart of GA is shown in fig 2.
Reproduction is the first operator applied to the population to select “good” design vectors that gives a minimal value to objective function.

Crossover is second operator applied to the population with initially defined probability $P_c$. The Crossover operator randomly selects two design vectors (called parents) from population and by changing its binary codes obtains two new design vectors although there are also methods that use only real numbers [11].

The crossover operator generate random integer number in interval from 1 to $n$, where $n$- length of binary code. By changing binary numbers of parents from $i$-th place, where $i$ is generated as random number, obtains two new binary codes (called child). The crossover operation is done only in case if child gives better value of fitness function.

For example if two components of design vector has values $x_1 = 105(mm)$ and $x_2 = 110(mm)$, $n=8$, $i=6$, the crossover modify components of design vector following obtaining a new values of components of design vector $x_1 = 106(mm)$, $x_2 = 109(mm)$:

\[
\begin{align*}
\{x_1 \} &= \{011010|01\} \\
\{x_2 \} &= \{010111|10\}
\end{align*}
\]

Crossover:

\[
\begin{align*}
\{x_1'\} &= \{01101010\} \\
\{x_2'\} &= \{01101101\}
\end{align*}
\]

The Mutation operator is applied to the new binary codes with a specific small probability $P_m$. This operator changes each number in binary code from value 1 to 0 or 0 to 1 with probability $P_m$, that is very small, usually $P_c<0.01$. In previously given example the Mutation operator gives following result:

\[
\begin{align*}
\{x_1\} &= \{01101001\} \\
\{x_1'\} &= \{01101001\}
\end{align*}
\]

III. OPTIMIZATION

The value of maximal total load $Q$ is calculated using following criteria’s:

1. Compressions stress in top flange of panel is less than compression strength of plywood (in direction of mostly orientated veneer fibers):

   \[
   g_1 = \frac{\sigma_{\text{max}}}{f_{c,0,k}} \left(\frac{k_{\text{mod}}}{\gamma_m}\right) - 1 \leq 0
   \]  
   (6)

2. Tension stress in bottom flange of panel is less than tension strength of plywood (in direction of mostly orientated veneer fibers):

   \[
   g_2 = \frac{\sigma_{\text{max}}}{f_{t,0,k}} \left(\frac{k_{\text{mod}}}{\gamma_m}\right) - 1 \leq 0
   \]  
   (7)

3. Deflection of panel is less than 1/200 of span. Deflection is calculated using Timoshenko beam theory that takes into account shear deformations. The problem is solved using Ritz method, approximating deflection function of simply supported beam by forth order polynomial. Obtained following equation for deflection in middle of span:

   \[
   \Delta(0.5L) = \frac{0.013 \cdot qL^6G_l + 0.167 \cdot qL^4E \cdot t \cdot h^5}{EGL^2l^2 + 16E^2I \cdot t \cdot h^5}
   \]  
   (8)

4. Shear stress in curved plywood shell and foams are less than shear strength. In numerical analysis foam material are Extended polystyrene (EPS). The EPS is reduced to equal thickness plywood rib by using radio of Shear modulus of EPS and plywood.

   \[
   g_4 = \frac{\sigma_{\text{max}}}{f_{\nu,k}} \left(\frac{k_{\text{mod}}}{\gamma_m}\right) - 1 \leq 0
   \]  
   (10)

5. Compression stress in top flange should not exceed buckling stress that is calculated using linear Euler analysis of buckling. Top flange is considered as a simply supported beam under axial load with span equal to distance between shear stiffness diaphragms.

   \[
   g_5 = \frac{\sigma_{\text{max}}}{\sigma_{\text{crit}}} - 1 \leq 0
   \]  
   (11)

6. Stress of top flange should be less than bending strength of plywood (in orthogonal direction of mostly orientated wood fibers). The top flange is considered as a beam on elastic foundation. The elastic foundation is EPS and it is assumed that it satisfies Vinkler hypothesis. The modulus of elasticity of EPS is calculated depending on density of EPS. In case if relative deformations are less than 10% density of EPS could be approximated by following linear relationship [12]:

   \[
   E_p = \frac{P_p - 6.096}{12.724}
   \]  
   (12)

   \[
   g_6 = \frac{\sigma_{\text{max}}}{f_{m,90,k}} \left(\frac{k_{\text{mod}}}{\gamma_m}\right) - 1 \leq 0
   \]  
   (13)

where $k_{\text{mod}} = 0.55$ modification factor for long term load [13]-[14], $k_{\text{sys}} = 1$, $\gamma_m = 1.2$ - material safety factor, $f_{c,0,k}$, $f_{t,0,k}$, $f_{\nu,k}$, $f_{m,90,k}$ - characteristic strength of Finnish birch plywood, $G = \frac{G_{v,\text{mean}}}{1 + k_{\text{def}}}$, $E = \frac{E_{1/1c,0,\text{mean}}}{1 + k_{\text{def}}}$, $k_{\text{def}} = 0.8$ - safety factor, that takes into account creep for services class 2 [13]-[14], I- second moment of area- calculated taking into account reduced width of compressed and tensioned flange according to [13]-[14], $t$- reduced thickness of plywood shell.
The weight of structure $W$ is calculated assuming that plywood average density is $700 \text{ kg/m}^3$.

Probability of crossover, probability of mutation was used $0.7$ and $0.001$. In the reproduction operator the only $3\%$ of design vectors with highest fitness ratio was kept. Numerical results showed that fast result convergence is obtained if the probability of crossover and mutation are in interval $0.65...0.75$ and $0.0005…0.0015$, respectively. The behavior of objective function in design space showed that global optimum could be found with high probability if reproduction operator keeps less than $3\%$ of design vectors with highest fitness ratio. In case if it was $5\%...7\%$ there is a big probability to identify local extremes.

The analysis showed that objective function $f(x)$ is strong nonlinear with many local extremes. Result s approved that classical optimization methods are difficult to use in this case. It could be done only in case if the design space is divided in many smaller subspaces.

The rational parameters of panel is obtained for the case if the span varies from $4000$ to $10000\text{ mm}$, height of panel varies from $200$ to $350 \text{ mm}$ and uniformly distributed load varies from $2$ to $7.5 \text{ KPa}$. The values of rational parameters and reserve of strength and stiffness for each criterion are showed in table II.

**TABLE II**

<table>
<thead>
<tr>
<th>Geometrical parameters, mm</th>
<th>$\rho_p$, kg/m$^3$</th>
<th>$f(x)$, kg/kN</th>
<th>Reserve of strength and stiffness for each criteria (see eq. (6)-(13)), %</th>
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<tbody>
<tr>
<td>$L$</td>
<td>$h$</td>
<td>$t_1$</td>
<td>$t_2$</td>
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<td><strong>q=2 KPa</strong></td>
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</table>

**Maximal deflection criteria is not satisfied**
The results showed that in almost all cases the leading criteria is maximal deflection $-g_3$. For the spans less than 6000 mm and total thickness of panel greater than 300 mm in some cases leading factor is maximal shear stress $-g_4$. For loads greater than 5 KPa, in some cases the leading factor is maximal compression stress in top flange $-g_1$. The criteria of maximal local bending stress in top flange are satisfied in all cases with reserve more than 50%. The compressed flange buckling criteria is satisfied in all cases with reserve more than 80 % when load is less than 5 KPa. In the case when span is 10000 mm and total thickness of panel $h=200$ mm, then deflection criteria is satisfied only in case when $q= 2$KPa.

The contour plot of objective function that is approximated by second order polynomial depending on span $L$ and total thickness $h$ when $q=2$KPa is shown in Fig 3. The plot shows that there are minimum point when $h=225$mm and $L=6500$mm.

During the process of optimization was discovered that the optimization procedure becomes more effective if used as two step optimization procedure. The first step is discrete parameter optimization with GA to identify the behavior of objective function in large design space. In the second step continuous parameter optimizations in the small design subspace that is identifier from the results of first step are done.

IV. CONCLUSION

Proposed a novel panel with curved plywood ribs structure and analysis method of its rational discrete geometrical and mechanical parameters by using Genetic algorithm optimization method.

Rational thickness of plywood elements, shape parameters of curved ribs with stiffening elements and density of foam of novel panel structure is obtained by using proposed method for the case if span varies form 4 to 10 m, total thickness of panel from 200 to 350 mm and uniformly distributed load on panel varies from 2 to 7.5 KPa. In the case if span is 8 m total thickness of panel is 250 mm and load is 3 KPa the rational parameters of panel are: thickness of curved plywood rib is 6.5 mm, thickness of compressed flange is 24 mm, thickness of tensioned flange is 9mm, thickness and width of plywood stiffener are 24 mm and 80mm, distance between shear stiffness diaphragms is 1000 mm, width of cylindrical shell is 400 mm and density of EPS foam is 9.9 kg/m$^3$.

The proposed analysis method of rational discrete parameters of novel panel structure that is based on Genetic algorithm becomes more effective if it is combined with classical optimization methods for continuous design variables obtaining two step optimization method.

ACKNOWLEDGMENT

The work was supported by EU Funds within the framework of project “Support of RTU PhD studies”.

REFERENCES


