A Parametric Study: Frame Analysis Method for Masonry Arch Bridges

M. E. Rahman, D. Sujan, V. Pakrashi, P. Fanning

Abstract**—**The predictability of masonry arch bridges and their behaviour is widely considered doubtful due to the lack of knowledge about the conditions of a given masonry arch bridge. The assessment methods for masonry arch bridges are MEXE, ARCHIE, RING and Frame Analysis Method. The material properties of the masonry and fill material are extremely difficult to determine accurately. Consequently, it is necessary to examine the effect of load dispersal angle through the fill material, the effect of variations in the stiffness of the masonry, the tensile strength of the masonry mortar continuum and the compressive strength of the masonry mortar continuum. It is also important to understand the effect of fill material on load dispersal angle to determine their influence on ratings. In this paper a series of parametric studies, to examine the sensitivity of assessment ratings to the various sets of input data required by the frame analysis method, are carried out.

Keywords**—**Arch Bridge, Frame Analyses Method, Masonry

I. Introduction

MASONRY arch bridges represent a significant percentage of bridges on the rail and road networks in Republic of Ireland. There are approximately 20,000 bridges in the Republic of Ireland. There are approximately 20,000 bridges in the Republic of Ireland and it is estimated that around 80% of these bridges are masonry arch bridges. Many of the masonry arch bridge in Ireland were built in the 16th to 17th centuries and are now carrying traffic loads far beyond those estimated by their designers. The weight of vehicles on bridges has increased steadily. European Union directives require that bridges do not constitute a barrier to free movement of goods and a 1999 directive requires that all bridges in the European economic area be capable of enabling safe passage of vehicles having a gross vehicle weight (GVW) of 40t. The minimum axle weight is specified as 11.5t [1], [2].

The assessment methods for masonry arch bridges are MEXE, ARCHIE, RING and frame analysis method. The results obtained varied widely among the methods, although most of the variation seems to be a result of differing factors of safety. The most consistent and reasonable results were yielded by the frame analysis method [1].

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Two dimensional analytical model for single-span arch bridges subjected generalized loading patterns and abutment movements has also been developed [3].

The behavior of masonry arch bridges are complex system whose structural response is a function of the composite masonry and mortar material, the contained fill material, backing and the interaction between these and the surrounding soil medium. Arch thickness and physical properties of the fill material have significant impact on ultimate capacity of the arch bridge. The material properties of the masonry and fill material are extremely difficult to determine accurately [2], [4], [5]. The frame analysis method first proposed by Boothby [6] and later modified by Fanning and Boothby [7] in light of service load tests and three dimensional finite element models is based on this approach. The method is based on a linear elastic analysis of the arch barrel modeled as two-dimensional assembly of beam elements.

The material properties used in this study are based on the recommendations by Boothby [6]; Fanning & Boothby [2] which demonstrated close correlation between three dimensional finite element model results for these bridges compared to service load test responses. Consequently, it is necessary to examine the effect of load dispersal angle through the fill material, the effect of variations in the stiffness of the masonry, the tensile strength of the masonry mortar continuum, the compressive strength of the masonry mortar continuum and the effect of fill material on load dispersal angle to determine their influence on ratings.

II. Frame Analysis Method

This method, which uses a linear elastic analysis, is used to find the load carrying capacity of a masonry arch bridge by determining axial force and moments throughout the arch barrel. Co-existing axial force and moments throughout the arch ring are then compared to an estimate of the strength of the arch ring cross section [6], [7], [8].

In this method, a unit width of the arch barrel is modelled as a series of straight elastic bars using a linearly elastic frame analysis routine in order to determine an admissible set of forces and moments in the arch barrel. The arch ring is divided into number of segments. The supports are considered as a rigid in the vertical direction and have elastic springs in the horizontal direction, allowing horizontal movement of the abutments but not vertical displacement or rotation.

The fundamental material stiffness property used in the analysis is an effective modulus of elasticity representing the combined effect of masonry units, mortar and joints. The self-weight of the arch ring is computed and superimposed dead loads include the weight of the fill and weight of the paving material. The live load is taken as a linearly varying vertical pressure on the back of the arch ring resulting from truck axle load. Each axle load is applied over a length of 30 cm and a width of one traffic lane, or 3 m. The load is dispersed through the fill at a slope of 2 vertical to 1 horizontal. After execution of the analysis for various axle patterns, and positions, predicted axial forces and moments are checked against a strength assessment of the arch cross section.

The original strength assessment procedure specified relatively low compressive strengths and no tensile strength for masonry. The compressive strength values were confirmed, by material testing, to be conservative and have been re-evaluated by following material tests in [2]. Modelling studies of bridges in a testing program in the US [9] and in Ireland [2] have indicated that the tensile capacity of well-constructed masonry in good condition may be as high as 1.0MPa. Using an ultimate strength assessment model with a ratio of tensile strength to compressive strength, specified as ß, an explicit expression for the compressive strength requirement of any cross section of an arch, of depth h, subjected to an axial load P and a moment M can be written as:

$$f_c = \frac{1}{2\beta h^2}\left[\sqrt{[ph(1-\beta)-2M(1+\beta)]^2+4\beta(ph)^2}-[ph(1-\beta)-2M(1+\beta)]\right] \quad (1)$$

Hence, at each cross section of the modelled bridge the required compressive strength f_c can be determined on the basis of the combinations of axial force and bending moment.

III. Study Bridges

Two bridges located in the Dublin area were considered in the study. Typically the bridges were rated with and without the backing material being modeled explicitly. Each of the bridges were in good condition and the compressive and tensile strengths of the masonry in the arch barrels were set at 15MPa and 0.75MPa respectively for the purposes of determining a safe axle load on a single axle bogey.

Griffith Bridge (Fig.1) is an elliptical arch canal bridge on the Grand Canal in Dublin. Grand Canal Company built most of the masonry arch bridges of the Grand Canal during seventeenth centuries with nearest span length. This bridge is dated 1791 and was named, like most canal bridges, after the builder (namely Richard Griffith) who joined the board in 1784. The Griffith Bridge has a span of 9.48 m, a rise over the abutments of 2.71 m, a rise of the arch barrel at the quarter points of 2.265m, an average depth of fill, at the quarter points of the transverse road profile, between the road surface and the arch barrel at the crown, including road surfacing of 0.125 m, a span rise ratio of 3.5, and an arch ring thickness of 450mm. The arch barrel has maintained its elliptical shape with no major distortions. The foundations of the bridge were not inspected. However, from springing levels taken and the absence of any distress in the arch barrel, it can be inferred that there is no relative settlement or horizontal movement of foundations an abutment. The spandrel walls are in good condition with no separation from the arch barrel and no lateral distress [1].

The Killeen Road Bridge built in 1791 is an elliptical masonry arch canal bridge like Griffith Bridge. It is located on the southwest side of the Dublin and links to Daingean road over the Grand Canal. The Killeen Bridge was named after Patrick Killeen. The Killeen Road Bridge has a span of 9.29 m, a rise over the abutments of 2.646 m, a rise of the arch barrel at the quarter points of 2.35m, an average depth of fill, at the quarter points of the transverse road profile, between the road surface and the arch barrel at the crown, including road surfacing of 0.25 m, a width of 7.17 m, a span rise ratio of 3.51 and an arch ring thickness at the key stone of 0.516 m and at the springing level of 0.43m. The arch ring is constructed of limestone on the face and in the barrel, with joints about 1 cm thick. The spandrel walls are also of ashlars limestone construction, with joint thickness of approximately 1 cm [1].

Fig. 1 Griffith Bridge – Dublin

IV. Effect of Load Dispersal Angle Through The Fill Material

The load dispersal angle is used to distribute the axle load longitudinally through the fill material in many assessment algorithms. In the Frame Analysis Method the load dispersal angle used was 26.56 degree, which is similar to BA 16/01 [11]. The load dispersal angles considered for examination are summarised in Table I.

In the case of AASHTO single axle the ultimate capacity of Griffith Bridge (Fig. 2) was 20.3 tonnes for a load dispersal angle of 26.56^0. For load dispersal angle of 34.6^0 and 45^0 the rating increased to 20.9 and 21.4 tonnes respectively. Changing the angle of dispersal from a slope of 2 vertical: 1 horizontal (26.56^0) to 1:1 (45^0) only results in an increase in rating of 5.4%.

Furthermore for the AASHTO double axle the ultimate capacity of Griffith Bridge (Fig. 2) was 11 tonnes for a load dispersal angle of 26.56^0. For load dispersal angle of 34.6^0 and 45^0 the rating increased to 11.4 and 12 tonnes respectively. Changing the angle of dispersal from a slope of 2 vertical: 1horizontal (26.56^0) to 1:1 (45^0) only results in an increase in rating of 9%.

In addition for the AASHTO single axle the ultimate capacity of Killeen Bridge (Fig. 3) was 23 tonnes for a load dispersal angle of 26.56^0. For load dispersal angle of 34.6^0 and 45^0 the rating increased to 23.8 and 24.2 tonnes respectively. Changing the angle of dispersal from a slope of 2 vertical: 1horizontal (26.56^0) to 1:1 (45^0) only results in an increase in rating of 5.2%.

Likewise for the AASHTO double axle the ultimate capacity of Killeen Bridge (Fig. 3) was 12.4 tonnes for a load dispersal angle of 26.56^0. For load dispersal angle of 34.6^0 and 45^0 the rating increased to 12.9 and 13.6 tonnes respectively. Changing the angle of dispersal from a slope of 2 vertical: 1horizontal (26.56^0) to 1:1 (45^0) only results in an increase in rating of 9.67%.

The variation of load dispersal angle (Fig. 2 & Fig. 3) led to up to a 10% variation on the ultimate capacities of the Griffith Bridge and the Killeen Bridge. The rating is increased with increasing load dispersal angle. The larger dispersal angle distributes concentrated axle loads over greater lengths of the arch barrel, which reduced extradous stresses and hence gave lower deformations and a higher capacity.

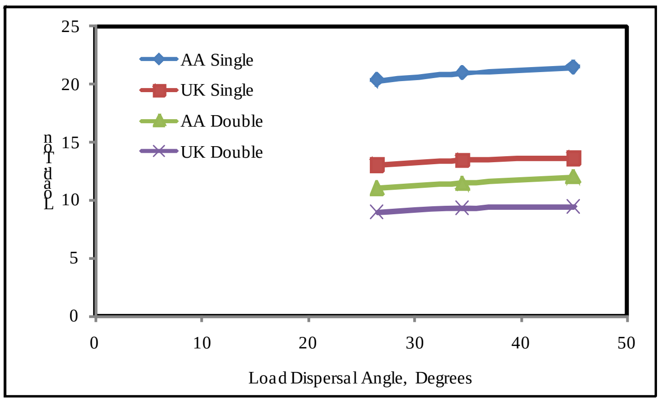

Fig. 2 Load vs Dispersal Angle, Griffith Bridge

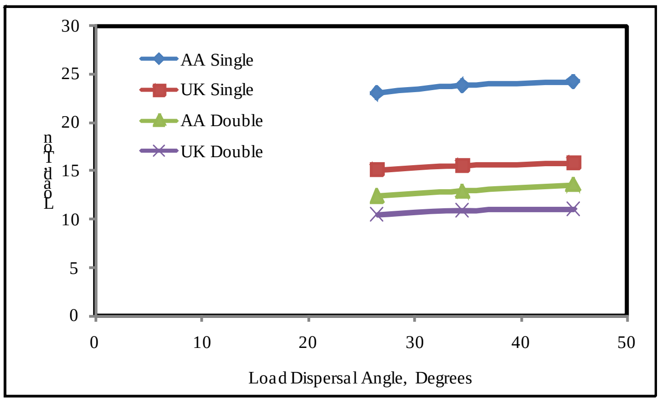

Fig. 3 Load vs Dispersal Angle, Killeen Bridge

TABLE I
LOAD DISPERSAL ANGLE; GRIFFITH & KILLEEN BRIDGE

Bridges	Angle (Degree)		
Griffith	26.56	34.6	45
Killeen	26.56	34.6	45

V. THE EFFECT OF VARYING THE ELASTIC MODULUS OF MASONRY ARCH

The material stiffness property used in the Frame Analysis Method is a combined value to cater for effects of masonry units and mortar. The resulting modulus of elasticity is significantly lower than the modulus of elasticity of the units alone [6]. It is very difficult to determine the value of the elastic modulus of the arch ring, as it is a composite of mortar and voussoir units. Any laboratory tests on this composite material may not give reliable results because replication of confining stresses pertinent to its in situ condition is tough. The data used in this study for Griffith and Killeen Bridge are summarised in Table II.

In the case of AASHTO single axle the ultimate capacity of Griffith Bridge (Fig. 4) was 20 tonnes for an elastic modulus of 1 GPa. For elastic modulus of 5GPa, 10GPa and 15GPa the rating increased to 20.1, 20.3 and 20.5 tonnes respectively. Changing the elastic modulus from 1GPa to 15GPa has only resulted in an increase in rating of 2.5%.

Additionally for the AASHTO double axle the ultimate capacity of Griffith Bridge (Fig. 4) was 10.75 tonnes for an elastic modulus of 1 GPa. For elastic modulus of 5GPa, 10GPa and 15GPa the rating increased to 10.85, 10.95 and 11.05 tonnes respectively. Changing the elastic modulus from 1GPa to 15GPa has only resulted in an increase in rating of 2.8%.

Likewise for the AASHTO single axle the ultimate capacity of Killeen Bridge (Fig. 5) was 22.5 tonnes for an elastic modulus of 1 GPa. For elastic modulus of 5GPa, 10GPa and 15GPa the rating increased to 22.8, 23 and 23.4 tonnes respectively. Changing the elastic modulus from 1GPa to 15GPa has only resulted in an increase in rating of 4%.

Finally for the AASHTO double axle the ultimate capacity of Griffith Bridge (Fig. 5) was 12.1 tonnes for an elastic modulus of 1 GPa. For elastic modulus of 5GPa, 10GPa and 15GPa the rating increased to 12.3, 12.4 and 12.6 tonnes respectively. Changing the elastic modulus from 1GPa to 15GPa has only resulted in an increase in rating of 4.1%.

It is clearly evident from Fig. 4 and 5 that the arch's elastic modulus has no significant effect on the ultimate capacity of Griffith or Killeen Bridge. The rating increased with increasing elastic modulus but the variation of ultimate capacity is insignificant compared to the variation of arch's elastic modulus. The arch system is rigid when the elastic modulus of the arch ring is higher and gives a higher rating and the system is flexible when the elastic modulus is lower and gives a lower rating.

TABLE II
STRENGTH AND STIFFNESS PARAMETER; GRIFFITH AND KILLEEN BRIDGE

Property	Value
Masonry Elastic Modulus (Varied)	1 GPa, 5 GPa, 10 GPa and 15 GPa
Masonry Density	2200 kg/m^3
Fill Density	1700 kg/m^3
Masonry Compressive Strength	15 MPa
Masonry Tensile Strength	0.75 MPa
Abutment Stiffness	5000 kN/mm

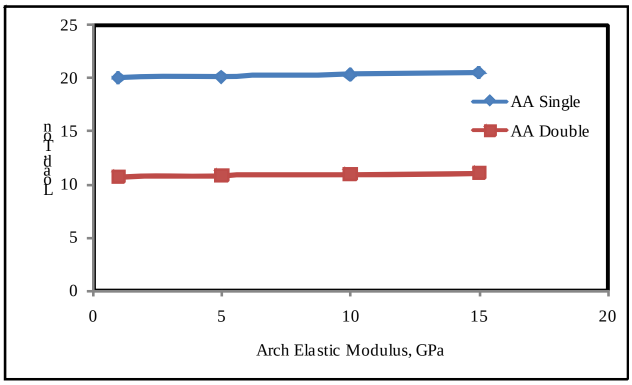

Fig. 4 The Effect of Arch Elastic Modulus, Griffith Bridge

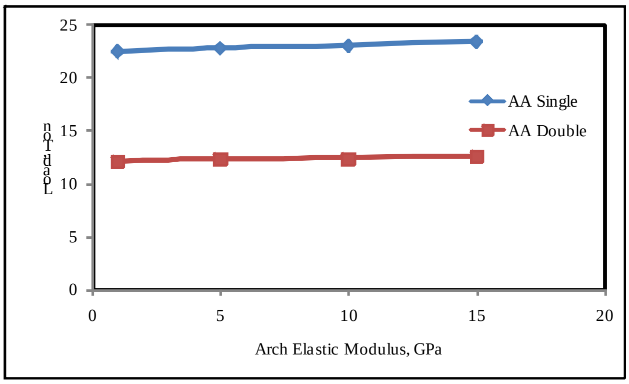

Fig. 5 The Effect of Arch Elastic Modulus, Killeen Bridge

VI. THE EFFECT OF VARYING FACTOR OF β MASONRY ARCH

β Factor is a ratio of tensile strength to compressive strength. Initially the frame analysis method is based on excluding any small tensile strength of masonry, but subsequent test results and 3D finite element result have estimated that it may be appropriate to incorporate a small tensile capacity for the masonry. The conventional method for the design of masonry arches is based upon the assumption that mortar masonry continuum must not be subjected to tensile stresses. This is very conservative since all mortars can resist some tension and moreover, even if a joint does crack the arch is still far from failure [12].

The data used in this study for Griffith, and Killeen Bridge are summarised in Table III.

In Fig. 6, the ultimate capacity of Killeen Bridge increased 2200% for AASHTO single axle and 3000% for AASHTO double axle configuration as the β factor increased from zero to 5%. In Fig. 7, the ultimate capacity of Griffith Bridge increased 577% for AASHTO single axle and 588% for AASHTO double axle configuration as the β factor increased from zero to 5%. The ratings (Fig. 6, Fig. 7) of Griffith Bridge and Killeen Bridge are increased with increasing β factor. The estimated ultimate capacities of bridges were sensitive to variations in β factor. The masonry must be considered capable of carrying a limited tensile stress to assure fidelity to experimental results and accuracy of bridge assessment [6]. Ignoring the tensile strength may lead to conservative results. Care however must be taken when selecting a value for the β factor. For an intact arch, higher value of β factor should be used.

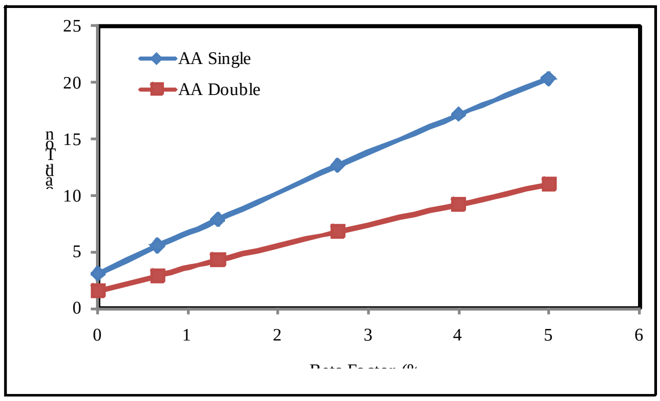

Fig. 6 The Effect of Arch β factor, Killeen Bridge

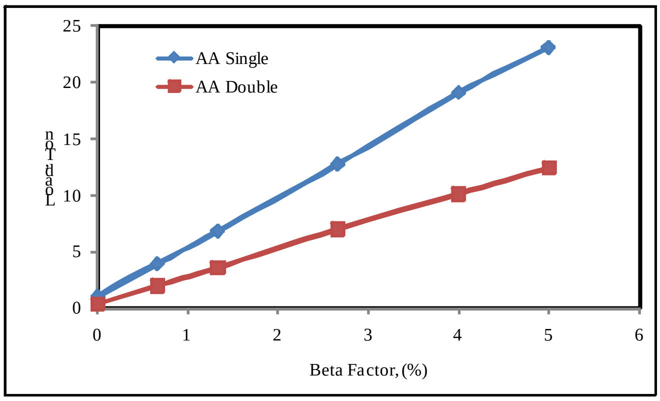

Fig. 7 The Effect of Arch β factor, Griffith Bridge

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TABLE III
STRENGTH AND STIFFNESS PARAMETER

Property	Value
Masonry Elastic Modulus	10 GPa
Masonry Density	2200 kg/m^3
Fill Density	1700 kg/m^3
Masonry Compressive Strength	15 MPa
β Factor (Varied)	0.0 %, 0.66%, 1.33%, 2.66%, 4% & 5%
Abutment Stiffness	5000 KN/mm

VII. The Effect of Varying the Compressive Strength of Masonry Arch

The Frame Analysis Method is based on considering the ultimate compressive strength. It is very difficult to determine the real compressive strength, as it is a composite of mortar and voussoir units. The data used in this study for Griffith and Killeen Bridge are summarised in Table IV.

In Fig. 8, the ultimate capacity of Griffith Bridge increased 1700% for AASHTO single axle with constant ratio of tensile strength to compressive strength of 0.05 as the compressive strength of masonry arch increased from 2 MPa to 20 MPa. In Fig. 8 the ultimate capacity of Griffith Bridge increased 1700% for AASHTO double axle with constant ratio of tensile strength to compressive strength is 0.5 as the compressive strength of masonry arch increased from 2 MPa to 20 MPa.

In Fig. 9, the ultimate capacity of Griffith Bridge increased from 204% for AASHTO single axle with constant tensile strength of 0.75 MPa. as the compressive strength of masonry arch increased from 2 MPa to 20 MPa. In Fig. 9 the ultimate capacity of Griffith Bridge increased 200% for AASHTO double axle configuration with constant tensile strength of 0.75 MPa as the compressive strength of masonry arch increased from 2 MPa to 20 MPa.

In Fig. 10, the ultimate capacity of Killeen Bridge increased 1331% for AASHTO single axle with constant ratio of tensile strength to compressive strength of 0.05, as the compressive strength of masonry arch increased from 3 MPa to 20 MPa. In Fig. 10 the ultimate capacity of Killeen Bridge increased from 1427% for AASHTO double axle with constant ratio of tensile strength to compressive strength of 0.05 as the compressive strength of masonry arch increased from 3 MPa to 20 MPa.

In Fig. 11, the ultimate capacity of Killeen Bridge increased from 96.7% for AASHTO single axle with constant tensile strength of 0.75MPa as the compressive strength of masonry arch increased from 3 MPa to 20 MPa. In Fig. 11 the ultimate capacity of Killeen Bridge increased 95% for AASHTO double axle configuration with constant tensile strength of 0.75MPa as the compressive strength of masonry arch increased from 3 MPa to 20 MPa.

In the Fig. 8, Fig. 10 the slope of the curve is constant with increasing compressive strength and tensile strength. In the Fig. 9, Fig. 11 it is gradually decreased with increasing compressive strength and constant tensile strength. In effect, with increasing compressive strength and constant tensile strength, the slope almost becomes asymptote at the higher values of compressive strength due to constant tensile strength.

Thus, indicating that the variation of tensile strength has the most prominent effect on the predicted ultimate capacity of masonry arch bridges. Variation of compressive strength exerts substantial impact on the ultimate capacity of masonry arch bridges only in the lower values of compressive strength.

TABLE IV
STRENGTH AND STIFFNESS PARAMETER

Property	Value
Masonry Elastic Modulus	10 GPa
Masonry Density	2200 kg/m^3
Fill Density	1700 kg/m^3
Masonry Compressive Strength	2MPa, 3MPa, 5MPa, 10MPa,15MPa and 20MPa
Masonry Tensile Strength	0.1MPa, 0.15MPa, 0.25MPa, 0.5MPa, 0.75MPa & 1.0MPa
Abutment Stiffness	5000 kN/mm

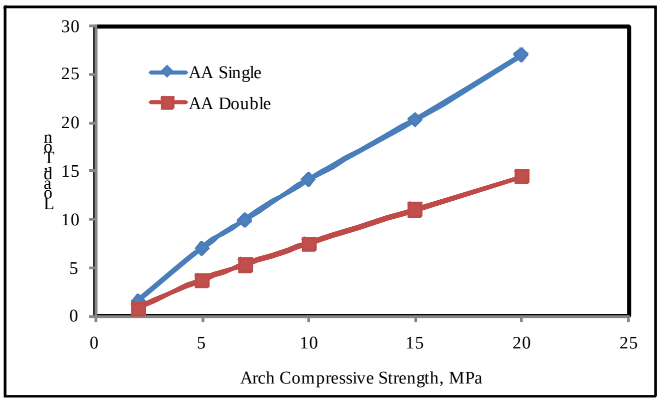

Fig. 8 The effect of arch compressive strength with constant ratio (0.05) of tensile strength to compressive strength, Griffith Bridge

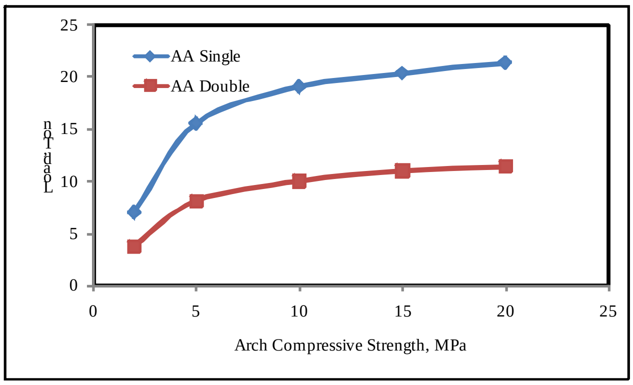

Fig. 9 The effect of arch compressive strength with constant tensile strength of 0.75MPa, Griffith Bridge

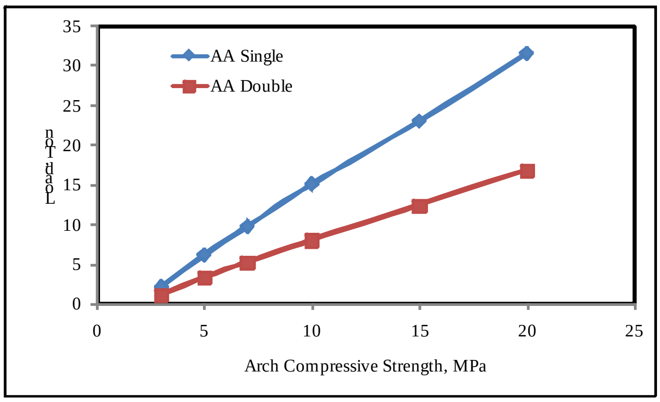

Fig. 10 The effect of arch compressive strength with constant ratio (0.05) of tensile strength to compressive strength, Killeen Bridge

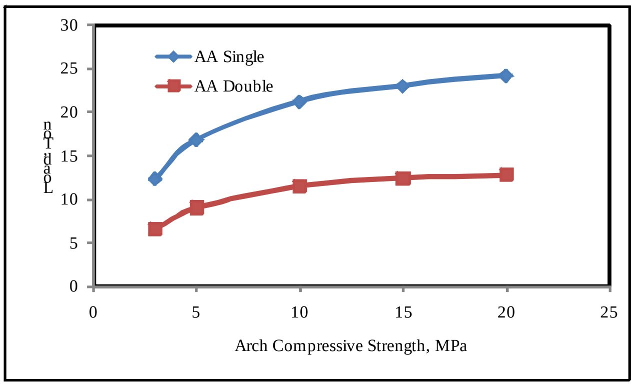

Fig. 11 The effect of arch compressive strength with constant ratio (0.75) of tensile strength to compressive strength, Killeen bridge

VIII. Determination of Back Fill's Load Dispersal Angle

The load dispersal angle is used to distribute the axle load longitudinally through the fill material in many assessment algorithms. The purpose of this study is the determination of load dispersal angle through the fill material. The graphical representation of the load dispersal through the fill material is shown in Fig. 12.

Load dispersal angle, α, $=\tan^{-1}[(L-0.30)/2H]$ (2)

For the determination of dispersal angle, four locations relative to the crown (0.0m, 0.5 m, 1.5m and 2.5m) were considered. For each case the individual single axle load was applied to the model. The graphical representation of contact pressure distribution is shown in Fig.13 to Fig. 14. The dispersal angles for different position of axle load were estimated using equation 2 are summarised in Table V.

These angles are larger than the angles that are suggested by Department of Transport for masonry arch bridges. The larger dispersal angle distributes concentrated axle load over greater lengths of the arch barrel, which reduced extradous stress and hence lower deformations and gives higher capacity.

In terms of factors of safety, the angle suggested by the Department of Transport gives more factor of safety than the predicted angle. This is because the angle suggested by the Department of Transport will give a conservative result. However, it is very difficult to determine the load dispersal angle through the fill material, as it is rarely possible to quantify fill properties accurately.

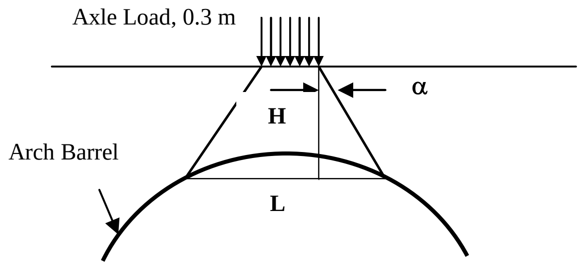

Fig. 12 Axle Load Distribution through the Fill Material

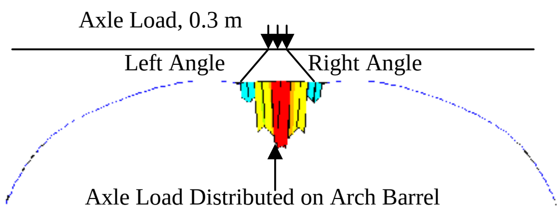

Fig. 13 Axle Load Distribution, at Crown

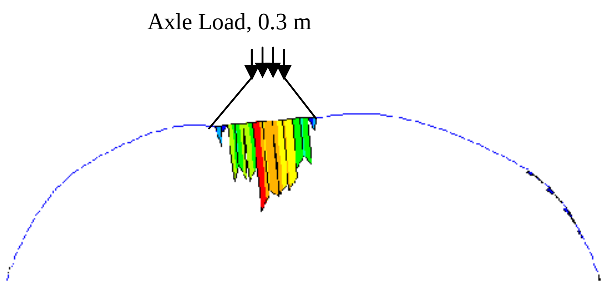

Fig. 14 Axle Load Distribution, at 0.5m

TABLE V
LOAD DISPERSAL ANGLE, GRIFFITH BRIDGE

Axle Load Position Relative to Crown	Dispersal Angle (Degree)	
	Left side	Right side
0.0 m	40.44	40.44
0.5 m	36	34.31
1.5 m	38	36
2.5 m	39.4	32

IX. CONCLUSIONS

1. Variations of load dispersal angle of axle load, through the fill material have no more than a 10% effect on the rating of the two bridges.
2. Variations of modulus of elasticity of masonry have little effect on the ultimate capacity of the arch barrel.
3. For the frame analysis method β factor and compressive strength of the masonry are important parameters when rating masonry arch bridges. The β factor has more significant effect than compressive strength.
4. The variation of tensile strength has the most prominent effect on the predicted ultimate capacity of masonry arch bridges. Variation of compressive strength exerts substantial impact on the ultimate capacity of masonry arch bridges only in the lower values of compressive strength.
5. The predicted backfills load distribution angles are larger than the angles that are suggested by Department of Transport for masonry arch bridges. The larger dispersal angle distributes concentrated axle load over greater lengths of the arch barrel, which reduced extradous stress and hence lower deformations and gives higher capacity. In terms of factors of safety, the angle suggested by the Department of Transport gives more factor of safety than the predicted angle. This is because the angle suggested by the Department of Transport will give a conservative result. However, it is very difficult to determine the load dispersal angle through the fill material, as it is rarely possible to quantify fill properties accurately.

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