Fluid Flow and Heat Transfer Structures of Oscillating Pipe Flows

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Abstract—The RANS method with Saffman’s turbulence model was employed to solve the time-dependent turbulent Navier-Stokes and energy equations for oscillating pipe flows. The method of partial sums of the Fourier series is used to analyze the harmonic velocity and temperature results. The complete structures of the oscillating pipe flows and the averaged Nusselt numbers on the tube wall are provided by numerical simulation over wide ranges of Re and kc. Present numerical code is validated by comparing the laminar flow results to analytic solutions and turbulence flow results to published experimental data at lower and higher Reynolds numbers respectively. The effects of Re and kc on the velocity, temperature and Nusselt number distributions have been di scussed. The enhancement of the heat transfer due to oscillating flows has also been presented. By the way of analyzing the overall Nusselt temperature and Nusselt number distributions have been di scussed. R

Oscillating pipe flows, Reynolds number, Keulegan-Carpenter number, Nusselt number, Oscillating pipe flows, Reynolds number

I. INTRODUCTION

OCEAN wave energy [1] and sea water cooling [2] have been potential sources as renewable energy with high energy densities [3]. The fluid mechanics and heat transfer of oscillating pipe flows are very important for ocean energy applications [4].

The early studies of oscillating flows were highly concentrated on flow structures [5-9]. The flow was observed to be laminar at low Re and become turbulent at high Re by Ohmi et al. [10]. They demonstrated that for Re > 8, the critical Reynolds number (Re)_cr for the onset of transition is independent of Re: the value of (Re)_cr = 4.00×10^4 corresponds to the onset of disturbed laminar flow superimposed with small perturbations, while (Re)_cr = 1.51×10^5 for the onset of intermittently locally-bursting turbulent flow. Akhavan et al. [11] also investigated experimentally the transition of oscillating flows in circular pipes. Hino et al. [7] observed that the value of (Re)_cr increased when Re < 5.12, because the viscous Stokes layers at the pipe wall become relatively thicker when Re decreases and the interaction of the Stokes layers from the wall restricts the viscous diffusion for boundary layer growth. Further decrease in Re leads to the limit case of a quasi-steady laminar Poiseuille flow. Conversely, the increase of Re to a very large value will lead to the other limit case where the Stokes layer becomes much thinner than the pipe radius. To investigate the flow transition as well as its associated wall shear stress, Blondeaux [12] investigated numerically the oscillating flows in a semi-infinite fluid domain over a flat plate by implementing the Reynolds Averaged Navier-Stokes (RANS) method with Saffman’s turbulence model [13, 14]. However, he reported only amplitude but no phase angle information on the oscillatory shear stress. The Lam-Bremhorst form of the low-Reynolds number k- turbulence model was chosen for oscillating-flow modeling by [15], while there are still some deficiencies due to the shortcomings of the low-Reynolds number computational model. Also the higher order harmonics are not decomposed from the first order. Hsu et al. [16] demonstrated that Saffman’s turbulence model is applicable for unsteady oscillating flows and they also provided a complete account for the oscillatory shear stress on the flat plate. Hsu et al. [17] revealed the flow structure of oscillating channel flows and obtained (Re)_cr = 2.00×10^4.

Recently, more studies are concentrated on heat transfer of oscillating pipe flows due to the increasing importance of the application in the ocean energy. Experimental studies show that the oscillating flows can enhance heat transfer. Experimental results of Chai et al. [18] show that the heat exchange capability of the oscillating heat pipe heat exchanger is about 3 times higher than that of a common tube heat exchanger. However their measurement is under the laminar flow region and they did not show the relationship of the enhancement of the heat transfer with governing parameters such as the Reynolds numbers. Wang and Lu [19] applied large eddy simulation (LES) technique to simulate heat transfer between the two constant temperature endplates of oscillating channel flow at Re = 350. They find out that the heat transfer takes place in a much thinner region near the wall at Pr = 100 than at Pr = 1. Due to the limitation of the computational speed of the LES method, they did not give out the full structures for heat transfer and fluid flow from laminar to transient and turbulent range. Also the assumption of the constant temperature difference between the two endplates does not fit for the present mode for sea water cooling heat exchangers, because the characteristic temperature difference is not on the pipe wall itself, but between inlet/outlet oscillating sea water and the pipe wall.

In the present study, the RANS method with Saffman’s
turbulence model was employed to study overall structures of the axis direction dominated flow and two dimensional heat transfer of oscillating flows in circular pipes. The experimental studies of [20] and [21] show that the entrance length of oscillating pipe flow can be approximated by $L_{\text{entrance}} = 8.76 \times 10^{-3} R_e$. So the entrance length is a small value comparing to the transportation length of the water pipe before the water coming into the heat transfer part. Thus the flow can be assumed to be fully developed one dimensional dominated oscillating flow. Experimental study of a pulse combustor tail pipe in [22] showed that the mean temperature was as high as 800 K, while the surface temperature oscillated only about 0.56K. So the two dimensional heat transfer model with constant wall temperature $\Theta_w$ and inlet/outlet temperature $\Theta_i$ boundary conditions is applied in the present study. Also the present model can speed up the simulation and make it possible for us to provide a complete picture of the oscillating flow structures and the overall heat transfer enhancement over a wide range of $Re_a$ and $Re_b$. Results of oscillating velocity and temperature fields are decomposed by the method of partial sums of the Fourier series. The overall heat transfer enhancement comparing to a baseline case of the oscillating flow structures and the potential application of the results in sea water cooling will also be discussed.

II. GOVERNING EQUATIONS AND GOVERNING PARAMETERS

Consider an oscillating flow in a pipe with radii $R$, the cylindrical coordinate is chosen such that $x$ is in the flow direction parallel to the centerline of the pipe. The pressure gradient in the $x$ direction that drives the flow is assumed to be cosinusoidal with a frequency $f$ as:

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \alpha_p \cos(2\pi ft)$$  \hspace{1cm} (1)

where $\rho$ is the fluid density, $p$ the pressure, and $\alpha_p$ is the amplitude of negative pressure gradient which is assumed to be constant. Using $\alpha_p$ and $f$, a displacement length scale $A$ is now defined as $A = \alpha_p/(2\pi f)^2$. Similarly as the oscillating channel flows discussed in [17], there are three length scales for oscillating flows in circular pipes: the displacement amplitude of fluid oscillation $A$, the pipe radius $R$, and the Stokes layer thickness $\delta$. The Stokes layer thickness $\delta = \sqrt{v/2\pi f}$ measures the viscous diffusion distance in one cycle of oscillation, where, $v$ is the fluid viscosity and $f$ is the oscillation frequency. The ratios of $A$ and $R$ to $\delta$ then give two important independent parameters defined respectively by $Re_a = A^2/\delta^2 = 2\pi f A^2/v$ and $Re_b = R^2/\delta^2 = 2\pi f R^2/v$. The oscillating period number, i.e. the Keulegan-Carpenter number $KC$ (based on the characteristic length scale $R$) is defined as: $KC = 2\pi f R = 2\pi (Re_a/Re_b)^{1/2}$, and the Reynolds number $Re$ is defined as $Re = (2\pi f) R / v = (Re_a Re_b)^{1/2}$. Thus, in previous literatures we can see two set of governing parameters ($Re_a$, $Re_b$) and ($KC$, $Re$) for oscillating flow studies. The characteristics of the oscillating pipe flows then depend entirely on ($Re_a$, $Re_b$) or ($KC$, $Re$). While for heat transfer in oscillating pipe flows will also have Prantel number $Pr$ as the third governing parameter. The coordinate systems for the two groups of governing parameters ($\log(Re_a)$, $\log(Re_b)$) and ($\log(KC)$, $\log(Re)$) differ $\pi/4$, so results can be presented in either of them.

Using $R$ as the length scale, $2\pi f A$ as the velocity scale, $R/(2\pi f A)$ as the time scale, $\rho A (2\pi f)^2$ as the scale for negative pressure gradient, $\Theta_w - \Theta_i$ as the temperature scale, and the scales $(2\pi f A)^2$ and $2\pi f A R$ for the pseudo-energy $\epsilon$ and the pseudo-vorticity $\omega$ respectively, the non-dimensional governing equations can be obtained based on Saffman’s turbulence model [13] as:

$$\frac{\partial u}{\partial t} = \frac{2\pi}{KC} \cos(2\pi f/KC) + \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{1}{Re} \left( \frac{\partial u}{\partial r} \right) \right]$$  \hspace{1cm} (2)

$$\frac{\partial \omega}{\partial t} = \alpha_p \frac{\partial \omega}{\partial r} - \beta \omega \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{1}{Re} \left( \frac{\partial \omega}{\partial r} \right) \right]$$  \hspace{1cm} (3)

$$\frac{\partial \Theta}{\partial t} + u \frac{\partial \Theta}{\partial x} = \frac{1}{Re Pr} \frac{\partial}{\partial r} \left[ (1 + \sigma \gamma) \frac{\partial \Theta}{\partial r} \right]$$  \hspace{1cm} (4)

where, the Keulegan-Carpenter number $KC = 2\pi f A / R$ is also the dimensionless period, and the Reynolds number $Re = (2\pi f A) / v$, the $Pr = 7.0$ is selected to simulate water cooling in the present study.

The proper boundary conditions are:

$$u = 0, \quad e = 0, \quad \omega = \frac{\alpha_p}{\alpha_e} \frac{\partial \Theta}{\partial r} \left. \right|_{r = \pm 1}$$  \hspace{1cm} (6)

$$\frac{\partial u}{\partial r} = \frac{\partial e}{\partial r} = \frac{\partial \omega}{\partial r} = \frac{\partial \Theta}{\partial r} = 0 \left. \right|_{r = 0}$$  \hspace{1cm} (7)

$$\Theta = 0 \left. \right|_{r = \pm 1}$$  \hspace{1cm} (8)

In (2)-(5), $\alpha_e$, $\alpha_p$, $\beta$, $\beta_e$, $\sigma_e$, $\sigma_w$, $\gamma$ and $\gamma_r$ are universal constants. In the present computation, we followed Saffman & Wilcox [14] and Jacobs [23] to use $\alpha_e = 0.3$, $\alpha_p = 0.18$, $\beta = 0.09$, $\beta_e = 0.15$, $\sigma_e = 0.5$, $\sigma_w = 0.5$, $\gamma = 1.0$ and $\gamma_r = 1/0.89$. The value of $S$ in the wall boundary condition (6) depends on the surface roughness and is equal to 100 for a smooth wall [14].

III. NUMERICAL PROCEDURE AND RESULT VALIDATION

A. Numerical Procedure

The equation system (2)-(5), subjected to boundary conditions (6)-(8), was solved numerically with the following procedures: (i) Central difference scheme for spatial
derivatives, (ii) Second order Adams-Bashforth scheme for time advancement of the source terms, and (iii) Implicit scheme for the viscous terms.

The main reason for adapting Saffman’s turbulence model rests on its applicability to flows over the entire range of Reynolds number to provide an estimate of flow transition. Meanwhile, the simplicity of RANS method, especially the less time-consuming feature, enabled us to compute the flow and heat transfer characteristics over wide ranges of Reynolds numbers to provide a complete picture of the oscillating flow structure and the overall heat transfer enhancement.

In the present simulations, the length of the tube is 8 times the diameter. A mesh with grid size 160×200 is used and the mesh is stretched by exponential function to provide more points near the wall and inlet/outlet of the pipe to resolve the Stokes layer near the tube wall and the entrance heat transfer. The dimensionless time step \( \Delta t \) was chosen as \( \Delta t / \kappa \tau = 10^{-6} \). The convergence error is less than \( 10^{-6} \) for the velocity field and less than \( 10^{-5} \) for the temperature field.

B. Validation by Comparing to Analytical Results of Laminar Oscillating Pipe Flows

When the Reynolds number \( \text{Re}_d \) is sufficiently low, the flow is laminar, i.e., \( \frac{\nu'\theta'}{\nu} = 0 \) or \( \gamma (\nu/\nu) = 0 \). In term of the complex expression, \( u = [\hat{u} \exp(2\pi i/\kappa) + c.c.]/2 \) where \( \hat{u} \) is the complex amplitude, \( i = \sqrt{-1} \), and c.c. denotes the complex conjugate, the solution to (2) with boundary condition (6) and (7) is given by

\[
\hat{u} = -i\left[1 - \frac{J_0(\sqrt{-i} \text{Re}_e)}{J_0(\sqrt{-i} \text{Re}_e)}\right]
\]

where, \( J_0 \) denotes the Bessel function of the first kind and of zero order. The amplitude and the phase angle of \( u \) are then obtained by taking the absolute value and the argument to \( \hat{u} \), respectively.

The friction coefficient defined by \( C_f = 2 \tau_w/\rho(2\pi A)^2 \) \( = [\hat{C}_f \exp(2\pi i/\kappa) + c.c.]/2 \) with \( \tau_w \) being the wall shear stress, can be obtained by taking the derivative to (9) with respect to \( r \) and evaluating the resultant equation at the wall to give the following expression:

\[
\hat{C}_f = -\frac{2}{\text{Re}} \left[ J_0(\sqrt{-i} \text{Re}_e) / J_0(\sqrt{-i} \text{Re}_e) \right]
\]

Also, the amplitude and the phase angle of \( C_f \) are the absolute value and the argument of \( \hat{C}_f \), respectively.

Two limit cases of high and low \( \text{Re}_e \) are of great interest. When \( \text{Re}_e \to \infty \), Eq. (9) reduces to

\[
\hat{u} = -i\left[1 - \frac{1}{r} \exp\left[-(1+i)\left(\frac{\text{Re}_e}{2}(1-r)\right)\right] \right]
\]

Equation (11) indicates that the oscillating velocity is composed of two Stokes layers near the walls (whose scale is of order \( \theta \)) and the centerline velocity has an amplitude one and 90° phase-lag to the negative pressure gradient. By the same token, Eq. (10) reduces to

\[
\hat{C}_f = \left(1 - i\right)\frac{2\sqrt{\text{Re}_e}}{\text{Re}} = \left(1 - i\right)\left[\frac{2}{\text{Re}_d}\right]
\]

which indicates that the amplitude of wall shear stress depends solely on \( \text{Re}_d \) and has the phase angle 45° leading the centerline velocity. On the other hand, when \( \text{Re}_e \to 0 \), Eq. (9) becomes the parabolic profile of a quasi-steady flow given by

\[
\hat{u} = \left(1 - r^2\right)\text{Re}_e/4
\]

which shows that the amplitude decreases with decreasing \( \text{Re}_e \) and the phase angle becomes in-phase with the negative pressure gradient. The wall shear stress of Eq. (10) now reduces to

\[
\hat{C}_f = \text{Re}_e/\text{Re} = \left(\text{Re}_e/\text{Re}_d\right)^{1/2}
\]

which shows that the amplitude of shear stress decreases with decreasing \( \text{Re}_e \) in 1/2 power if \( \text{Re}_d \) is fixed (i.e., when the amplitude of negative pressure gradient is fixed) and the phase angle is in-phase with the velocity (or negative pressure gradient).

To validate the present numerical code, computational results of the amplitudes and phase angles of centerline velocity \( u_c \) at \( r = 0 \) and wall shear stress \( C_f \) obtained by present code are compared with the analytical results from (9) and (10). As shown in Fig. 1, five cases for various \( \text{Re}_e \) (<0.8×10^4) are in excellent agreement with those predicted from a laminar-flow analytic solution calculated from (9) and (10). From Fig. 1, we can see that when \( \text{Re}_e \) is larger than 10^3 and less than 0.8×10^4, \( |\tilde{u}_c| \) equals to 1.0 and \( \theta_{u_c} \) maintains constant at -90°. Similarly, \( |\tilde{C}_f|\text{Re}_d^{1/2} \) is constant equal to 2.0 and \( \theta_{C_f} \) remains constant at -45°. These amplitudes and phase angles equal to those predicted by (11) and (12) for the limit case of \( \text{Re}_e \to \infty \). For laminar oscillating flows in two flat plate channels [17], as \( \text{Re}_e \) decreases, the amplitudes \( |\tilde{u}_c| \) and \( |\tilde{C}_f|\text{Re}_d^{1/2} \) (shown in dashed lines in Fig. 1) overshoot to the maximum values greater than 1.0 and 2.0 respectively. For the present laminar oscillating flows in circular pipes, the amplitudes \( |\tilde{u}_c| \) overshoot to the maximum values greater than 1.0, however there is not overshooting for \( |\tilde{C}_f|\text{Re}_d^{1/2} \). When \( \text{Re}_e \) becomes very low, the amplitudes of both \( u_c \) and \( C_f \) decrease and their phase angles approach zero. In fact, \( |\tilde{u}_c| \) varies linearly with \( \text{Re}_e \) and \( |\tilde{C}_f|\text{Re}_d^{1/2} \) varies with \( (\text{Re}_e)^{1/2} \), which are agree with (13) and (14). The agreement of the numerical results
and the analytical solution shown in Fig. 1, show that the present code works very well at low $Re$ number range.

![Fig. 1 Variations of centerline velocity $u_C$ and wall frictional coefficient $C_F$ with $Re_R$ for oscillating laminar flow.](image)

C. Validation by Comparing to Experimental Data of Turbulent Oscillating Pipe Flows

In order to validate the present code at high Reynolds number range, a comparison between our numerical result for one turbulent flow case ($Re_f = 224.767$ and $Re_A = 6.2 \times 10^5$) and the experimental data of Akhavan (1991) has been done and shown in Fig. 2. The good agreement between the numerical results and experimental data on the transient velocity for the eight phase angles in a period guaranteed that the present code is also valid in high Reynolds number range.

![Fig. 2 A comparison between our numerical result and the experimental data of Akhavan [11]](image)

IV. RESULT AND DISCUSSION

The method of partial sums of the Fourier series is used to decompose the velocity and temperature results. The velocity and temperature can be decomposed based on the dimensionless period of the oscillating pressure (i.e. the $KC$ number) as:

$$u(r,t) = u_0(r) + \sum_{k=1}^{N} u_k(r) \cos(k \pi / KC + \theta_k(r))$$ \hspace{1cm} (15)

and

$$\Theta(x,r,t) = \Theta_0(x,r) + \sum_{k=1}^{N} \Theta_k(x,r) \cos(k \pi / KC + \theta_k(x,r))$$ \hspace{1cm} (16)

where, $k$ is the order of the harmonic term and $\theta_k$ is phase angle difference comparing to the pressure phase angle. Zero order harmonic terms $(u_0, \Theta_0)$ are the cycle averaged values. The integer $N$ larger than 3 is enough for decomposition of present numerical results, and higher orders of harmonic terms have maximum values of amplitude less than $10^{-5}$ and $10^{-3}$ for velocity and temperature field respectively, which are negligible.
A. Numerical Results for Fluid Flow

Numerical simulations for five different values of Re_{\infty}, the velocity profiles of three different values of Re_{\infty}, corresponding to laminar, transitional and fully turbulent flows, are plotted in Fig. 3 for the two extreme cases of Re_{\infty} = 5255 (solid lines) and 3.974 (dashed lines). For the case of Re_{\infty} = 5255, Fig. 3 shows that the velocity profile at Re_{A} = 10^3 is of a typical laminar oscillating flow with a thin Stokes layer near the wall and a potential core. When the flow becomes transitional at Re_{A} = 10^3, the velocity profile shown in Fig. 3 indicates that the turbulent mixing is still confined in the turbulent boundary layer near the wall whose thickness is much thicker than the laminar Stokes layer. The locations of amplitude overshoot and phase-angle undershoot are shifted toward the pipe centerline due to turbulent mixing, even though the potential flow remains in the core region. At Re_{A} = 10^6, the turbulent boundary layer apparently has occupied the whole pipe, the amplitude overshoot disappears. Alternatively, this can be interpreted as the location of overshoot has moved to the pipe centerline. The phase angle \theta_u then becomes quite uniform across the pipe, but remains to be close to -90^\circ. On the other hand, for the case of Re_{A} = 3.974 the velocity profile shown in Fig.3 indicates that the flow is laminar and nearly quasi-steady when Re_{A} = 10^3, with an almost parabolic profile in \[u\] and phase angles \theta_u ranging from -23.5^\circ to -37.4^\circ. When the flow becomes transitional at Re_{A} = 10^6, the enhancement of the fluid diffusion by turbulent eddy viscosity apparently has flattened the velocity profile near the pipe core region to result in lower velocity amplitude. Meanwhile, the phase angle \theta_u ranges from -18.1^\circ to -21.8^\circ, which indicates that the eddy viscosity effect renders the flow to approach toward the quasi-steady state. Further increase of the Reynolds number seems only to provide high eddy viscosity to further enhance the turbulent mixing effect toward a fully developed quasi-steady turbulent pipe flow, as shown by Re_{A} = 10^8 in Fig. 3 and the phase angle \theta_u ranges from -6.5^\circ to -8.1^\circ. The data also show that second order harmonic term of velocity is negligible comparing to the first order term and its phase angle is not as regular as that of the first hand harmonic term.

A.1 Velocity Distribution along the r Direction

As our objective is to explore the flow structure for transition rather than non-linearity, only the amplitudes and the phase angles of the first order harmonic of the velocity results are presented in this paper.

To illustrate the flow transition under different conditions of Re_{\infty}, the velocity profiles of three different values of Re_{\infty}, corresponding to laminar, transitional and fully turbulent flows, are plotted in Fig. 3 for the two extreme cases of Re_{\infty} = 5255 (solid lines) and 3.974 (dashed lines). For the case of Re_{\infty} = 5255, Fig. 3 shows that the velocity profile at Re_{\infty} = 10^3 is of a typical laminar oscillating flow with a thin Stokes layer near the wall and a potential core. When the flow becomes transitional at Re_{\infty} = 10^3, the velocity profile shown in Fig. 3 indicates that the turbulent mixing is still confined in the turbulent boundary layer near the wall whose thickness is much thicker than the laminar Stokes layer. The locations of amplitude overshoot and phase-angle undershoot are shifted toward the pipe centerline due to turbulent mixing, even though the potential flow remains in the core region. At Re_{\infty} = 10^6, the turbulent boundary layer apparently has occupied the whole pipe, the amplitude overshoot disappears. Alternatively, this can be interpreted as the location of overshoot has moved to the pipe centerline. The phase angle \theta_u then becomes quite uniform across the pipe, but remains to be close to -90^\circ. On the other hand, for the case of Re_{\infty} = 3.974 the velocity profile shown in Fig.3 indicates that the flow is laminar and nearly quasi-steady when Re_{\infty} = 10^3, with an almost parabolic profile in \[u\] and phase angles \theta_u ranging from -23.5^\circ to -37.4^\circ. When the flow becomes transitional at Re_{\infty} = 10^6, the enhancement of the fluid diffusion by turbulent eddy viscosity apparently has flattened the velocity profile near the pipe core region to result in lower velocity amplitude. Meanwhile, the phase angle \theta_u ranges from -18.1^\circ to -21.8^\circ, which indicates that the eddy viscosity effect renders the flow to approach toward the quasi-steady state. Further increase of the Reynolds number seems only to provide high eddy viscosity to further enhance the turbulent mixing effect toward a fully developed quasi-steady turbulent pipe flow, as shown by Re_{\infty} = 10^8 in Fig. 3 and the phase angle \theta_u ranges from -6.5^\circ to -8.1^\circ. The data also show that second order harmonic term of velocity is negligible comparing to the first order term and its phase angle is not as regular as that of the first hand harmonic term.

A.2 Full Structures of Velocity and the Wall Shear Stress

To obtain overall flow structures, the results of centerline velocity \nu_c and the wall shear stress \tau_r for all computed cases of Re_{\infty} and Re_{\infty} are plotted in Figs. 4 and 5, respectively, for (a) amplitude and (b) phase angle. In Figs. 4 and 5, the solid lines represent the analytical laminar flow results of low Re_{\infty} from Eq. (9) and (10). We now first examine the result of \nu_c given in Fig. 4. For the two sets at high Reynolds numbers of Re_{\infty} = 5255, the amplitudes \nu_1 as shown in Fig. 4a maintain at one and the corresponding phase angles as shown in Fig. 4b have the value of -90^\circ, except at very high Re_{\infty}. This suggests that the turbulent oscillating boundary layer for high Re_{\infty} is still thinner than \tau and is unable to produce noticeable effect on the centerline velocity, until Re_{\infty} becomes very high. For the cases of Re_{\infty} = 328.4, 15.90, 3.974 and 1.131, the values of \nu_1 however drop monotonically with increasing Re_{\infty} in the turbulent regime. This is accompanied by the continuing shift of phase angle from -90^\circ toward 0^\circ as indicated in Fig. 4(b). Apparently, when Re_{\infty} is sufficiently small, the oscillating turbulent boundary layer
becomes quasi-steady as $Re_d \to \infty$. Assuming the flow is quasi-steady, the expression for $|\dot{u}_r|$ at very high $Re_d$ can be devised by following Saffman's derivation \[13\] to give:

$$|\dot{u}_r| = \frac{1}{\kappa} \left( \frac{Re_k}{Re_d} \right)^{0.25} \left[ 0.25 \ln(Re_k) + 0.75 \ln(Re_d) + 1.92 \right]$$ \hfill (17)

where, $\kappa$ (0.38<\kappa<0.47) is the von Kármán constant. The results calculated from (17) for high $Re_d$ and low $Re$. are shown as the dashed lines in Fig. 4a. They agree very well with the numerical results. It is recalled that in the quasi-steady limit where the shear layer covers the entire pipe, the pressure force is balanced totally by the shear. Therefore, we conclude that at very high $Re_d$, the eddy viscosity effect has greatly enhanced the shear force to render the transient inertia force negligible.

![Fig. 4](image.png)

Fig. 4 Variations of (a) amplitude and (b) phase angle of $\dot{u}_r$ with $Re_d$ for five values of $Re_d$. Solid lines: laminar solution; Dashed lines: quasi-steady analytical solution using Saffman’s model \[13\]

For a better understanding of the flow structure, we shall examine the wall shear stress shown in Fig. 5. Attention is first given to the case of $Re_d = 328.4$, i.e., when $R$ is about eighteen times the Stokes layer thickness $\delta$. When $Re_d$ is sufficiently low, say $Re_d < 0.8 \times 10^4$ before flow transition, the oscillating flows are laminar. The results of $C_F$ as computed according to the RANS method with Saffman's turbulence model agree excellently with the analytical predictions from (10), i.e., $\dot{C}_F Re = 35.54$ and $\theta_{C_F} = -43.84^\circ$. As $Re_d$ increases, the transition from laminar to turbulent occurs approximately at $\left(Re_d\right)_tr = 0.8 \times 10^4$ as shown in Fig. 5b where $\theta_{C_F}$ starts to decrease from -43.84°. Under the condition of $Re_d = 328.4$, the oscillating turbulent flow after transition remains as a boundary layer flow confined near the wall, with a potential flow in the pipe core region. Interestingly, the amplitude $\dot{C}_F Re$ does not change noticeably until $Re_d = 7.5 \times 10^5$, and hence is not a good indicator for flow transition. It is found out that the phase angle of the wall shear stress is a
more sensitive gauge than amplitude for the determination of flow transition. Further increase in Re<sub>t</sub> leads to higher value of the eddy viscosity υ<sub>T</sub> that thickens the thickness δ<sub>T</sub> of the oscillating turbulent boundary layer. The phase angle θ<sub>cT</sub> continues to decrease with increasing δ<sub>T</sub>, until reaches a minimum value of about θ<sub>cT</sub> = -71° at Re<sub>t</sub> = 7.5 × 10<sup>5</sup> where δ<sub>T</sub> has become sufficiently thick that the effect due to the mutual interaction of the boundary layers at top and bottom of the channel is appreciable. For Re<sub>t</sub> > 7.5 × 10<sup>5</sup>, the phase angle θ<sub>cT</sub> increases with increasing Re<sub>t</sub>. In the limit of Re<sub>t</sub> → ∞, R becomes the governing length scale since δ<sub>T</sub> >> R and the oscillating turbulent flow becomes a quasi-steady turbulent flow where θ<sub>cT</sub> → 0. The amplitude of wall shear stress given in Fig. 5a for Re<sub>T</sub> = 328.4 shows that |C<sub>F</sub>Re<sub>T</sub>| increases with increasing Re<sub>T</sub> and asymptotically reaches a constant when Re<sub>T</sub> → ∞. For a fully developed turbulent flow in a channel that is steady in mean and driven by a mean pressure gradient ∂p/∂x, a simple momentum balance results in τ<sub>u</sub> / R = −∂p/∂x which in terms of the friction coefficient becomes (14). This implies that Eq. (14), which was originally obtained for quasi-steady oscillating laminar flows, applies equal well to quasi-steady oscillating turbulent flows. The asymptotic value of |C<sub>F</sub>Re<sub>T</sub>| is Re<sub>T</sub>. This gives |C<sub>F</sub>Re<sub>T</sub>| = 328.4 if Re<sub>T</sub> = 328.4, which as shown as dashed line in Fig. 5a agrees very well with the computed result.

With the above comprehension of the flow for Re<sub>T</sub> = 328.4, we now examine the flows at different Re<sub>T</sub>. For higher Re<sub>T</sub> such as the cases of Re<sub>T</sub> = 5255, the oscillating laminar Stokes layer at low Re<sub>T</sub> is much thinner than R. The transition from laminar to turbulent still occurs near (Re<sub>T</sub>)<sub>cr</sub> = 0.8 × 10<sup>4</sup>; however, after the transition it requires much higher Re<sub>T</sub> than that of Re<sub>T</sub> = 328.4 for δ<sub>T</sub> to become comparable with R. Fig. 5b indicates that θ<sub>cT</sub> reaches a minimum of -71° at Re<sub>T</sub> = 7.5 × 10<sup>5</sup> for Re<sub>T</sub> = 328.4 and is still decreasing at Re<sub>T</sub> = 1.5 × 10<sup>6</sup> for Re<sub>T</sub> = 5255. Figure 8a also shows that the amplitude results of this study never reach the asymptotic values of |C<sub>F</sub>Re<sub>T</sub>| for Re<sub>T</sub> = 5255. Apparently, for the cases of Re<sub>T</sub> = 5255 the computed range of Re<sub>T</sub> in this study covers only the laminar Stokes layer flow and the oscillating turbulent boundary layer flow regimes. On the other hand, for the cases of low Re<sub>T</sub> the thickness of the Stokes layer is already comparable with R when Re<sub>T</sub> = 3.974 and much thicker than R when Re<sub>T</sub> = 1.131. At low Re<sub>T</sub> the flow is laminar with θ<sub>cT</sub> = -23.914° for Re<sub>T</sub> = 3.974 and θ<sub>cT</sub> = -7.968° for Re<sub>T</sub> = 1.131. The oscillating laminar flow is already in the quasi-steady flow regime. As Re<sub>T</sub> increases passing the critical value (Re<sub>T</sub>)<sub>cr</sub>, the oscillating flow moves directly from the quasi-steady laminar flow regime to the quasi-steady turbulent flow regime. Therefore, the phase angle θ<sub>cT</sub> increases from its respective laminar flow value toward 0°. Fig. 5a shows that for both cases of Re<sub>T</sub> = 3.974 and 1.131 the amplitudes |C<sub>F</sub>Re<sub>T</sub>| increases with increasing Re<sub>T</sub> starting from (Re<sub>T</sub>)<sub>cr</sub> and reaches the asymptotic values of |C<sub>F</sub>Re<sub>T</sub>| = 3.974 and 1.131, respectively, as plotted again as dashed lines. There is a delay in flow transition depending on Re<sub>T</sub>. The lower the Re<sub>T</sub>, the higher will be the (Re<sub>T</sub>)<sub>cr</sub> and the earlier the oscillating turbulent flow will reach the asymptotic results of quasi-steady state.

### A.3 Flow Regimes

From the results given above, the structure of the oscillating pipe flows is constructed using parameters (Re<sub>T</sub>, Re<sub>T</sub>) as shown in Fig. 6. Each point on Fig. 6 represents one computed case. The open circle represents the laminar flow, the star represents the oscillating turbulent boundary layer flow and the triangle represents the quasi-steady turbulent flow. The transition from laminar regime to turbulent regime is marked by the sudden change of θ<sub>cT</sub> from the constant laminar values.

![Flow regimes of oscillating flows in channels in term of coordinates (Re<sub>T</sub>, Re<sub>T</sub>).](image)

At low Re<sub>T</sub>, say Re<sub>T</sub> < 0.8 × 10<sup>4</sup>, the oscillating flows are laminar and two flow regimes showing low and high Re<sub>T</sub> respectively, are identified. The domain of Re<sub>T</sub> << 1 represents the quasi-steady laminar flow regime where the velocity profile is parabolic and the domain of Re<sub>T</sub> >> 1 represents the Stokes layer laminar flow regime where the velocity profile decays exponentially from the wall. As Re<sub>T</sub> increases, the Stokes layer at high Re<sub>T</sub> becomes unstable. The transition from laminar to turbulent occurs approximately at (Re<sub>T</sub>)<sub>cr</sub> = 0.8 × 10<sup>4</sup> and is plotted as the dashed line in Fig. 6. For Re<sub>T</sub> > (Re<sub>T</sub>)<sub>cr</sub> and high Re<sub>T</sub>, the flow is in the oscillating turbulent boundary layer flow regime where the thickness δ<sub>T</sub> of turbulent boundary layer remains thinner than R, with a potential flow in the channel core. The increase in Re<sub>T</sub> will lead to thicker δ<sub>T</sub> and, in the limit of Re<sub>T</sub> >> (Re<sub>T</sub>)<sub>cr</sub> but still of high Re<sub>T</sub>, the thickness δ<sub>T</sub> becomes much thicker than R so that the flow is governed by R and belongs to the quasi-steady turbulent flow regime. The flow transition is delayed to
higher ($Re_c$), when $Re_R$ decreases. In the limit of $Re_T \ll 1$ where flows become quasi-steady, the transition is expected to occur at the same critical condition of a fully developed steady channel flow at \( \frac{U}{2v_{rms}} = 2300 \), where $U$ is the mean velocity. For full developed pipe flow \( \frac{U}{2v_{rms}} = U_{max} / 2 \). We have $U_{max} = \frac{2\pi d}{4}(Re_c) / 4$ from (13) and \( \frac{U_{rms}}{2v_{rms}} = (Re_{Re_c}) / 4 = 2300 \); hence \( (Re_{Re_c}) / 4 = 9200 \). This limit case of critical condition is also plotted in Fig. 6. Since the critical value of 9200 was found from experimental observation, which is less sensitive than our classification using phase angle change, the dashed line predicts a slightly higher value of \( (Re_{Re_c}) \). As $Re_A$ passes $(Re_{Re_c})$, the oscillating flows move directly from the quasi-steady laminar flow regime into the quasi-steady turbulent flow regime.

**B Numerical Results of Heat Transfer**

Numerical results for the same five values of $Re_A (= 5255, 328.4, 15.90, 3.974$ and $1.313)$ and $Re_R$ (from $10^{1.5}$ to $10^{7.5}$) have been presented to cover the heat transfer regimes from nearly conduction to laminar and turbulent flow convection. Also a pure conduction case is simulated as a base line for the study of heat transfer enhancement.

**B.1 Temperature Distribution**

It is noted that the present numerical simulation results for the temperature field contain higher orders of harmonics than the velocity field, because the fluid field will affect the temperature field as shown in (5). Based on the decomposition of the transient results of the temperature field, as shown in (16), we find out that the temperature field is dominated by the zero order of harmonic. The amplitude of the temperature harmonics will decrease with the increase of the order of harmonics. Phase angles are zero at the inlet/outlet of the pipe, while the phase angles at the center of the pipe ($x=0$) are the maximum value along the x direction. The phase angles will increase with the order of harmonics, and the center phase angle is larger than $360^\circ$. The amplitude of temperature harmonics will decrease with the increase of $Re$. The phase angles are zero at the inlet/outlet of the pipe (x=0) are the maximum value along the x direction.

**B.2 Axis Direction Distribution of Nusselt Number**

The axis direction distributions of the Nusselt Number on the pipe wall for $Re_R = 328.4$ and $Re_A$ from $10^2$ to $10^7$ are plotted as solid lines in Fig. 7. The dash line is the pure conduction result. With $Re_A$ increased from $10^2$ to $10^7$, the center point Nusselt number $Nu_R(x=0)$ will increase from 0 to 222. There is always a heat transfer leading edge near the inlet/outlet of the pipe due to the assumption of the constant inlet/outlet water temperature. The leading edge Nusselt numbers are much larger than the center ones. From Fig. 7, we can also see that the center point of Nusselt number equals 0 for the cases $Re_A < 0.8 \times 10^7$, which is for nearly conduction to laminar flow regions. For the oscillating turbulent boundary layer flow regime, when $Re_A > 0.8 \times 10^7$, the center point Nusselt numbers will increase with $Re_A$.

**B.3 Averaged Nusselt Number based on various (Re_A, Re_R)**

The averaged Nusselt Numbers along x direction are plotted in Fig.8 in form of $Re_A$ and $Re_R$. At lower $Re_A$ and $Re_R$, $Nu_R$ is near to the pure conduction value (1.7031). While the value increase with both $Re_A$ and $Re_R$. Those $Nu_R$ lines in coordinate $(Re_A, Re_R)$ never cross each other, which shows $Re_R$ and $Re_A$ are the basic two independent governing parameters for heat transfer as well as the structure of the flows in oscillating pipe flows. If $Nu_R$ is plotted in form of $Re$ and $KC$, we can see overlap of the data, which is similar as the overlap of data shown in the experimental study of [24].
heat transfer enhancement ratio. It is obviously that if we use larger pipe diameter, the oscillating turbulent boundary layer flow region, heat transfer enhancement is not clear in Fig. 9, so in the following section, we will present a clear picture, which shows the effect of the Keulegan–Carpenter number on the heat transfer, with Re\(_A\), i.e. increase of the pipe diameter such as Re\(_A\) and Re\(_R\) number. Comparing to the pure conduction x axis averaged Nusselt Number (Nu\(_c\)) is about 1.7031, the heat transfer enhancement ratio Nu\(_R\)/Nu\(_c\) is always greater than 1 in all ranges of Re\(_R\) and Re\(_A\), i.e. increase of the pipe diameter R and the amplitude of the oscillating flow A, can definitely increase the heat transfer. Thus, we can get more effective heat transfer. Thus, we can remove thermal energy more quickly through sea water cooling heat exchangers on sea floors, where both the frequency f and the amplitude A of the ocean wave are as large as possible. The circular white balls in Fig. 10 show the positions of the ocean wave amplitude should be D/A = 4π/10\(^{1.8}\). So the optimal ratio of the inner diameter of the heat exchanger pipe over the ocean wave oscillating amplitude is D/A = 0.2.

**V. CONCLUSIONS**

Fluid flow and heat transfer in oscillating pipe flows have been studied using RANS method with Saffman’s turbulence model over wide ranges parameters to reveal the full structures of the fluid flow and heat transfer in oscillating pipe flows. For low Re\(_A\), the flows are laminar and the present computed results are in excellent agreement with those predicted from a laminar-flow analytical solution. For laminar flow, the flow characteristics depend solely on Re\(_A\) with Re\(_R\) >> 1 corresponding to the Stokes layer flow limit and Re\(_R\) << 1 to the quasi-steady laminar flow limit. As Re\(_A\) increases, the high Re\(_R\) Stokes layer flow becomes unstable at approximately (Re\(_A\))\(^{cr}\) = 0.8×10\(^4\) and experiences a turbulent boundary layer flow regime before reaching a quasi-steady turbulent flow regime as Re\(_A\) → ∞, while the low Re\(_R\) quasi-steady laminar flow transits directly, with a delay, to the quasi-steady turbulent flow. This value of (Re\(_A\))\(^{cr}\) = 0.8×10\(^7\) agrees very well with the experimental results of Ohmi [10]. For turbulent oscillating flow, present numerical results (Re\(_A\) = 224.767 and Re\(_R\) = 6.2×10\(^5\)) and the experimental data of Akhavan [11] agree very well on both amplitude and phase.

**B.4 Heat Transfer Enhancement**

Comparing to the pure conduction x axis averaged Nusselt Number (Nu\(_c\)) = 1.7031, the heat transfer enhancement ratio Nu\(_R\)/Nu\(_c\) is always greater than 1 in all ranges of Re\(_R\) and Re\(_A\). Fig. 11 show the distribution of log(Nu\(_R\)/Nu\(_c\)) in term of coordinates (log(Re\(_A\)), log(Re\(_R\))). From Fig. 11, we can see that the enhancement of heat transfer increased form near 1 at lower Re\(_A\) value less than 10\(^4\) and lower Re\(_R\) value less than 4. The enhancement ratio is less than 10 for Re\(_A\) < 0.8×10\(^4\) or Re\(_R\) < 10. It means that in laminar flow and quasi-steady turbulent regions, the heat transfer enhancement is small and in order of 1. This similarity is consistent with the previous discussion on the quasi-steady turbulent flow has similar drag formation as laminar flow in (14). For oscillating turbulent boundary layer flow region, heat transfer enhancement ratio can be in order higher than 10. At higher Re\(_A\) and Re\(_R\) region such as Re\(_R\) = 5.25.5.0 and Re\(_A\) = 10\(^7\)-5, the heat transfer enhancement ratio Nu\(_R\)/Nu\(_c\) can be as much as 500. From Fig. 9, we can see that increase of both Re\(_R\) and Re\(_A\), i.e. increase of the pipe diameter R and the amplitude of the oscillating flow A, can definitely increase the heat transfer. Thus, we can remove thermal energy more quickly through sea water cooling heat exchangers on sea floors, where both the frequency f and the amplitude A of the ocean wave are as large as possible. The circular white balls in Fig. 10 show the positions of the maximum Nu\(_R\) for a wide range of fixed Re\(_R\) number from 5 to 10\(^5\). It is clear that Nu\(_R\) has a maximum value near the line of log(Re\(_A\)) ≈ 1.8, i.e. the optimal Keulegan–Carpenter number is about 10\(^{1.8}\) for heat transfer. Hence, in order to enhance the heat transfer between the tube wall and the sea water, the optimal KC number is about 63. Thus, the optimal size for the heat exchanger pipes should be around R/A ≈ 2π/10\(^{1.8}\), i.e. the inner diameter of the heat exchanger pipe over the wave oscillating amplitude should be D/A = 4π/10\(^{1.8}\). So the optimal ratio of the inner diameter of the heat exchanger pipe over the ocean wave oscillating amplitude is D/A = 0.2.

**B.5 Optimal Keulegan-Carpenter number and Tube Diameter**

In order to show the effects of the Keulegan–Carpenter number KC and the Reynolds number Re on the heat transfer, log(Nu\(_R\)) in form of coordinates (log(KC), log(Re)) is shown in Fig. 10. From Fig.10 we can see that log(Nu\(_R\)) increase dramatically with log(Re). Now that Re is linear to the velocity scale 2πfA, we should put sea water cooling heat exchangers on sea floors, where both the frequency f and the amplitude A of the ocean wave are as large as possible. The circular white balls in Fig. 10 show the positions of the maximum Nu\(_R\) for a wide range of fixed Re\(_R\) number from 5 to 10\(^5\). It is clear that Nu\(_R\) has a maximum value near the line of log(Re\(_A\)) ≈ 1.8, i.e. the optimal Keulegan–Carpenter number is about 10\(^{1.8}\) for heat transfer. Hence, in order to enhance the heat transfer between the tube wall and the sea water, the optimal KC number is about 63. Thus, the optimal size for the heat exchanger pipes should be around R/A ≈ 2π/10\(^{1.8}\), i.e. the inner diameter of the heat exchanger pipe over the wave oscillating amplitude should be D/A = 4π/10\(^{1.8}\). So the optimal ratio of the inner diameter of the heat exchanger pipe over the ocean wave oscillating amplitude is D/A = 0.2.

**V. CONCLUSIONS**

Fluid flow and heat transfer in oscillating pipe flows have been studied using RANS method with Saffman’s turbulence model over wide ranges parameters to reveal the full structures of the fluid flow and heat transfer in oscillating pipe flows. For low Re\(_A\), the flows are laminar and the present computed results are in excellent agreement with those predicted from a laminar-flow analytical solution. For laminar flow, the flow characteristics depend solely on Re\(_A\) with Re\(_R\) >> 1 corresponding to the Stokes layer flow limit and Re\(_R\) << 1 to the quasi-steady laminar flow limit. As Re\(_A\) increases, the high Re\(_R\) Stokes layer flow becomes unstable at approximately (Re\(_A\))\(^{cr}\) = 0.8×10\(^4\) and experiences a turbulent boundary layer flow regime before reaching a quasi-steady turbulent flow regime as Re\(_A\) → ∞, while the low Re\(_R\) quasi-steady laminar flow transits directly, with a delay, to the quasi-steady turbulent flow. This value of (Re\(_A\))\(^{cr}\) = 0.8×10\(^7\) agrees very well with the experimental results of Ohmi [10]. For turbulent oscillating flow, present numerical results (Re\(_A\) = 224.767 and Re\(_R\) = 6.2×10\(^5\)) and the experimental data of Akhavan [11] agree very well on both amplitude and phase.
angle. By the way of analyzing the overall Nusselt Number over wide ranges of the Reynolds number and Keulegan-Carpenter number, we know that $N_{Re}$ increase dramatically with the Re number and we also obtain the optimal KC number is about 10^{-8} for a wide range of Re from 5 to 10^{5}. So the best place to put pipe heat exchangers for the application of sea water cooling is to choose the ocean floor, where both the frequency $f$ and the amplitude $A$ of the ocean water waves are as large as possible. The optimal ratio of the inner diameter of the heat exchanger pipe $D$ over the wave oscillating amplitude $A$ is $D/A \approx 0.2$.

ACKNOWLEDGMENT

The authors thank Prof. C. T. Hsu for his help on turbulent models. This study was supported by the Multi-Year Research Grant MYRG151(V1-L2)-FST11-SY and the Start-Up Research Grant SRG009-FST11-YS of the Macau University. This study was also supported by the University of Minnesota Supercomputing Institute and the Information and Communication Technology Office of University of Macau.

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