Abstract—The lecture represents significant advances in understanding of the transfer processes mechanism in turbulent separated flows. Based upon experimental data suggesting the governing role of generated local pressure gradient that takes place in the immediate vicinity of the wall in separated flow as a result of intense instantaneous accelerations induced by large-scale vortex flow structures similarity laws for mean velocity and temperature and spectral characteristics and heat and mass transfer law for turbulent separated flows have been developed. These laws are confirmed by available experimental data. The results obtained were employed for analysis of heat and mass transfer in some very complex processes occurring in technological applications such as impinging jets, heat transfer of cylinders in cross flow and in tube banks, packed beds where processes manifest distinct properties which allow them to be classified under turbulent separated flows. Many facts have got an explanation for the first time.

Keywords—impinging jets, packed beds, turbulent separated flows, ‘two-thirds power law’

I. INTRODUCTION

A CONTINUING demand for high heat rates between a solid and flow in many thermal-processing applications invites development of heat transfer enhancement techniques based on heat transfer surfaces of intricate shapes with turbulent flows and separation zones in flow passages. Turbulent separated flows occur at sufficiently high Reynolds numbers in many important technological applications. It is observed in applications of interest to mechanical, aerospace, chemical, and civil engineers, and in flows of interest to meteorologists and earth scientists such as flows at the rear of bluff bodies, in ducts with abrupt geometry change, and in boundary layers upstream of step-like discontinuities in wall profile. Phenomena of separation and reattachment are also realized in the presence of strong adverse pressure gradients in many aerodynamic flows of interest; for example, flow in diffusers, on airfoils, and in cavities. Although this phenomenon is frequently observed, and despite of the extensive experimental studies there are no theoretical concepts of their generalization. The laws of momentum, heat and mass transfer obeyed by turbulent separated flows exhibit characteristic features which have escaped explanation. The main approach heretofore is accumulation of experimental data followed by generalization based on frequent occasions on a ‘common sense’ and on the experience rather than on a deep insight into the mechanism of phenomenon. As a result many of recommendations available are represented whether as tables and nomograms or quite cumbersome correlations valid within the range of parameters of particular experiments. Any extension in application of such a type of correlations may cause significant errors in design.

The objective of this lecture is to present fundamental transport mechanisms in turbulent separated flows and to discuss its perspectives and the experience gained for developing limiting laws, regularities and recommendations on construction of correlating equations for heat and mass transfer in some very complex processes occurring in technological applications of great importance which manifest distinct properties allowing them to be classified under turbulent separated flows.

A. Turbulent separated flows: experimental facts and modeling approaches

By turbulent separated flow, we mean the entire process associated with turbulent flow separation and reattachment onto a solid surface. It may be generated by strong adverse pressure gradients in or on smoothly varying geometries: flow in a diffuser, on airfoils; and flows that separate due to abrupt changes in geometry with geometrically fixed separation position such as flows in cavities and over a single-sided backward-facing step, flows downstream of the leading edge of a blunt plate or a blunt circular cylinder.

Separation of the turbulent boundary layer is reported in experimental studies to be an unsteady phenomenon consisting of local events of unsteady separations which occurs chaotically in the vicinity of the mean flow separation line. Each such separation is accompanied by the departure of the shear layer which in turn generates new large eddies, which grow rapidly and agglomerate thereby decreasing the average frequency. These eddies supply turbulent energy to the near-wall flow.

Experimental studies of turbulent flows over sharp-edged bluff bodies demonstrate the occurrence of the persistent large-scale eddies that are developed in the separated free-shear line. The instantaneous reattachment (the impingement of the separated shear layer on the wall) location fluctuates with frequency which corresponds to that of wall pressure fluctuations with maximum energy content.

Structurally, a separation-and-reattaching region of flow is considered as containing a shear layer that on one side bounds a main stream and on the other, next to a wall, there is a region of instantaneous mean, relatively slow, reverse flow with a thin near-wall zone within which the flow experiences a direct influence of the surface. The thickness of the near-wall zone corresponds to the distance from the wall to the peak mean reverse flow. Namely, the flow pattern and
structural characteristics of the near-wall zone are responsible for heat and mass transfer in separated flows.

Among some the most important properties of the near-wall zone of turbulent separated flows and regularities of heat and mass transfer as it has been shown in experimental studies (see for references reviews [1] - [3]) are: breakdown of the logarithmic law-of-the-wall, so that the friction velocity is not the characteristic velocity for mean velocity profiles generalization outside of the linear sublayer; there is no a peak in turbulence intensity profiles in the vicinity of a wall; the Reynolds shear stress remains small in the near-wall region; the intensity of velocity fluctuations parallel to the wall in the near-wall region is of the order of magnitude of the local mean velocity; the intensity of fluctuations normal to the wall is nearly an order of magnitude smaller; the Nusselt number is usually proportional to the Reynolds number to the power close to 2/3 in contrast to attached turbulent boundary layers with an exponent close to 0.8; the Nusselt number reaches its maximum value at the point where the mean skin friction passes through zero, this shows that the Reynolds analogy fails in this type of flow.

Attempts of the theoretical description of complex turbulent flows based on Reynolds averaged Navier-Stokes approach, large eddy simulation, direct numerical simulation or hybrid approaches (an extensive list of references can be found in [4] - [6] have not furnished any substantial advance to understanding of transport mechanism. The prediction of fluid dynamics and heat and mass transfer in such flows is still a challenging problem for modern turbulence theory and computational fluid dynamics. To provide asymptotic solutions, a physically justified turbulence model taking into consideration mechanism and main governing parameters should be developed.

B. Similarity laws and heat and mass transfer in TSF

Our estimates [7] of the scales of turbulent motion in the near-wall zone of separated flow on a flat plate (data [8] show that the spatial dimensions of low-frequency fluctuations of the longitudinal velocity in the near-wall zone are much greater than the layer thickness. The backflow velocity has a maximum at a distance $N$ from the wall that is far smaller than the local boundary layer thickness - in experiments [8] it constituted a few percent of the local boundary layer thickness. On the other hand ([3]), the root-mean-square fluctuation of the transverse velocity in the near-wall zone is an order of magnitude lower than the longitudinal fluctuations.

These facts indicate that the large-scale motion in the near-wall zone is of the character of an unsteady turbulent boundary layer. In this case large-scale unsteady recirculation flow supported by large-scale vortices from the mixing layer above plays the role of external flow.

And so turbulent motion in the boundary layer can be divided into two components: a large-scale component with a length scale much greater than the near-wall thickness $N$ and a small-scale component which is composed of vortices with a size of the order of $N$ or less.

Averaging the Navier-Stokes equations with the Reynolds averaging rules over small-scale fluctuations and assuming no correlations between large-scale and small-scale velocity fluctuations, one can obtain the equations of large-scale motion in the near-wall layer and put forward three hypotheses summarizing the physical model of the transfer processes in the near-wall layer of turbulent separated flows and serving as a basis for theoretical analysis and dimensional considerations [7], [9].

Hypothesis 1. When the Reynolds number is high enough, the statistical regimes of large- and small-scale velocity fluctuations are independent. The statistical regime of small-scale velocity fluctuations is determined uniquely in the near-wall zone $y << N$ by the local values $\tau_w(\bar{x}, t)$, $\alpha(\bar{x}, t)$ and $\nu$.

The constant across the boundary layer kinematical pressure gradient $\bar{a} = \nabla p$ created by the large-scale outer motions (the near-wall root-mean-square pressure gradient $\alpha_{rms}$ is

$$\alpha_{rms} = \left[\frac{1}{\rho} \frac{\partial P}{\partial x}\right] + \alpha_{rms}^2,$$

where $\alpha$ is the pressure gradient fluctuation.

One can show [9] that for $P_+ = \nu \alpha/\tau_w^{3/2} >> 3 \cdot 10^{-2}$, and this is the case for the backflow region, the wall shear stresses become negligible, which corresponds to the flow near the separation; fluid acceleration in the near-wall layer that is produced by large-scale vortices of separated flows prevents formation of a sublayer of constant shear stress, and the friction velocity is not the governing parameter of the near-wall layer of turbulent separated flows. The statistical regime of turbulent flow in this region is determined by the local pressure gradient (local instantaneous accelerations) and distance from surface. The root-mean-square wall pressure gradient is a quantitative characteristic of these accelerations, thus

Hypothesis 2. When $P_+$ is high enough ($P_+ >> 10^2$) the statistical regime of the small-scale fluctuations in the near-wall region $y << N$ is determined uniquely by the values $\nu$ and $\alpha(\bar{x}, t)$.

From this hypothesis and dimensional considerations, it follows that the thickness of the viscous sublayer is given by

$$l \sim \nu^{-2/3} \alpha^{-1/3}$$

For distances from the wall far greater than the viscous layer thickness it would appear reasonable to formulate the third hypothesis:

Hypothesis 3. If the conditions of the first and second hypotheses are met with, there is a range of distance from the wall $\nu^{-2/3} \alpha^{-1/3} << y << N$ for which the statistical regime of the small-scale fluctuations in the near-wall region is determined uniquely by the value $\alpha(\bar{x}, t)$ and does not depend on viscosity $\nu$.

For analysis on a scalar substance transfer which is temperature we propose that

Hypothesis 4. If the conditions of the first and second hypotheses are met with, there is a range of distance from the wall $l_1 << y << N$ for which the statistical regime of the small-scale fluctuations in the near-wall region is determined uniquely by the value $\alpha(\bar{x}, t)$ and $q(\bar{x}, t)/(\rho c_p)$. 
From dimensional considerations [7], [9], similarity laws for the near-wall mean velocity and temperature, and small-scale spectra of longitudinal velocity and pressure fluctuations were obtained (and which are very different from those for isotropic turbulence – the Kolmogorov’s similarity laws):

\[ \bar{u} = \frac{a}{\alpha} \left[ K(\alpha)^{1/2} + K_{1}(\alpha^{1/2}) \right], \]

\[ T = K_{1} \frac{q}{\rho \alpha^{1/2} \rho C_{p}^{1/2}} + K_{11} \frac{q}{\rho C_{p}^{1/2}} \left[ \rho \alpha^{1/2} \rho C_{p}^{1/2} \right]^{1/3}, \]

\[ E_{ij} \propto \alpha_{rms} k^{-2}, \quad E_{p} \propto \rho^{1/2} \alpha_{rms}^{2} k^{-3} \]

where \( K, K_{1}, K_{11} \) are constants and \( K_{1} \) and \( K_{11} \) are the universal functions of Prandtl number \( Pr \).

The heat transfer law after averaging over the ensemble of large-scale fluctuations of wall pressure gradient is obtained in the form

\[ Nu = constC_{a}^{1/3} Re^{2/3} Pr^{\beta} \]  

(2)

Since the boundary layer accelerations due to large-scale vortices \( \alpha_{rms} \propto \bar{u}^{2} / L \), the coefficient \( C_{a} = \alpha_{rms} L / \bar{u}^{2} \) is independent of the Reynolds number and law (2) represents the “2/3-power law” (\( Nu \propto Re^{2/3} \)) for turbulent heat transfer in separated flows.

Thus, the “2/3-power law”, which was found experimentally in the early 1960s ([1]) and is encountered in almost all heat and mass transfer experiments with turbulent separation, and similarity laws for structural characteristics of the near-wall region of turbulent separated flows have received theoretical justification for the first time. Experimental data available support our similarity laws [7], [9].

To compare the heat transfer law (2) with experimental data the near-wall pressure gradient \( \alpha_{rms} \) needs to be related to the parameters measured in experiments. It seems to be logical to relate \( \alpha_{rms} \) to the level of wall pressure fluctuations \( p'_{rms} \) and the characteristic length scale of pressure disturbances \( l_{p} \):

\[ \alpha_{rms} = p'_{rms} / (\rho l_{p}^{2}) \],

so that law (2), with \( C_{p} = 2p'_{rms} / (\rho l_{p}^{2}) \) as the coefficient of wall pressure pulsations, becomes (9)

\[ Nu = constl_{p}^{1/3} C_{p}^{1/3} Re^{2/3} \]

The relationship \( h \propto (p'_{rms})^{1/3} \) is in excellent agreement with experiments [10] on heat transfer from a cylinder and a square prism and [11] on the local heat transfer on a blunt flat plate generalized by the equation

\[ Nu = (0.21 + 0.23)C_{p}^{1/3} Re^{2/3} \]  

(3)

A large body of experimental research on turbulent separated flows provides support for the similarity laws and the “2/3-power law” for heat and mass transfer [12], [13].

In what follows we employ this approach to analysis of some very complex processes occurring in many technological applications. These processes manifest distinct properties, which allow them to be classified under turbulent separated flows.

II. IMPINGING JETS

Due to Kelvin-Helmholtz instability of thin shear layer initiated from the nozzle edge the large vortices are formed and grow in the downstream direction of the jet owing to their interactions and coalescence and then impinge on the wall inducing secondary peaks in the heat/mass transfer coefficient distribution. During this interaction of vortices with the wall strong instantaneous adverse pressure gradients are induced causing the unsteady flow separations of the wall boundary layer [14]. Jets attain very high intensities of turbulence by virtue of large-scale eddies produced in the shear layer. Additionally, as this is the case for a round impinging jet, the growing ring vortices surrounding the jet cause alternative acceleration and deceleration of the potential core resulting in potential core fluctuations [15].

These facts incite to the analysis of transfer processes in impinging jet in the context of turbulent separated flows. But namely the structure of the near-wall zone gives us the decisive argument in favor of this approach. The graphs at the left of Fig. 3 representing the mean velocity and temperature profiles in the near-wall zone of round impinging jet (\( H/d=2.0; Re = 35000 \)) [16] in terms of the characteristic TSF variables confirm the similarity laws (1) for the velocity and temperature profiles and analogy between physical mechanisms of transfer processes in the near-wall zone in impinging jets and turbulent separated flows.

Fig. 3 Velocity and temperature profiles in a round impinging jet

Fig. 4 represents experimental data of various authors for single round nozzles (SRN), single slot nozzles (SSN) and in-line array nozzles. All experimental values of exponents, in fact, represent “2/3 power law”.
The heat and mass transfer in turbulent flow over tube bundles depend on the flow pattern and the level of large-scale turbulence. The coefficient of heat/mass transfer associated with an individual tube is determined as well by its position in the bundle and the bundle arrangement (either staggered or aligned in the direction of fluid flow). The front face of a cylinder is under action of impinging jets. The boundary layers at the cylinder after separation from both sides behave as a free shear layer, which generates shedding downstream large-scale vortices forming a wake behind the cylinder. In the case of high Reynolds numbers the turbulent boundary layer separation occurs resulting in a significant narrowing of the turbulent wake and increase of shedding frequency. The length of the vortex formation region shortens with increasing in Reynolds number: in the Reynolds number range $10^3 \leq Re \leq 10^5$ for the in-line arrangement it is about $2.5D$ and for higher Reynolds numbers it decreases according to [20] and [21]. This shortening of the length of the vortex formation region is the cause of the heat transfer enhancement in the vortex region due to turbulence generated in the shear region of separated flow with the maximum value of the turbulence intensity of about 25 per cent and higher, and is an important parameter controlling the process. Experimental values of the maximum and average Nusselt number behavior as a function of Reynolds number for a cylinder in the wake of the first one strictly follow the “two-thirds power law” resulting from the similarity properties of transfer processes in TSF (Fig. 5).

Experimental data [20] on heat transfer from a cylinder from a second and third rows in a bundle of tubes agreed well with the “two-thirds power law” as well (Fig. 6). This behavior does not depend on the row number starting from the second one.

Pressure drop in the tube bank in cross flow as a measure of energy dissipation into the vortex generation in the separation zones behind every tube the wall pressure pulsations might be represented in terms of the base pressure coefficient, $C_{p} \propto Eu$.
The implementation of the similarity variables developed for turbulent separated flows above allows the generalization of experimental turbulent energy spectra, temperature and velocity profiles in a channel packed with spherical balls.

Fig. 8 represents spectral characteristics of the turbulent pulsations in a cell at the symmetry axis with the local Strouhal number, $\overline{f}D/\overline{u_m}$ along the x-axis and the normalized energy $\bar{u}^2_{\Delta f}$ in a broad frequency band, $\phi_{\Delta f} / \bar{u}^2_{\Delta f}$, where $\phi = \overline{u^2_{\Delta f}}$ is the energy density in the frequency band, along the y-axis.

These spectra of the longitudinal velocity fluctuations are strictly faithful to the “-2 power law”.

Experimental velocity profiles in the near-wall zone of a duct filled in with spherical balls [25], [26] are represented in Figure 9 in the turbulent separated flow similarity variables. The figure clearly demonstrates that velocity field has the similarity properties and it shows as well, the velocity profiles measured in the near-wall zone follow the similarity law, $u \propto y^{1/2}$ developed for the velocity profile in turbulent separated flows ($y$ here is the distance measured from the wall).

Experimental non-dimensional temperature profiles $\theta$ in the near-wall zone of cubically packed spheres at the seven most typical cross-sections of the duct are represented in [27] in terms of TSF similarity variables.

The experimental data support the similarity law, $T \propto y^{1/2}$ for the temperature profile developed for turbulent separated flows.
So the intensity of local pressure pulsations in packed beds can be estimated as \( p' \propto U^2 / d \) from which it follows that

\[
C_a \propto \frac{U^2}{d} \frac{d}{U^2} = \text{const}
\]

resulting in, bearing in mind a great quantity of pores in a packed bed, the asymptotic universal law

\[
\text{Nu} = \text{const} \, \text{Re}^{2/3} \, \text{Pr}^{\beta}
\]

The constant is to be determined from experimental data. The grain diameter or equivalent pore dimension can be used as a characteristic length scale in this law.

![Convective heat transfer of air and different liquids in tubes filled with spherical balls](image)

Experimentation data by the author and other authors are in excellent agreement with data by various authors, and with the asymptotic universal “two-thirds power law” (see for details references [28] – [30]), Fig.10.

VI. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
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<tbody>
<tr>
<td>( a )</td>
<td>thermal diffusivity, m²/s</td>
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<tr>
<td>( c_p )</td>
<td>specific heat, J/(kg·K)</td>
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<tr>
<td>( d )</td>
<td>ball, bubble, cylinder, and nozzle diameter, m</td>
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<tr>
<td>( \text{Eu} )</td>
<td>( = \Delta p / (pU^2) ) - Euler number, [-]</td>
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<tr>
<td>( g )</td>
<td>acceleration of gravity, m/s²</td>
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<tr>
<td>( H )</td>
<td>distance from a nozzle to a target plate, m</td>
</tr>
<tr>
<td>( h )</td>
<td>heat/mass transfer coefficient, W/(m²·K)</td>
</tr>
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<td>( k )</td>
<td>longitudinal wave-number, m⁻¹</td>
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<tr>
<td>( L )</td>
<td>characteristic size, m</td>
</tr>
<tr>
<td>( N )</td>
<td>near-wall thickness, m</td>
</tr>
<tr>
<td>( \text{Nu} )</td>
<td>( = hL/k ) - Nusselt number, [-]</td>
</tr>
<tr>
<td>( p )</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>( \text{Pr} )</td>
<td>( = v/a ) - Prandtl number, [-]</td>
</tr>
<tr>
<td>( q )</td>
<td>wall heat flux, W/m²</td>
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<tr>
<td>( r )</td>
<td>radial distance from a stagnation point, m</td>
</tr>
<tr>
<td>( \text{Re} )</td>
<td>( = UL/v ) - Reynolds number, [-]</td>
</tr>
<tr>
<td>( \text{Sc} )</td>
<td>( = D/\nu ) - Schmidt number, [-]</td>
</tr>
<tr>
<td>( \text{Sh} )</td>
<td>( = hL/D ) - Sherwood number, [-]</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature, °C</td>
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<tr>
<td>( u )</td>
<td>velocity, m/s</td>
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<tr>
<td>( U )</td>
<td>characteristic velocity, m/s</td>
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<tr>
<td>( y )</td>
<td>distance from a wall</td>
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<tr>
<td>( \alpha )</td>
<td>-kinematical pressure gradient, m/s²</td>
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<tr>
<td>( \kappa )</td>
<td>thermal conductivity, W/(m·K)</td>
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<td>( \nu )</td>
<td>kinematic viscosity m²/s</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density, kg/m³</td>
</tr>
<tr>
<td>( \tau )</td>
<td>wall shear stress, m²/s²</td>
</tr>
</tbody>
</table>

**Subscripts**

- \( f \) - fluid
- \( \text{max} \) - corresponds to a minimal value
- \( \text{min} \) - corresponds to a maximal value
- \( w \) - wall

REFERENCES


