Performance Analysis of Space-Time Trellis Coded OFDM System

Yi Hong and Zhao Yang Dong

Abstract—This paper presents the performance analysis of space-time trellis codes in orthogonal frequency division multiplexing systems (STTC-OFDMs) over quasi-static frequency selective fading channels. In particular, the effect of channel delay distributions on the code performance is discussed. For a STTC-OFDM over multiple-tap channels, two extreme conditions that produce the maximum possible diversity gain are highlighted. The analysis also proves that the corresponding coding gain increases with the maximum tap delay. The performance of STTC-OFDM, under various channel conditions, is evaluated by simulation. It is shown that the simulation results agree with the performance analysis.

Keywords: Space-time trellis code, OFDM, delay profile.

I. INTRODUCTION

Space-time trellis coding (STTC) technique has been proposed to achieve both the diversity and coding gains in multi-input multi-output (MIMO) fading channels [1]. The orthogonal frequency division multiplexing (OFDM) technique is widely used to combat intersymbol interference (ISI) by transforming a frequency selective fading channel into a set of parallel correlated flat fading channels. Recently, various STTCs in OFDM systems, referred to as STTC-OFDMs, in frequency selective fading channels have been investigated [2][3][4][5].

The diversity gain of STTC-OFDM systems is investigated in [3] and [4]. It was pointed out in [4] that the performance of space-time coded OFDM systems depends on the channel delay profile. To reduce this dependence and simplify the code design, ideal interleaving was usually used to scramble the coded symbols.

In contrast to the analysis in [4] with ideal interleaving, the worst case, where no interleaving is employed in the transmitter, was considered in [5], as the optimization for the worst case can provide a robust system design [6]. For STTC-OFDMs in quasi-static frequency selective fading channels, the maximum possible diversity gain is the product of the number of transmit antennas $n_T$, the number of receive antennas $n_R$ and the number of the channel taps $L$ [3]. Since the low memory order STTCs in OFDM systems cannot achieve the maximum possible diversity gain [3], the performance of STTC-OFDM is analyzed in terms of the coding gain [5]. However this analysis only applies to the STTCs with the minimum error event length $p_{min}$ of 2. This results in a restriction since some STTCs, especially high memory order codes, have the minimum error event length $p_{min}$ greater than 2 [6].

In this paper, we will address this issue. We extend the analysis in [5] to the general case, in which STTCs have the minimum error event length $p_{min}$ no less than 2. Then the effect of the channel delay distribution on the coding gain is discussed. The code performance of STTC-OFDM over quasi-static frequency selective fading channels is evaluated by simulations. It is shown that the simulation results agree with the performance analysis.

This paper is organized as follows. Section II introduces the system model. Section III presents the pairwise error probability (PWEP) of the STTC-OFDMs in quasi-static frequency selective fading channels. In Section IV, the code performance of STTC-OFDM based on the diversity and coding gains has been investigated. In particular, the effect of various channel delay distributions on the code performance is discussed in terms of the coding gain. Section V presents the simulation results. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider an OFDM system with $n_T$ transmit antennas, $n_R$ receive antennas and $K$ subcarriers in a quasi-static frequency selective fading channel. Each OFDM frame consists of $n_T K$ M-PSK STTC symbols, where the encoded symbol $x_i(k), i \in \{1, 2, ..., n_T\}, k \in \{1, 2, ..., K\}$, is transmitted on the $k$-th subcarrier from the $i$-th transmit antenna. After matched filtering, sampling and fast Fourier transform (FFT), the received signal at the $j$-th receive antenna and on the $k$-th subcarrier is given by

$$r_j(k) = \sum_{i=1}^{n_T} H_{ij}(k)x_i(k) + n_j(k), (1)$$

where $H_{ij}(k), j \in \{1, 2, ..., n_R\}$, denotes the channel frequency response from the transmit antenna $i$ to receive antenna $j$ and subcarrier $k$, $n_j(k)$ is the noise component at receive antenna $j$ through subcarrier $k$, which is an independent complex Gaussian random variable with zero-mean and variance $N_0/2$ per dimension.

The quasi-static fading channel in this paper is assumed to be static during one OFDM frame but varies from one frame to another. The fading channels between different transmit and receive antennas are assumed to be uncorrelated. Assuming the fading channel has $L$ non-zero taps, the time-domain channel impulse response can be modeled by an $L$ tap-delay line [7].

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Dr. Yi Hong is with Institute for Telecommunications Research, University of South Australia. Email: Yi.Hong@unisa.edu.au
Dr. Zhao Yang Dong is with School of Information Technology and Electrical Engineering University of Queensland, Australia. Email: zdong@itee.uq.edu.au.
The channel impulse response from the $i$-th transmit antenna to the $j$-th receive antenna is expressed as [3]

$$h_{ij}(\tau, t) = \sum_{l=0}^{L-1} \tilde{h}_{ij}(l, t) \delta \left( \tau - \frac{nt}{K \Delta f} \right),$$

(2)

where $\delta(\cdot)$ denotes the Dirac delta function, $\tilde{h}_{ij}(l, t)$ denotes the complex amplitude of the $l$-th non-zero tap with the delay of $t$. The $\tilde{h}_{ij}(l, t)$'s are modeled by the wide-sense stationary (WSS) and narrowband complex Gaussian processes, which are independent for different paths with $E[|\tilde{h}_{ij}(l, t)|^2] = \sigma_l^2$. We normalize the channel power such that we have $\sum l \sigma_l^2 = 1$.

In (2), $n_t$ is the normalized time delay for the $l$-th tap and it is given by $n_t = t_l K \Delta f = t_1 T_s$, where $l \in \{0, 1, \ldots, L-1\}$, $\Delta f$ is the subcarrier separation, and $T_s$ is the sampling interval of the OFDM systems. We call the delays $t_0, t_1, \ldots, t_1, \ldots, t_{L-1}$ of the $L$ non-zero taps the channel delay distribution. Let $t_{L-1}$ denote the delays of the $l_2$-th and $l_1$-th taps, respectively, where $l_1, l_2 \in \{0, 1, \ldots, L-1\}, l_2 > l_1$ and $t_{L-1} > t_l$. The interval $t_{L-1} - t_l$ is assumed to be not less than the sampling interval $T_s$.

Since the performance analysis is done within one OFDM frame, the time index $t$ in (2) is omitted hereafter. A cyclic prefix (CP) with the length of $T_{cp}$, where $T_{cp} > L-1$, is appended to each OFDM frame to avoid the ISI. With proper cyclic extension and tolerable leakage, the channel frequency response between the $i$-th transmit antenna and the $j$-th receive antenna is given by [3]

$$H_{ij}(k) = \sum_{l=0}^{L-1} \tilde{h}_{ij}(l) e^{-j2\pi kn_t/K}$$

(3)

$$= \mathbf{h}_{ij}^*(\mathbf{w}(k)),$$  

(4)

where $\mathbf{h}_{ij} = [\tilde{h}_{ij}(0), \tilde{h}_{ij}(1), \ldots, \tilde{h}_{ij}(L-1)]^*$ is the channel vector, $\mathbf{w}(k) = [e^{-j2\pi kn_t/K}, e^{-j2\pi kn_t/K}, \ldots, e^{-j2\pi kn_t/L-1/K}]^*$ is the FFT coefficient vector, $*$ and $T$ denote the Hermitian and transpose operation, respectively. That the delay of the first channel tap is $t_0 = 0$. Thus, the FFT coefficients can be rewritten as

$$\mathbf{w}(k) = [1, e^{-j2\pi k1\Delta f}, e^{-j2\pi k2\Delta f}, \ldots, e^{-j2\pi kL-1\Delta f}]^T.$$  

(5)

### III. PAIRWISE ERROR PROBABILITY (PWE) OF STTC-OFDM

Assuming that the perfect channel state information (CSI) is known to the receiver, a codeword $x = (x(1), \ldots, x(k), \ldots, x(K))$, where $x(k) = (x_1(k), x_2(k), \ldots, x_{n_T}(k))$, $k \in \{1, 2, \ldots, K\}$, is transmitted and erroneously decoded as $\hat{x} = (\hat{x}(1), \ldots, \hat{x}(k), \ldots, \hat{x}(K))$, where $\hat{x}(k) = (\hat{x}_1(k), \hat{x}_2(k), \ldots, \hat{x}_{n_T}(k))$. The pairwise error probability (PWE) of deciding erroneously using the maximum likelihood decoder (MLD), conditioned on $H_{ij} = [H_{ij}(1), H_{ij}(2), \ldots, H_{ij}(K)]$, $i \in \{1, 2, \ldots, n_T\}$, $j \in \{2, \ldots, n_R\}$, is upper bounded by [6]

$$P_r(x \rightarrow \hat{x} | H_{ij}) \leq \exp \left( -\frac{E_s}{4N_0} \sum_{j=1}^{n_R} h_j^* \mathbf{D}(x, \hat{x}) h_j \right),$$

(6)

where

$$h_j^* = [h_{ij}^*, h_{ij}^*, \ldots, h_{ij}^*]_{1 \times n_{T_{RT}}}$$

and $\mathbf{D}(x, \hat{x}) = \sum_{k=1}^{L} \mathbf{W}(k) \Delta(k) \Delta^*(k) \mathbf{W}^*(k)$,

(7)

$$\Delta(k) = \begin{bmatrix} x_1(k) - \hat{x}_1(k) \\ x_2(k) - \hat{x}_2(k) \\ \vdots \\ x_{n_T}(k) - \hat{x}_{n_T}(k) \end{bmatrix}_{n_{T_{RT}} \times 1}$$

and $E_s$ is the energy per symbol at each transmit antenna.

Averaging the conditioned PWE in (6) with respect to the Rayleigh fading coefficients, the upper bound of the averaged PWE is given by [3]

$$P_r(x \rightarrow \hat{x}) \leq \left( \prod_{n=1}^{\xi} \left( 1 + \lambda_n E_s/(4N_0) \right) \right)^{-n_R}$$

(8)

$$= \left( \det \left( \mathbf{D}_o(x, \hat{x}) \right) \right)^{-\frac{n_R}{\xi}} \alpha^{-\xi n_R}$$

where $\xi$ is the rank of matrix $\mathbf{D}(x, \hat{x})$, $\lambda_n$, $n \in \{1, 2, \ldots, \xi\}$, are the non-zero eigenvalues of the matrix $\mathbf{D}(x, \hat{x})$, $\alpha = E_s/(4N_0)$, $\mathbf{D}_o(x, \hat{x}) = \frac{1}{2} \mathbf{I} + \mathbf{D}(x, \hat{x})$, $\mathbf{I}$ is the identity matrix, and $\det(\mathbf{A})$ denotes the determinant of the matrix $\mathbf{A}$. As discussed in [1], we call the minimum value of $\left( \det \left( \mathbf{D}_o(x, \hat{x}) \right) \right)^{1/\xi}$ the coding gain and the minimum value of $\xi n_R$ the diversity gain of the system.

In (7), $\Delta(k) \Delta^*(k)$ is a rank-one matrix [6]. If the symbols of the codewords $x$ and $\hat{x}$ corresponding to the $k$-th subcarrier in the given OFDM frame are the same, e.g. $x(k) = \hat{x}(k)$, $\Delta(k) \Delta^*(k)$ is an all zero matrix. Otherwise, we obtain $\Delta(k) \Delta^*(k) \neq 0$.

Let $p$ denote the length of the pairwise error event path, which is the number of the time instances in the code trellis such that $\Delta(k) \Delta^*(k) \neq 0$. The minimum value of $p$ over all possible codeword pairs is denoted by $p_{\min}$. Note that $\xi = \text{rank}(\mathbf{D}(x, \hat{x}))$. We thus obtain $\min_{x, \hat{x}} \xi \leq \min(p_{\min}, n_T L)$ [6]. To achieve the maximum possible diversity gain $n_R n_T L$, it requires that $p_{\min} \geq n_T L$. This relation states that increasing the number of channel taps $L$ results in a larger diversity gain. Otherwise, if $p_{\min} < n_T L$, the maximum possible diversity gain $n_R n_T L$ cannot be obtained [3]. As a consequence, to minimize the error probability, the coding gain, or equivalently, the minimum determinant $\left( \det \left( \mathbf{D}_o(x, \hat{x}) \right) \right)^{1/\xi}$ needs to be maximized over all codeword pairs.

Considering (7), it is obvious that both the matrix $\mathbf{D}(x, \hat{x})$ and its individual matrices $\mathbf{W}(k) \Delta(k) \Delta^*(k) \mathbf{W}^*(k)$, where
\(k \in \{1, 2, ..., K\}\), are non-negative symmetric Hermitian. According to Minkowski inequality [8], the determinant of the matrix \(\tilde{D}_\alpha(x, \tilde{x})\) has the following property:

\[
\det (\tilde{D}_\alpha(x, \tilde{x})) = \det \left( \frac{1}{\alpha} I + D(x, \tilde{x}) \right)
\]

\[
= \det \left( \frac{1}{\alpha} I + \sum_{k=1}^{K} W(k) \Delta(k) \Delta^*(k) W^*(k) \right)
\]

\[
\geq \det \left( \frac{1}{\alpha} I + \sum_{k=1}^{K-1} W(k) \Delta(k) \Delta^*(k) W^*(k) \right)
\]

\[
\geq ... \geq \det \left( \frac{1}{\alpha} I + \sum_{k=1}^{p_{\text{min}}} W(k) \Delta(k) \Delta^*(k) W^*(k) \right).
\]

Let \(\det (\tilde{D}_\alpha(x, \tilde{x}))\) denote the minimum determinant of the matrix \(\tilde{D}_\alpha(x, \tilde{x})\) with the minimum length of the pairwise error event paths, \(p_{\text{min}}\). Therefore, to maximize the coding gain, the value of \(\det (\tilde{D}_\alpha(x, \tilde{x}))\) needs to be maximized.

IV. THE EFFECT OF CHANNEL DELAY DISTRIBUTION ON THE CODE PERFORMANCE

In order to investigate the effect of the channel delay distributions on the code performance, we assume that the powers of the multi-paths are fixed while their relative delays are variable.

Consider the determinant \(\det (\tilde{D}_\alpha(x, \tilde{x}))\) with \(n_T \geq 2\) and \(L \geq 2\) in (10) [8], where \(t_L - t_1\) is the time interval between the \(1\)-th and \(2\)-th taps, \(t_2 > t_1, t_2 > t_1, t_1, t_2 \in \{1, ..., n_T\}\), and \(k_1, k_2 \in \{1, ..., p_{\text{min}}\}\) and \(k_2 > k_1\) in (10). The coefficients \(\beta, \eta\) are positive real values, which are determined by the STTC only. Note that the determinant consists of two parts, PART I and PART II, where PART I is a positive constant for a given STTC and only PART II is related to the channel delays. In order to evaluate the effect of channel delay distribution on the performance of the given STTC-OFDM, we focus on PART II.

In PART II, defining \(\varpi = \pi (k_2-k_1) \Delta f\), we have

\[
\sin^2(\pi (k_2-k_1) \Delta f (t_z)) = \sin^2(\varpi (t_z)).
\]

Note that the maximum delay of the channel is \(t_{L-1}\), where \(t_{L-1} < T_c\). Considering that \(t_L - t_1 \leq t_{L-1}\) and \(k_2 - k_1 \leq p_{\text{min}} - 1\), we have

\[
\varpi (t_L - t_1) \leq \pi (p_{\text{min}} - 1) \Delta f / t_{L-1},
\]

with \(p_{\text{min}} = \lceil v/2 \rceil + 1\) for STTCs with the memory order \(v\) [6, p. 122], where \(\lceil v/2 \rceil\) denotes the maximum integer not greater than \(v/2\). The maximum delay for indoor communications environment, such as Wireless LAN, is less than 500 ns [9][10]. For most wireless OFDM systems and all the STTCs designed in the literature [1][6][11], we have \(1^1\)

\(1^1\)For Wireless LAN and HiPerLAN OFDM systems [12][13], the value of \(\varpi (t_L - t_1)\) is in the range of \((0, 0.45\pi]\). For wireless OFDM systems in the research literature, such as [2][4][6], the value of \(\varpi (t_L - t_1)\) is in the range of \((0, 0.46\pi]\).

It is obvious that the value of \(\sin^2(\varpi (t_L - t_1))\) increases monotonically with \(\varpi (t_L - t_1)\) in the range of \((0, \pi/2]\). Now we can rewrite PART II as

\[
\text{PART II} = \sum_{k_2-k_1=1}^{p_{\text{min}}-1} \gamma_{k_1, k_2} \left( \sum_{t_1, t_2=0}^{L-1} \sin^2(\varpi (t_L - t_1)) \right).
\]

Considering the two-tap \((L = 2)\) channels, (14) can be further rewritten as

\[
\text{PART II} = \sum_{k_2-k_1=1}^{p_{\text{min}}-1} \gamma_{k_1, k_2} \sin^2(\varpi (t_L)).
\]

Then we have the following observation.

**Observation 1:** Consider a given STTC-OFDM over the two-tap channels. Note that the positive constant \(\gamma_{k_1, k_2}\) is determined by the STTC and the value of PART II increases with \(\varpi (t_L)\) in the range of \((0, \pi/2]\). Hence, the minimum determinant \(\det (\tilde{D}_\alpha(x, \tilde{x}))\) increases with the maximum tap delay \(t_L\).

Consider the given STTC-OFDM with the positive constant coefficients \(\gamma_{k_1, k_2}\), where the channels with \(L\) taps, where \(L > 2\). To simplify the analysis, we assume that the maximum delay of the channel \(t_{L-1}\) is fixed first. Then we have the following lemma.

**Lemma 1:** Since the time interval \(t_L - t_1\) is within the open set \((0, \pi/2]\), where \(t_2, t_1 \in \{1, ..., L - 2\}\), and \(t_2 > t_1\), the cost function \(\sum_{k_2-k_1=1}^{p_{\text{min}}-1} \gamma_{k_1, k_2} \left( \sum_{t_1, t_2=0}^{L-1} \sin^2(\varpi (t_L - t_1)) \right)\) is a non-decreasing function of time differences. The points that the maximum value of this cost function are located at the extreme points (This results from the application of the extreme points [14].)

**Proof:** Consider the objective function

\[
f(\Delta t_Z) = \sum_{|k_2-k_1|=1}^{p_{\text{min}}-1} \gamma_{k_1, k_2} \left( \sum_{t_1, t_2=0}^{L-1} \sin^2(\varpi (t_Z - t_1)) \right),
\]

where \(\Delta t_Z = t_2 - t_1, z = 1, 2, ..., Z\) and \(Z \) is the total number of all the possible time differences \(t_2 - t_1\). We thus have the following non-linear programming problem:

\[
\text{maximize } f(\Delta t_Z),
\]

subject to \(\Delta t_Z \in [T_c, T_{L-1} - T_c]\), and \(f(\Delta t_Z) \in (0, 1)\).

Let \(\hat{x}_Z\) be a maximizing solution for the problem \(\{\max f(\Delta t_Z) : t_2 \in S\}\), provided that \(\Delta t_Z \in S\) and \(f(\Delta t_Z) \geq f(\Delta t_Z)\), where \(S = \{\Delta t_Z : \Delta t_Z \leq T_{L-1} - T_c\}\) is a special case of a polyhedral set [14, p. 54]. In such a case, according to Theorem 3.4.7 [14, p. 107], we say that a maximum solution \(\Delta t_Z\) exists and \(\Delta t_Z\) is an extreme point of \(S\). Since the time interval \(t_L - t_1\) is within the open set \((0, \pi/2]\) and the objective function is a non-decreasing function of time differences, theoretically, the maximum solutions are given by \(\Delta t_1 = ... = \Delta t_Z = T_{L-1} - T_c\) [14, p. 55].
\[
\det \left( \tilde{D}_\alpha(x, \tilde{x}) \right) |_{p_{\text{min}}} = \frac{1}{\alpha^{N/2}} + \sum_{t=1}^{\infty} \sum_{k=1}^{P_{\text{max}}} \beta |\Delta_t(k)|^2 + \cdots + \sum_{k_2-k_1=1}^{P_{\text{max}}-1} \gamma_{k_1,k_2} \left( \sum_{l=1}^{L-1} \sin^2(\pi (k_2-k_1) (t_{l_2}-t_{l_1}) \Delta f) \right), \quad (10)
\]

**Property 1:** For a given STTC-OFDM over the channels with the same maximum delay \(t_{L-1}\), the minimum determinants in (10) under the channel delay distributions of (16) and (17) should be same.

**Proof:** Let \(C\) and \(D\) denote the values of

\[
\sum_{l=1}^{L-1} \sin^2(\varpi (t_{l_2}-t_{l_1})) \quad \text{under the channel delay distributions of (16) and (17), respectively. As shown in Fig. 1, defining}
\]

\[
a_l = \begin{cases} t_{l-1} - (l-2)T_s, & l = 1, \\ T_s, & l = 2, \ldots, L - 1. \end{cases}
\]

and

\[
b_l = \begin{cases} t_{l-1} - (l-2)T_s, & l = 1, \\ T_s, & l = 2, \ldots, L - 2. \end{cases}
\]

respectively. Then, \(C\) and \(D\) are rewritten as

\[
C = \sum_{l=1}^{L-1} \sin^2(\varpi a_l) + \sum_{l=1}^{L-2} \sin^2(\varpi (a_{l+1} + a_l)) + \cdots + \sin^2(\varpi (a_1 + \ldots + a_{L-1})),
\]

and

\[
D = \sum_{l=1}^{L-1} \sin^2(\varpi b_l) + \sum_{l=1}^{L-2} \sin^2(\varpi (b_{l+1} + b_l)) + \cdots + \sin^2(\varpi (b_1 + \ldots + b_{L-1})),
\]

respectively.

In (18) and (19), we can see that \(a_l\) and \(b_l\), where \(l = 1, 2, \ldots, L - 1\), take the same set of values but in different orders. Thus, it is clear that

\[
\sum_{l=1}^{L-1} \sin^2(\varpi a_l) = \sum_{l=1}^{L-1} \sin^2(\varpi b_l),
\]

\[
\sum_{l=1}^{L-2} \sin^2(\varpi (a_{l+1} + a_l)) = \sum_{l=1}^{L-2} \sin^2(\varpi (b_{l+1} + b_l)),
\]

\[
\vdots
\]

\[
\sin^2(\varpi (a_1 + \ldots + a_{L-1})) = \sin^2(\varpi (b_1 + \ldots + b_{L-1})).
\]

Then, we have

\[C = D.\]

In this case, for the given STTC-OFDM, the minimum determinants in (10) should be same.

**Property 2:** For a given STTC-OFDM under the channel delay distributions of (16) or (17), the corresponding minimum determinant in (10) increases with the maximum delay \(t_{L-1}\).
it is clear that

Similarly, defining $A = \sum_{l=1, j_2=0}^{L-1} \sin^2(\varpi(t_2 - t_1))$ with delays of $t_1, t_2, ..., t_{L-1}(1)$, we have

$$A = \sum_{l=0}^{L-2} \sin^2(\varpi(t_{L-1}(1) - t_1)) + \sum_{l=0}^{L-3} \sin^2(\varpi(t_{L-2} - t_1))$$

$$+ ... + \sum_{l=0}^{L-3} \sin^2(\varpi(t_2 - t_1)) + \sin^2(\varpi(t_1)).$$

Similarly, defining $B = \sum_{l_1, j_2=0}^{L-1} \sin^2(\varpi(t_2 - t_1))$ with delays of $t_1, t_2, ..., t_{L-1}(2)$, we have

$$B = \sum_{l=0}^{L-2} \sin^2(\varpi(t_{L-1}(2) - t_1)) + \sum_{l=0}^{L-3} \sin^2(\varpi(t_{L-2} - t_1))$$

$$+ ... + \sum_{l=0}^{L-3} \sin^2(\varpi(t_2 - t_1)) + \sin^2(\varpi(t_1)).$$

Then,

$$A-B = \sum_{l=1}^{L-1} \sin^2(\varpi(t_{L-1}(1)-t_1)) - \sum_{l=0}^{L-1} \sin^2(\varpi(t_{L-1}(2)-t_1)).$$

As we have $t_{L-1}(1) > t_{L-1}(2)$ and $\varpi(t_2 - t_1) \in (0, \pi/2)$, it is clear that

$$\sin^2(\varpi(t_{L-1}(1)-t_1)) > \sin^2(\varpi(t_{L-1}(2)-t_1)),$$

where $l \in \{0, 1, ..., L-2\}$. Hence, we have

$$A > B.$$

This means the minimum determinant $\det\left(\tilde{D}_\alpha(x, \tilde{x})\right)|_{p_{\text{min}}}$ increases with the maximum delay $t_{L-1}$ under the channel delay distribution of (17). Similarly, according to Property 1, we can prove that the above conclusion is true for the STTC-OFDM under the channel delay distribution of (16).

V. SIMULATIONS

The code performance of STTC-OFDMs over frequency selective fading channels with various channel delay distributions is evaluated by simulations. In the simulations, the OFDM system is assumed to have a bandwidth of 1 MHz and 256 OFDM subcarriers. The subcarrier separation $\Delta f$ is 3.9KHz. The OFDM frame duration is 256$\mu$s and a guard interval is 40$\mu$s. The quasi-static frequency selective fading channels with equal gain taps but different delay distributions are assumed. Two transmit and three receive antennas are employed in the STTC-OFDMs.

Fig. 2 shows the minimum determinant $\det\left(\tilde{D}_\alpha(x, \tilde{x})\right)|_{p_{\text{min}}}$ of 4 and 8-state 4-PSK STTCs [11] in the OFDM system over the two-tap channels with different delays $t_1-t_0$, where $t_1 \in [5, 39\mu s]$ and $t_0 = 0$. In this case, we have $\alpha \approx 1$ and $p_{\text{min}} = 2$. Note that $\varpi t_1 \in [0.06, 0.48]$. According to Observation 1 in SECTION 4, the minimum determinant $\det\left(\tilde{D}_\alpha(x, \tilde{x})\right)|_{p_{\text{min}}}$ increases with the delay $t_1$, as illustrated in Fig. 2.

The frame error rate (FER) performance of the 8 and 64-state 4-PSK STTCs [11] in the OFDM system over the two-tap channels with delays of $t_1=5\mu s$ and $t_1=39\mu s$ is shown in Fig. 3. Note that the 8-state ($p_{\text{min}} = 2$) and 64-state ($p_{\text{min}} = 4$) codes have $\varpi t_1 \in [0.06, 0.48]$ and $\varpi t_1 \in [0.18, 1.4]$, respectively. It is shown that the 8 and 64-state codes in the fading channel with a delay of 39$\mu$s outperform the corresponding codes in the channel with a delay of 5$\mu$s by 0.5 dB and 1.3 dB at the FER of $10^{-3}$, respectively, which agrees with the previous discussion.

Fig. 3 also shows that the 64-state STTC is more sensitive to various channel delays than the 8-state one, as the 64-state code has a larger value of $p_{\text{min}}$ and therefore a larger number of positive additive terms $\gamma_{k_1,k_2}$ and $(\sum_{i,j} \sin^2(\varpi(t_1)))$ in (14).

Fig. 4 shows the performance of 16-state 4-PSK STTC [11] in the OFDM system over three-tap frequency selective fading channels with different delay distributions. Note that $p_{\text{min}} = 3$ for the 16-state code and the positive coefficients $\gamma_{k_1,k_2}$ defined in PART II are constant for the given code. From Fig. 4, we can see that the STTC-OFDM under both the channel delay distributions of $(t_0 = 0\mu s, t_1 = 35\mu s, t_2 = 39\mu s)$ and $(t_0 = 0\mu s, t_1 = 4\mu s, t_2 = 39\mu s)$, corresponding to (16) and (17), respectively, has the same performance and outperforms the one under the channel delay distribution of $(t_0 = 0\mu s, t_1 = 20\mu s, t_2 = 39\mu s)$ by 0.4 dB at the FER of $10^{-3}$. In addition, it is shown that the STTC-OFDM under the channel delay distributions (17) with the maximum delay of $t_2 = 39\mu s$ outperforms the one under the delay distribution (17) with the maximum delay of $t_2 = 20\mu s$ by 1.1 dB at the FER of $10^{-3}$. The simulation results are all consistent with...
the analysis in Section IV.

VI. CONCLUSION

In this paper, we consider the STTC-OFDM systems with no interleavers over quasi-static frequency selective fading channels. In order to provide a robust system design, we presented the performance analysis of STTC-OFDMs under various channel conditions in terms of the coding gain. In particular, the effect of various channel delay distributions on the code gain is investigated. Through this analysis, we point out two extreme conditions that produce the largest minimum determinant for a STTC-OFDM over multiple-tap channels. The analysis also proves that the corresponding coding gain increases with the maximum tap delay. The performance of STTC-OFDM under various channel conditions is evaluated by simulation. It is shown that 1) the minimum determinant of STTC in OFDM systems increases with the maximum tap delay of the channel; 2) the STTC-OFDM under two extreme channel conditions outperforms that under other channel conditions; and 3) the high memory order STTCs are more sensitive to the channel delays since they have a larger value of error event length $p_{min}$. Hence, we can see that all the simulation results are consistent with the performance analysis.

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Yi Hong received PhD degree in School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, Australia, in 2004. From 2004, she is a research fellow in Institute for Telecommunications Research, University of South Australia. Her current research interest includes wireless communications, error control coding and space-time coding techniques, multiuser detection and multiple-input-multiple-output (MIMO) systems.

Zhao Yang Dong received his PhD in Electrical and Information Engineering from The University of Sydney, Australia in 1999. He is now a senior lecturer at the School of Information Technology and Electrical Engineering, The University of Queensland, Australia. His research interest includes systems and control research, evolutionary computing, artificial intelligence and their applications in power engineering, and signal processing.