Abstract—We suggest a novel method to incorporate long-
term redundancy (LTR) in signal time domain compression
methods. The proposition is based on block-sorting and curve
simplification. The proposition is illustrated on the ECG
signal as a post-processor for the FAN method. Test
applications on the new so-obtained FAN+ method using the
MIT-BIH database show substantial improvement of the
compression ratio-distortion behavior for a higher quality
reconstructed signal.

Keywords—ECG compression, Long-term redundancy,
Block-sorting, Curve Simplification.

I. INTRODUCTION

D igital signals compression is a requirement in modern
applications for efficient storage and or transmission over
the network. Basically, compression consists in the principle
of eliminating redundancy. Ideally, we desire the smallest
representation of the original data as a result of compression,
i.e. a fully decorrelated form where ‘no structure in the data is
discernible’ [1]. This objective is yet to be attained since all
existing compression methods exploit partially the existing
redundancy in the data. For instance in the case of ECG
signal, Jalaledine et al [2] classify compression methods in
three categories: Direct data compression methods, like
AZTEC [3] and FAN [4]; transform methods, e.g. [5] and
parameter extraction methods, e.g. [6]. From another point of
view, signals compression can be achieved in the time domain
or in the frequency domain. This study concentrates
particularly on a specific class of time domain compression
methods known as piecewise linear approximation (PLA).
This approach is widely used in many computer areas
including time series representation for analysis, compression
and data mining and, although no assumption is made on the
nature of the signal, we concentrate specifically on the ECG
signal.

Time domain compression methods (PLA) have in common
the principle of reducing redundancy through selection of a set
of characteristic points (CP) on the signal trace. These points
are selected based on wisely predetermined rules so as the
most significant points are selected. The so-obtained set of
CPs stand for the reduced form of representation of the
original signal. The reconstruction process is achieved through
interpolation between successive CPs in this set. This type of
lossy compression is acceptable in many biological signals,
including the ECG.

It can be noted that in PLA methods, the CPs are selected
on the basis that all samples between two successive CPs are
correlated. Therefore, in this type of compression, short-term
redundancy (STR) only is considered. Yet, in biological
signals (especially the quasi-periodic signals like ECG), in
addition to short-term redundancy (within a period), there
exists also long-term redundancy (LTR, between periods).
This is the main reason for existing PLA methods low
compression performances. But, since these methods consider
few points only at a time, they have the advantage of
possessing linear temporal complexity allowing effective real-
time implementation.

The objective of the proposed method is to incorporate
LTR in this category of methods. Our objective is clearly
justified since the so-enhanced methods would have competing
compression performances and at the same time
would allow real-time implementation. Our proposition is
based on two main tools: block-sorting through a variant of
the quite recent Burrows-Wheeler algorithm (BWA) [7] and
curve simplification through a variant of the Douglas-Peucker
algorithm [8]. Our algorithm is coupled as a post-processing
step to the FAN method in the specific case of ECG signal, as
an illustration.

The rest of this work is organized as follows. In section 2,
tools and methods used in the proposed approach are
presented. In section 3 experiments on the novel method are
illustrated on selected ECG records from the MIT-BIH
II. MATERIALS AND METHODS

A. Curve Simplification

Given a discrete curve, formally expressed by polyline
\( P^n(p_i), i=1..N, \) where \( p_i=(x_i,y_i), \) with \( x_i \) the horizontal coordinate and \( y_i \) the vertical coordinate of \( p_i, \) the simplification of \( P \) consists in the computation of another polyline \( Q^n(q_j), j=1..M, \) satisfying the following conditions:

1. \( M<N; \)
2. \( q_1=p_1 \) and \( q_M=p_N; \)
3. Let \( \|P\|, \|Q\|<\varepsilon, \) with \( \varepsilon>0, \) a preset threshold on the tolerance of the simplification error.

If \( Q \) satisfies conditions a-c, it is said to be an \( \varepsilon \)-Simplification of \( P. \)

The compression ratio associated with the STR reduction (first simplification), as expressed in terms of number of samples reduction, is given by Eq. (3):

\[
q_k = \text{Arg Max}_{q_k \in [p_{i-1}, p_i]} \| q_k \|
\]

with

\[
\| q_k \| = \sqrt{\sum_{i=0}^{\infty} (y_i - \hat{y}_i)^2}
\]

In Eqs. (2) and (4), \( y_i \) is the magnitude of point \( q_i \) and \( \hat{y}_i \) that of \( \hat{q}_i. \)

B. Compression Through Block-Sorting

Block sorting is quite a recent trend as fare as compression is concerned. The Burrows and Wheeler Algorithm (BWA) [7] is one of the first compression algorithms using this technique. The original BWA is a lossless compression method, reported to yield excellent results on images, text and sound [1].

In the case of ECG, the main features P, QRS and T are of clinical importance. It can be shown also that for a binary tree of segmentation where the most perceptible points are selected in the upper levels, Fig. 2 illustrates this property in the case reported in Fig. 1. It is easy to derive from the binary tree of segmentation that the process temporal complexity is of \( \sim O(N\log(N)) \) order. The simplification algorithm is formally described in Fig. 3.
The reconstructed magnitude \( \hat{y} \) as an approximation to the original magnitude \( y \) is realized by linear interpolation between successive CPs. This step yields the output tuples \((x_i, \hat{y}_i)\) for \( i = 1..N \), where \( \hat{y}_i \) is computed by Eq. 8.

\[
\hat{y}_i = \begin{cases} 
  y_i', & k = x_i' \quad i = 1.L \\
  y_i' + (k-x_i') \frac{y_i' - y_{i-1}'}{x_i' - x_{i-1}} , & k = x_i' + L, x_i' - L \quad i = 1.L-1 \\
  \hat{y}_i', & k = x_i \\
  \hat{y}_i + (k-x_i) \frac{\hat{y}_i - \hat{y}_{i-1}}{x_i - x_{i-1}} , & k = x_i + L, x_i + 1 - L \quad i = 1.M-1
\end{cases}
\]

### III. APPLICATION TO ECG SIGNAL

We apply particularly the proposed method to the ECG signal. The ECG (Electrocardiogram) is a biological signal reflecting the heart activity. Samples \( y_i \) of this signal represent the difference in potential as measured at the temporal index \( x_i \) between two electrodes positioned at specific positions on the body skin. Due to its quasi-periodic nature, a typical ECG signal is composed of a sequence of cardiac cycles. A normal cycle is itself composed of three clinically significant features, in this order: P wave, QRS complex and T wave. It may be interesting to mention that compression of the ECG has been under way during the last four decades. As illustration of our method, we propose to enhance the compression capability of the classical FAN method [4]. FAN is a popular time domain-STR compression technique dedicated to the ECG signal, reported to yield quite high compression ratios in its category [2]. The FAN method uses a sequential strategy in selecting the CPs. Starting with the first input point as CP, FAN selects the next CP as the furthest point s.t. the maximal deviation between the original points and the points obtained by linear interpolation between successive CPs is below a preset threshold (\( \varepsilon \)). The deviation is measured by the absolute difference of magnitudes. Therefore, FAN can be viewed as a simplification process using a sequential strategy that reduces the distance \( ||P,Q|| = \max_{i=1..N} ||x_i - \hat{x}_i|| \). It can easily be shown that the FAN temporal complexity is of linear order, which allows effective real-time implementation of this method. Our proposed algorithm for LTR reduction is coupled as a post-processing step to this method. The so-enhanced FAN method is denoted herein FAN+. Evaluation of the FAN and FAN+ methods is performed on carefully selected records from the Massachusetts Institute of Technology – Beth Israel Hospital (MIT-BIH) ECG database. The MIT-BIH database is a collection of 48 records sampled at 360 Hz. Each record is 30 minutes long and each sample is coded on 12 bits. This base serves as a cross-reference for researchers. The evaluation is performed on the numerical level through the compression ratios \( CR_0 \) (Eq. 5) for FAN and \( CR_1 \) (Eq. 6) for FAN+ and computation of the respective distortions upon reconstruction expressed by the percent root difference \( PRD_0 \) for FAN and \( PRD_1 \) for FAN+ where \( PRD_0 \) is given by Eq. 9 and \( PRD_1 \) by Eq. 10 with \( \bar{y} \) representing the mean original magnitude.
The first application is a detailed illustration of the proposed method and is reported in Fig. 4. This figure shows in (a) a 1000 samples ECG from the beginning of rec. 105. Obtained results are as follows: M=71, then CR0=1000/(2x71)=7.04:1, with a reconstruction distortion of PRD0=7.18%. The 71 CPs were processed with our proposed algorithm yielding L=10 CPs at the same precision. The new compression ratio is then CR1=1000/(71+2x10)=11:1 and the new distortion is PRD1=7.90%. The same figure shows in (a) the 71 CPs obtained by FAN plotted on the original segment as small squares. Fig. 4 (b) shows the reconstructed signal using the 71 CPs of FAN. Fig. 4 (d) shows the sorted magnitudes of the 71 CPs. Fig. 4 (e) shows the simplification of this last curve with the 10 CPs reported as small squares. Fig. 4 (b) shows the reconstructed signal of this last curve with the 10 CPs reported as small squares. Fig. 4 (b) shows the reconstructed signal using the 71 CPs of FAN. Fig. 4 (c) shows the reconstructed signal by the proposed method and is reported in Fig. 4. This figure shows in (a) a 1000 samples ECG from the beginning of rec. 105. Obtained results are as follows: M=71, then CR0=1000/(2x71)=7.04:1, with a reconstruction distortion of PRD0=7.18%. The 71 CPs were processed with our proposed algorithm yielding L=10 CPs at the same precision. The new compression ratio is then CR1=1000/(71+2x10)=11:1 and the new distortion is PRD1=7.90%. The same figure shows in (a) the 71 CPs obtained by FAN plotted on the original segment as small squares. Fig. 4 (b) shows the reconstructed signal using the 71 CPs of FAN. Fig. 4 (d) shows the sorted magnitudes of the 71 CPs. Fig. 4 (e) shows the simplification of this last curve with the 10 CPs reported as small squares on Fig. 4 d. Fig. 4 (c) shows the reconstructed signal by the FAN+ method.

\[ PRD_i = \frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2} \times 100\% \quad (9) \]

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Fig. 4 illustrates the enhancement of the FAN method through incorporation of LTR reduction. For instance, for a distortion of PRD0=7.18% FAN simplified the 1000 samples with 71 CPs, thus yielding a CR0=7.04:1. Post-processing of the 71 CPs by the proposed method gave 10 CPs on the sorted magnitudes curve with 71 temporal indexes, thus a new CR1=11:1 and a new distortion of PRD1=7.90%. The gain in compression ratio is more than 56% for a small additional distortion. Note that the block size has great impact on LTR in the proposed method. In the specific case of the MIT-BIH database, the samples of which are coded on 12 bits, the maximal block size is $2^{12}$ samples, when expressing the compression ratio in terms of samples reduction. This constitutes no barrier, since the compression ratio can be expressed in terms of bits reduction (bit rate). The advantage of expressing the compression ratio in terms of samples reduction is to appreciate the true compression capabilities of a given method regardless of coding considerations. It is also of major importance to link the compression capability to a metric of distortion, for it is established that the highest compression ratios are obtained at higher distortion prices. We would then be interested in methods yielding higher compression ratios for lower distortions, ideally, in the operational rate-distortion sense (ORD). Fig. 5 clearly shows the enhancement of the RD behavior for all used records. For example, for a distortion of 10%, the compression ratios for the three records are as follows: 4.5:1, 7.75:1 and 10:1 for FAN and 8.5:1, 13.25:1 and 16.5:1 for FAN+, which represents gains in compression ratio for the same distortion respectively as follows: 88%, 71% and 65%. It can be checked that this gain varies for the different records as follows: Rec. 108 : 79%-87%, rec. 105 : 77%-79% and rec. 119 : 65%-71%. It can also be checked that the highest gains are obtained for the lowest distortions, i.e. for near-lossless compression. This is due to the fact that the more the FAN method goes to near-lossless compression, the more there are CPs to be post-processed, thus more LTR. In the case of higher compression ratios, it is all the way around. Nevertheless, the compression ratio is at most doubled. It is essential that the numerical evaluation be accompanied with visual inspection of the reconstructed signals in this type of applications. For this purpose, Fig. 6 shows the original signals (a), the reconstructed signals by FAN (b) and the reconstructed signals by FAN+ (c) for the three records. Note that in this case all plots are obtained around CR1=10:1. The plots clearly show the enhancement of the reconstructed signals due to taking into account LTR in the reconstruction process. This is an important property for our method with regard to classically used coding techniques to take into account LTR where the reconstructed signal is unchanged.

**Fig. 4 Illustration of STR and LTR compression on a small ECG segment.**

IV. DISCUSSION

Fig. 4 illustrates the enhancement of the FAN method through incorporation of LTR reduction. For instance, for a distortion of PRD0=7.18% FAN simplified the 1000 samples with 71 CPs, thus yielding a CR0=7.04:1. Post-processing of the 71 CPs by the proposed method gave 10 CPs on the sorted magnitudes curve with 71 temporal indexes, thus a new CR1=11:1 and a new distortion of PRD1=7.90%. The gain in compression ratio is more than 56% for a small additional distortion. Note that the block size has great impact on LTR in the proposed method. In the specific case of the MIT-BIH database, the samples of which are coded on 12 bits, the maximal block size is $2^{12}$ samples, when expressing the compression ratio in terms of samples reduction. This constitutes no barrier, since the compression ratio can be expressed in terms of bits reduction (bit rate). The advantage of expressing the compression ratio in terms of samples reduction is to appreciate the true compression capabilities of a given method regardless of coding considerations. It is also of major importance to link the compression capability to a metric of distortion, for it is established that the highest compression ratios are obtained at higher distortion prices. We would then be interested in methods yielding higher compression ratios for lower distortions, ideally, in the operational rate-distortion sense (ORD). Fig. 5 clearly shows the enhancement of the RD behavior for all used records. For example, for a distortion of 10%, the compression ratios for the three records are as follows: 4.5:1, 7.75:1 and 10:1 for FAN and 8.5:1, 13.25:1 and 16.5:1 for FAN+, which represents gains in compression ratio for the same distortion respectively as follows: 88%, 71% and 65%. It can be checked that this gain varies for the different records as follows: Rec. 108 : 79%-87%, rec. 105 : 77%-79% and rec. 119 : 65%-71%. It can also be checked that the highest gains are obtained for the lowest distortions, i.e. for near-lossless compression. This is due to the fact that the more the FAN method goes to near-lossless compression, the more there are CPs to be post-processed, thus more LTR. In the case of higher compression ratios, it is all the way around. Nevertheless, the compression ratio is at most doubled. It is essential that the numerical evaluation be accompanied with visual inspection of the reconstructed signals in this type of applications. For this purpose, Fig. 6 shows the original signals (a), the reconstructed signals by FAN (b) and the reconstructed signals by FAN+ (c) for the three records. Note that in this case all plots are obtained around CR1=10:1. The plots clearly show the enhancement of the reconstructed signals due to taking into account LTR in the reconstruction process. This is an important property for our method with regard to classically used coding techniques to take into account LTR where the reconstructed signal is unchanged.
V. CONCLUSION

A novel method for incorporation of long-term redundancy (LTR) in signal time domain compression methods has been proposed. The novel method is based on curve simplification and block sorting. The method is quite a general-purpose one-dimensional signals compression method, with more efficiency for quasi-periodic signals. It was implemented as a post-processing step for the FAN method in the specific case of the ECG signal. Results of the enhanced FAN+ method

Fig. 5. RD behavior of FAN (triangles) and FAN+ (stars).

Fig. 6. Original (a) and reconstructed signals by FAN (b) and by FAN+ (c) of MIT-BIH records 108, 105 and 119.
confirm the substantial improvement of the compression ratio-distortion behavior with respect to that of the FAN method for a better reconstructed quality signal. Note that the proposed method takes into account LTR on the magnitude axis only. It also needs no specific segmentation like R wave detection in the ECG signal. These two properties grant our method to be a general-purpose signal compression tool. Yet, for quasi-periodic signals, like ECG for instance, we believe that LTR reduction can be incorporated on the temporal axis too. Note also that the authors used the Douglas-Peucker algorithm as a unified tool for ECG baseline correction, features detection and STR compression in previous works [9]-[12].

REFERENCES


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