A Method of Planar-Template-Based Camera Self-Calibration for Single-View

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Abstract—Camera calibration is an important step in 3D reconstruction. Camera calibration may be classified into two major types: traditional calibration and self-calibration. However, a calibration method in using a checkerboard is intermediate between traditional calibration and self-calibration. A self-calibration algorithm is proposed based on a square in this paper. Only a square in the planar template, the camera self-calibration can be completed through the single view. The proposed algorithm is that the virtual circle and straight line are established by a square on planar template, and the relationship between them are be used, in order to obtain the image of the absolute conic (IAC) and establish the camera intrinsic parameters. To make the calibration template is simpler, as compared with the Zhang Zhengyou’s method. Through real experiments and simulation experiments, the experimental results show that this algorithm is feasible and available, and has certain precision and robustness.

Keywords—Absolute conic, camera calibration, circle point, vanishing point.

I. INTRODUCTION

In the process of acquiring three-dimensional (3D) information from two-dimensional (2D) image, the camera calibration is an indispensable step. Camera calibration usually is divided into two classes: traditional calibration and self-calibration. The traditional calibration approach needs to prepare a high precision geometric calibration object, and then uses the relationship between world coordinate and image coordinate of calibration object to complete the camera calibration. In 1992, Hartley and Faugeras first put forward the thought of camera self-calibration, which was called camera self-calibration by using corresponding relationship of images. Now, camera self-calibration has become one of hot spot in computer vision [1-7]. Zhang Zhengyou (2000) [8] put forward one kind more flexible and simple calibration method which was to use a dot matrix template instead of traditional calibration object to solve linearly camera intrinsic parameters via homography between dot matrix and image. On the basis of Zhang Zhengyou’s calibration method, Meng Xiaaqiao and Hu Zhan (2002) [9] put forward a kind of self-calibration method based on planar circle, which first introduced the concept of circular points into the technology of camera self-calibration. From then on, the technology of camera self-calibration enters a new field—the self-calibration by using circular points. Then, many self-calibration methods appear in succession based on Zhang Zhengyou and Meng Xiaaqiao. Wu Fuchao et al. (2003) [10], a kind of camera self-calibration method was proposed by using two non-parallel rectangles in space plane to solve circular points. In addition, Li Xinju et al. (2004) [11] analyzed the shortage of Zhang Zhengyou’s calibration method and put forward a kind of camera self-calibration method according to the similarity of plane scene image. Wang Guanghui (2008) [12] proposed a kind of camera self-calibration method using calibration object similar to Zhang Zhengyou’s checkerboard based on Kruppa equation. In recent years, a large number of self-calibration methods based on circular points or vanishing points have emerged [13-16]. However, in this paper, we use a square as calibration object to solve camera intrinsic parameters by circular points, vanishing point and projective invariance.

II. CAMERA MODEL

In this paper, we use perspective projection model which corresponds to a pinhole camera model (Fig.1).

![Fig. 1 Pinhole camera model](image)

Let the world coordinate of a point \((X)\) in space be \((X, Y, Z)\) and the corresponding image coordinates be \((u, v)\). \((u_o, v_o)\) is the coordinates of principal point and \(s\) is skew factor. \(f_s, f_r\) represent the focal length of the lens. \((r_s)\) and \((t_s, t_r, t_z)\) are rotation matrix and translation vector. So, the perspective projection matrix may be expressed as

\[
\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}
\]
where $K$ is camera intrinsic parameters in Equation (1). In this paper, the calibration object is a square on a planar template, so the planar template containing square may be defined as $X-Y$ plane of world coordinate system (WCS) and the coordinate $Z$ of space points is always zero. Now, the simplified expressions of perspective projection model can be obtained

$$
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
f_x & s & u_0 & 0 \\
0 & f_y & v_0 & 0 \\
0 & 0 & 1 & 1
\end{bmatrix} \begin{bmatrix}
x_1 \\
y_1 \\
z_1
\end{bmatrix},
$$

(1)

where $H$ is called homography matrix from 3D to 2D coordinates.

### III. CAMERA SELF-CALIBRATION

#### Lemma 1

If in Fig. 3, let the side length of square be $a$ on the planar template, the equations of virtual circle and virtual lines can be determined, and the virtual lines cut the virtual circle at $e,f,g,h,O$ and the circular points of the planar template can be obtained in image coordinate $2D$ coordinates.

#### Lemma 2

Assuming that homography matrix $H$ from 3D to 2D coordinates is known, image coordinates of any 3D points of the planar template can be obtained in image coordinate system (ICS), particularly, circular points also can be gotten in ICS.

#### Proof

From Equation (2), Lemma 2 is obviously true. Particularly, $[1 \pm i \ 0]^T$ are two circular points of the calibration plane in the WCS, so the image coordinates of circular points are unique

$$
\begin{bmatrix}
 m_i \\
m_j
\end{bmatrix} \approx H [1 \pm i \ 0]^T
$$

Obviously, homography matrix $H$ can be estimated by at least four matching points. In Fig. 3, the image coordinates of four vertices $a,b,c,d$ in the square can be obtained by Harris Corner Detection [17] and the world coordinates of $a,b,c,d$ are obviously known, so $H$ is estimated by four pairs of matching points. Combining Lemma 1 and Lemma 2, the image coordinates $e,f,g,h,O$ and the circular points $m_i,m_j$ can be gotten.
Lemma 3. Suppose $v_1, v_2$ are vanishing points on mutually orthogonal directions, $v_2^\top \omega v_2 = 0$ ( $\omega = K^{-1}K^{-1}$is IAC).

Proof. Let $p_{sa}, p_{sb}$ be points at infinity on mutually orthogonal directions, of which image coordinates are $v_1, v_2$. According to the camera model with imaging principles, we have

$$\lambda_1 v_1 = K [ R \ T ] p_{sa}, \lambda_2 v_2 = K [ R \ T ] p_{sb},$$

and Equation (7) can be rewritten as

$$(R \ T ) p_{sa} = \lambda_1 K^{-1} v_1, (R \ T ) p_{sb} = \lambda_2 K^{-1} v_2.$$

From orthogonality in projective geometry, we have

$$\lambda_1 \lambda_2 v_1^\top K^{-1} K^{-1} v_2 = v_2^\top [ R \ T ]^\top [ R \ T ] p_{sa} = 0,$$

and therefore

$$v_2^\top \omega v_1 = 0.$$

Lemma 4. Supposing $P_i \in l(P)$ $(i = 1, 2, 3, 4)$ and $(P_1 P_2 P_3 P_4) = k$ ($k \neq 0, 1, \infty$), where three of $P_1, P_2, P_3, P_4$ and $k$ are known, the fourth point can be determined. ($l(P)$ is point-set of line $l$.)

Proof. Supposing that homogeneous coordinates of $P_1, P_2, P_3, P_4$ are $a, b, a + \lambda_1 b, a + \lambda_2 b$, have

$$(P_2 P_3 P_4) = \lambda_1 = \lambda_2 = k.$$ (9)

Without loss of generality, assuming that known $P_1, P_2, P_3$ are $a, b, a + \lambda_1 b$, now the solution of the fourth point is transformed to determine $\lambda_2$. From Equation (9), we have

$$\lambda_2 = \frac{\lambda_1}{k}.$$ (10)

Supposing that homogeneous coordinates of $P_1, P_2, P_3$ are equal to $(u_1, v_1, 1), (u_2, v_2, 1), (u_3, v_3, 1)$, combining with $P_3 = a + \lambda_2 b$, we have

$$u_3 = \frac{u_1 + \lambda_2 u_2}{1 + \lambda_2}.$$ (11)

Equation (11) can be rewritten as:

$$\lambda_3 = \frac{u_1 - u_3}{u_3 - u_2}.$$ (12)

Combining with Equation (10) and (12), $\lambda_2$ can be obtained, and then $P_4$ can be determined.

As shown in Fig. 3, take line $ac$ as an example to discuss how to solve a vanishing point of diameter of the virtual circle. In Lemma 4. If $l(P) = a, P_2 = c, P_3 = \infty$, it is reasonable assumption that the length of $P_1 P_4$ and $P_3 P_4$ are equal, and $\frac{P_1 P_4}{P_3 P_4} = 1$. Based on perspective projection, have

$$\begin{align*}
(P_1 P_2 P_3 P_4) = \frac{P_2 P_1}{P_3 P_2} = (P_2 P_1),
\end{align*}$$

where $P_2$ is the diameter and $P_3$ is the centre of the virtual circle, respectively, and $\frac{P_1 P_4}{P_3 P_4} = -1$. Additionally, under the projective transformation group, some graphics have projective properties and projective invariants. Combining with Lemma 4, we have

$$\begin{align*}
(P_1 P_2 P_3 P_4) = -1, \quad P_{13} \times P_{23} = 0.
\end{align*}$$ (13)

Supposing the image coordinate of $P_\infty$ is $(u_\infty, v_\infty, 1)$, the matrix expression of Equation (13) is

$$\begin{pmatrix}
1 & 0 & -u_\infty & -u_1 \\
0 & 1 & -v_\infty & -v_1 \\
1 & 1 & u_\infty & u_1 \\
v_1 - v_\infty & u_2 - u_1 & v_1 - v_\infty & v_1 - v_2
\end{pmatrix} = \begin{pmatrix}
\frac{u_\infty (u_\infty + u_1) - 2 u_\infty u_2}{2 u_\infty - u_1 - u_2} \\
\frac{v_\infty (v_\infty + v_1) - 2 v_\infty v_2}{2 v_\infty - v_1 - v_2} \\
u_\infty - u_1 \\
v_\infty - v_2
\end{pmatrix}.$$ (14)

The image coordinate of $P_\infty$ can be obtained by the method of least squares to solve Equation (14). Similarly, points at infinity of any diameter of the virtual circle are gotten.

Lemma 5 [19, 20]. The sufficient and necessary conditions of which two non-isotropic lines are mutually orthogonal are that points at infinity of the two non-isotropic lines are conjugated with two circular points of the plane where the two non-isotropic lines are.

Fig. 4 is an image of a plane at infinity, where $\omega$ is absolute conic, $m_1, m_2$ are two circular points of the plane $\pi$ with the line at infinity $pq \cdot P$ is the point at infinity of one non-isotropic line in the plane $\pi$. From Lemma 5, $q$ corresponds to the point at infinity, at which the line on the plane $\pi$ is perpendicular to
the line containing the point \( p \) at infinity,
\[
(m_m, p_q) = -1.
\]

Taking line \( eg \) as an example, we will discuss how to solve the vanishing point of a line which is orthogonal to one of diameter of virtual circle. Suppose \( v_s \) is the vanishing point of line \( eg \) which is orthogonal to line at vanishing point \( v_v \). \( v_v \) can be gotten based on Lemma 4, Lemma 5 and cross-ratio invariance, and then the equation is as follows
\[
v_v^Tev_v^o = 0.
\]

Combining with Equation (8) and (15), three constraint equations can be established about \( \omega \). Similarly, \( n \) constraint equations with the form of Equation (10) can be obtained from \( n \) diameters of a virtual circle
\[
v_v^Tev_v^o = 0,
\]
where \( v_v \) is a vanishing point of one diameter of a virtual circle, and \( v_v \), \( v_v \) are two vanishing point on their mutually orthogonal directions. Let \( \omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix} \), we have
\[
\begin{align*}
&\begin{bmatrix}
(\omega_1 + \omega_2)V_1 + (\omega_3 + \omega_4)V_2 + (\omega_5 + \omega_6)V_3 + (\omega_1 + \omega_5)V_4 + \omega_2 &= 0 \\
(\omega_2 + \omega_3)V_1 + (\omega_4 + \omega_5)V_2 + (\omega_6 + \omega_1)V_3 + (\omega_5 + \omega_2)V_4 + \omega_3 &= 0 \\
&\vdots \\
(\omega_4 + \omega_5)V_1 + (\omega_6 + \omega_1)V_2 + (\omega_1 + \omega_2)V_3 + (\omega_3 + \omega_6)V_4 + \omega_4 &= 0
\end{align*}
\]

Let
\[
f = [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6]^T,
\]
\[
A = \begin{bmatrix}
u_{u_1}u_{u_2} + u_{v_1}v_1 + u_{v_2}v_2 & u_{u_1}u_{v_1} + u_{v_2}v_1 & u_{u_2}u_{v_2} + u_{v_1}v_2 & v_1v_2 & v_1v_2 & 1 \\
u_{u_1}u_{v_1} + u_{v_2}v_1 & u_{u_1}u_{v_1} + u_{v_2}v_1 & u_{u_2}u_{v_2} + u_{v_1}v_2 & v_1v_2 & v_1v_2 & 1 \\
&\vdots \\
u_{u_1}u_{v_1} + u_{v_2}v_1 & u_{u_1}u_{v_1} + u_{v_2}v_1 & u_{u_2}u_{v_2} + u_{v_1}v_2 & v_1v_2 & v_1v_2 & 1
\end{bmatrix}.
\]

Equation (16) can be rewritten as \( Af = 0 \). When taking \( n \) images of the planar template under different views, the multiple constraint equations can be gotten about \( C \), and \( f \) can be solved by the method of least squares to obtain \( \omega \). By Zhang’s method [8], the intrinsic parameters can be calculated.

Outline of the algorithm is presented below:

Step 1. Print a square, stick it on the plane and at least take a picture.

Step 2. Obtain image coordinates of four vertexs of the square by Harris, and estimate homography matrix \( H \).

Step 3. According to the Lemma 3, determine image intersection coordinates of virtual lines and a virtual circle, and image coordinates of circular points.

Step 4. Obtain vanishing points of parallel sides of the square by solving image intersections of parallel sides.

Step 5. According to Lemma 4, obtain vanishing points of two diagonals of the square.

Step 6. According to Lemma 4 and Lemma 5, obtain any of virtual lines and its vanishing point of orthogonal lines.

Step 7. Obtain \( \omega \) by the method of least squares, and then obtain \( K \) by Zhang’s method [8].

IV. EXPERIMENTS

A. Experiments with Simulated Data

The values of camera intrinsic parameters are set to \( f_x = 1000, f_y = 1100, u_n = v_o = 0, s = 0 \) in simulation experiments. Simulations adopt three images which correspond to extrinsic parameters
\[
R = \begin{bmatrix}
0.8080 & 0.4106 & -0.4225 \\
-0.0695 & -0.7785 & -0.6238 \\
0.9993 & 0.0065 & 0.0361
\end{bmatrix}, \quad T = \begin{bmatrix}
-142.56 & -34.891 & 521.456 \\
-0.5850 & 0.4747 & -0.6576
\end{bmatrix}.
\]

respectively. The origin of WCS is the intersection of two diagonals of the square. We adopted three virtual lines, of which the corresponding slopes are \( \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{3} \). In order to test the robustness of our proposed method, add the Gaussian noise where the mean value is zero and the mean square error is from 0.1 pixels to 5 pixels on the position of pixels. 200 independent experiments can be executed for each noise level, and solve their averages.

In the same camera parameters setting, carry out a comparative study between ours and Wang Guanghui’s method [12] with skew factor \( s = 0 \). The algorithm is denoted by Square in this paper, and Wang Guanghui’s algorithm is denoted by Hinf. Comparisons of two algorithms are as follows (Fig. 4):
B. Experiments with Real Images

The factual data can be used for comparing and testing two proposed algorithms. The kind of camera is a CCD digital camera where the size of the image is $320 \times 240$. Fig. 5 (a) and (b) are two photos from different calibration templates as follows.

We carried out a comparative test with Wang Guanghui’s calibration algorithm. The results are as follows (Table I):

<table>
<thead>
<tr>
<th>Camera Intrinsic Parameters</th>
<th>Hinf</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_x$</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>$f_y$</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>$s$</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>$u_0$</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>$v_0$</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
</tbody>
</table>

In addition, we executed the 3D reconstruction by the calibration results as follows.
V. CONCLUSIONS

In this paper, use a square as calibration template. In the square, two diagonals as the diameter, the intersection of two diagonals as the center of a circle creates a virtual circle, and the intersection of diagonals are used as the beam-center of beams of virtual lines. First, the homography matrix $H$ is estimated from the four vertexes of the square to obtain image coordinates of the circular points and the intersections of the virtual circle and virtual lines. In additional, based on the cross-ratio invariance of projective transformation and conjugation between orthogonal vanishing points and circular points, the vanishing points of any of lines and the orthogonal vanishing points of them can be solved on the beam of virtual lines. So, IAC can be obtained by solving the constraint equations which are made up of mutually orthogonal vanishing points. Further, intrinsic parameters are determined through Zhang’s decomposing method. Experimental data show that the algorithm is feasible and available, have a certain precision and robustness. In this paper, the homography matrix $H$ is determined only by four vertices of a square, so the accuracy of four vertices affect the results of calibration. Therefore, in order to effectively improve the accuracy and the robustness of the algorithm, we can consider checkerboard as calibration plane.

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