Capacity of Overloaded DS-CDMA System on Rayleigh Fading Channel with Timing Error

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Abstract—The number of users supported in a DS-CDMA cellular system is typically less than spreading factor (N), and the system is said to be underloaded. Overloading is a technique to accommodate more number of users than the spreading factor N. In O/O overloading scheme, the first set is assigned to the N synchronous users and the second set is assigned to the additional synchronous users. An iterative multistage soft decision interference cancellation (SDIC) receiver is used to remove high level of interference between the two sets. Performance is evaluated in terms of the maximum number acceptable users so that the system performance is degraded slightly compared to the single user performance at a specified BER. In this paper, the capacity of CDMA based O/O overloading scheme is evaluated with SDIC receiver. It is observed that O/O scheme using orthogonal Gold codes provides 25% channel overloading (N=64) for synchronous DS-CDMA system on an AWGN channel in the uplink at a BER of 1e-5. For a Rayleigh faded channel, the critical capacity is 40% at a BER of 5e-5 assuming synchronous users. But in practical systems, perfect chip timing is very difficult to maintain in the uplink. We have shown that the overloading performance reduces to 11% for a timing synchronization error of 0.02Tc for a BER of 1e-5.

Keywords—DS-CDMA, Interference Cancellation, Multiuser Detection, Orthogonal codes, Overloading.

I. INTRODUCTION

CELL overloading is an efficient way to increase the number users in a given bandwidth, which is of practical interest to mobile system operators. Infact this type of channel overloading is provisioned in the 3G standard [1]. Among the approaches described in the literature, the most efficient ones use multiple sets of orthogonal codes [2]. The number of active users (K) in a conventional synchronous orthogonal CDMA environment is limited by the spreading factor N, which is WT where W is the transmission bandwidth and T is the duration of a symbol. In synchronous CDMA system, synchronization between signatures can be maintained in the downlink of cellular systems with relative ease and hence, orthogonal signatures (Walsh functions) are used in the downlink of IS-95 and UMTS mobile radio standards.

Even in the uplink of UMTS, usage of orthogonal signatures has been advocated to realize multi-code channelization. Also, with multicarrier-CDMA, the signal alignment can be maintained for much weaker synchronization requirements, by application of an appropriate cyclic prefix and single-tap equalization.

When K exceeds N, the system becomes overloaded and the signatures are no longer orthogonal. This leads to multiple access interference (MAI). In an overloaded system, a conventional matched filter receiver is not optimal, due to the high level of MAI. Multiuser detection (MUD) is required in order to obtain a satisfactory performance of the users. The Maximum Likelihood (ML) detection is not an option because of its complexity that is exponential in the number of users. The nonlinear MUD’s such as multistage parallel interference cancellation (PIC) and successive interference cancellation (SIC) [3], have good complexity- performance trade-off as compared to other MUD’s. Hence these MUDs are suitable for overloaded systems.

It is interesting to note that several studies have been made in the recent past to understand, analyze and counter the detrimental effects of overloading. Almost all studies consider the uplink or reverse link and several studies suggest usage of appropriate multiuser detection (MUD) schemes at the base station receiver. For example, a method of accommodating K = N + M users in an N-dimensional signal space that does not compromise the minimum Euclidean distance of the orthogonal signaling has been presented in [4] for AWGN channel. A tree-like correlation coefficient structure of user signatures suitable for optimal multiuser detection has been proposed in [5]. In another approach, two sets of orthogonal codes which are orthogonal within the sets is introduced in [6]. In this paper, the orthogonal sets are generated using Walsh Hadamard (WH) codes, where the same WH code set is scrambled with set specific scrambling sequence (s-O/O). An iterative multistage detection technique has been proposed to cancel the interference between the two sets of user. In [7], it is shown that for uncoded BPSK modulated CDMA signal with N=64, an overloading of 11% can be achieved in an AWGN channel for s-O/O scheme. Recently, the performance of an overloading scheme where only one orthogonal set is scrambled is evaluated in [8]. Another kind of receiver simplification is presented in [9], where signals are divided...
into groups that are orthogonal to each other. A new overloading scheme using hybrid techniques has been proposed in [10], where the spreading codes and transmission modes are different for the two sets to increase the overloading performance. The attractive property of overloading scheme was the incentive to integrate a particular type of O/O, called quasi-synchronous sequences (QOS) [11], into cdma2000 standard [12].

In [13], a new method for generating different orthogonal sets of same length has been proposed. The new algorithm generates (N-1) distinct, orthogonal sets of N sequences of length N. It has been shown that the peak value of crosscorrelation between different sets of same length is less than half the sequence length for N ≥ 32. Such sequence sets would offer low intracell interference when users are synchronous in an overloaded environment. But as it is very difficult to get perfect synchronization in the uplink, in this paper, we have shown the impact of imperfect timing synchronization error. The performance is evaluated in terms of number of users supported for a given BER using SDIC receiver.

This paper is organized as follows. In the next section, we describe the system model for the O/O overloading scheme on an AWGN channel. First we have discussed synchronous users and then it is extended to users received with timing error. Next we discuss O/O scheme on a Rayleigh fading channel. In section-4 we explain the SDIC receiver operation and describe the process of iterative interference cancellation. Simulation results are presented and discussed in Section-5. Finally, we present the conclusion of this paper.

II. SYSTEM MODEL ON AWGN CHANNEL

A. System Model with Perfect Synchronization

In the sequel we will consider the DS-CDMA system with processing gain N and the number of users K (=M+N). We assume that the channel is a nondispersive additive white Gaussian noise (AWGN) channel and that the different user signals are in perfect time synchronism. The signal s_{g,k}(t) is the signature waveform of the k-th user in set-g, where g ∈ {1, 2}, k1 ∈ {1, 2, 3,........N} for set-1 and k2 ∈ {1, 2, 3,........M} for set-2 users (M ≤ N). The signature waveform may be expressed as:

\[ s_{g,k}(t) = \sum_{j=1}^{N} s_{g,k}^j P_{c}(t - jT_c) \]  

(1)

where \( s_{g,k}^j \in \{-1,1\} \), \( T_c \) is chip duration and \( P_{c}(t) \) is the rectangular pulse shape of the chip waveform with unit energy. We assume that all set-1 users are operational and hence \( N=\)Maximum number of users in set-1 (=Spreading factor). All users signatures are normalized to have unit energy i.e.,

\[ \left\| s_{g,k}(t) \right\| = 1 \]  

(2)

The transmitted signal for set-1, \( k_1 \)-th user is given as

\[ u_{1,k_1}(t) = A_{g,k_1} b_{1,k_1}(t) s_{1,k_1}(t) \cos(\omega_0 t + \phi_{1,k_1}) \]  

(3)

and for set-2, \( k_2 \)-th user

\[ u_{2,k_2}(t) = A_{g,k_2} b_{2,k_2}(t) s_{2,k_2}(t) \cos(\omega_0 t + \phi_{2,k_2}) \]  

(4)

where, \( A_{g,k_1} \) is the amplitude of transmitted signal, \( \omega_0 \) is the carrier frequency, and \( \phi_{g,k} \) the phase parameter in the carrier. The delay \( \tau_{g,k} \) represents the phase parameter in the carrier. The delay and phase parameters, \( \tau_{g,k} \) and \( \phi_{g,k} \) are i.i.d. uniform random variables in the interval [0, \( T_b \)] and [0,2\pi] respectively. Note that all set1-users are synchronous, i.e., \( \tau_{1,k_1} = \tau_{1,1} \). Similarly, set-2 users are also synchronous. \( \phi_{g,k} \) is shown to be

\[ \phi_{g,k} = \left( \theta_{1,k_1} - \omega_0 \tau_{1,k_1} \right) \mod 2\pi \]  

(5)

\[ \left( \theta_{2,k_2} - \omega_0 \tau_{2,k_2} \right) \mod 2\pi \]  

(6)

The transmitted signal for set-1, \( k_1 \)-th user is given as

The transmitted signal for set-2, \( k_2 \)-th user

where \( \{1, 1\} \) represents the phase parameter in the carrier. The delay and phase parameters, \( \tau_{g,k} \) and \( \phi_{g,k} \) are i.i.d. uniform random variables in the interval [0, \( T_b \)] and [0,2\pi] respectively. Note that all set1-users are synchronous, i.e., \( \tau_{1,k_1} = \tau_{1,1} \). Similarly, set-2 users are also synchronous. \( \phi_{g,k} \) is shown to be

\[ \phi_{g,k} = \left( \theta_{1,k_1} - \omega_0 \tau_{1,k_1} \right) \mod 2\pi \]  

(7)
If the first stage of the receiver is a conventional correlation CDMA receiver matched to \( s_{g_1, k_1} (t) \), then the decision statistics for the \( i \)th bit at stage 1 is given by

\[
Z_{g_1, k_1} = \int_{t_1 + \tau_{g_1, k_1}}^{t_1 + \tau_{g_1, k_1} + \varepsilon} r(t) s_{g_1, k_1} (t - \tau_{g_1, k_1} + \varepsilon) \cos(\omega_d t + \phi_{g_1, k_1}) dt
\]

where \( 0 \leq \varepsilon \leq T_c \) represents the timing error associated with the locally generated version of the desired user’s spreading code. For ease of analysis, we assume that each user is subject to the same timing error. Simplifying (13) leads to the expression for set 1 users as

\[
Z_{1, k_1} = \sqrt{\frac{P_{1, k_1}}{2}} b_{1, k_1}^i (NT_c - (2B_{1, k_1} + 1 - Q_{1, k_1})\varepsilon) + I_{k_1, 1} + I_{k_1, 2} + \eta
\]

where \( \eta \) is a Gaussian random variable with zero mean and variance \( N_\text{e} T_c / 4 \), \( B \) is a binomial random variable with mean \( N - 1 / 2 \) representing the number of chips alteration in the desired user’s spreading code, \( Q_{g_1, k_1} \) is a binary random variable with values \( \left\{ \pm \frac{1}{2} \right\} \) occurring with equal probability.

\( I_{k_1, 1} \) is the multiple access interference of set 1 user on \( k_1 \)-th set 1 users and \( I_{k_1, 2} \) is the interference of set-2 users on \( k_1 \)-th set 1 user due to the non-orthogonality of the simultaneous users in set 1 and set 2 respectively. The first term represents the imperfect correlation between the desired user’s received signal and the desired user’s spreading code and the last term represents the contribution to the decision statistics due to thermal noise.

The signal-to-noise ratio (SNR) for set1-users is given by

\[
\text{SNR}_1 = \frac{(E[D_{1, k_1}])^2}{\text{Var} [\hat{N}] + \text{Var} [I_{k_1, 1} + I_{k_1, 2}]}
\]

and for set2-users it is given as

\[
\text{SNR}_2 = \frac{(E[D_{2, k_1}])^2}{\text{Var} [N] + \text{Var} [I_{k_1, 1} + I_{k_1, 2}]}
\]

Since BPSK signaling is used for both the sets, the BER for each set becomes

\[
\text{BER}_{g_1} = Q\left(\sqrt{\text{SNR}_g}\right), \text{ for } g = 1, 2
\]

III. SYSTEM MODEL FOR RAYLEIGH FADING CHANNEL

We will consider the DS-CDMA system with processing gain \( N \) and the number of users \( K (=M+N) \) on a Rayleigh fading channel. We assume that signal spreading is performed by means of user-specific spreading sequences. Let us denote by \( S = [s_1, s_2, \ldots, s_N] \) the \( N \times K \) matrix containing the \( K \) signature sequences of length \( N \) associated to the \( K \) users (denoted by \( s_k = (s_{1k}, s_{2k}, \ldots, s_{nk})^T \)). We can also write: \( S = [S_1, S_2] \) with \( S_1 \) and \( S_2 \) respectively of size \( N \times N \) and \( N \times M \) corresponding to the matrices of sequences attributed to set 1 and set 2 users. Here, we consider schemes where the sequences assigned to users in the same group are orthogonal. In this work, we have considered QOS O/O scheme, where the signatures of the set 1 users are the Walsh- Hadamard
sequences $W H_1^{(k)}$ of order $N$, and the signatures of set 2 users are obtained by the same Walsh-Hadamard sequences by means of a (quasi-) bent sequence $Q \in \{1, -1\}^N$ [11]. QOS are balanced O/O sets, so that the correlation between set 1 and set 2 users is equalized. These QOS minimize the maximum correlation between the set 1 and set 2 users.

The following notations are used to describe the transmission model:

- $x = (x_1, x_2, ..., x_K)^T$ is the set of BPSK transmitted bits associated to the $K$ users during a given CDMA bit period,
- $A = \text{diag}(\{a_k, k \in \{1, ..., K\}\})$ is the matrix whose coefficients $a_k$ denote the $k$th user’s complex channel attenuation,
- $r = (r_1, r_2, ..., r_K)^T$ is the received signal block.

Assuming that the channel attenuations do not vary within one symbol interval, the received signal block can be written as follows:

$$r_n = \sum_{k=1}^{K} s_{kn} a_k x_k + z_n \quad n = 1, ..., N \quad (18)$$

This is equivalent to:

$$r = SAx + z \quad (19)$$

where $z = (z_1, z_2, ..., z_N)^T$ is the vector of AWGN samples of variance $\sigma_z^2 = N_0$.

We notice that the case of an AWGN channel is obtained by taking $A = I_K$. The Rayleigh fading channel model can be described by fading amplitudes generated according to $a_k = a_k^{(i)} + j a_k^{(0)}$, where $a_k^{(i)}$ and $a_k^{(0)}$ are independent zero-mean real Gaussian distributed random variables with variance $\sigma_{a_k}^2 = \sigma_{a_k^{(i)}}^2 = 1/2$.

IV. ITERATIVE MULTISTAGE DETECTION

In this work, an iterative interferences cancellation receiver is used to remove the MAI between the two sets. The basic principle of this receiver is to iteratively remove the estimated interference from each set due to the users of other set in multiple stages such that near single user performance is achieved.

Let us consider the $k$th user to be detected. If we correlate the received signal with the sequence assigned to the considered user (dispreading operation), we obtain:

$$v_k = \sum_{n=1}^{N} r_n s_{kn}^* a_k^* = |a_k|^2 x_k + I(k) + z_k \quad (20)$$

where

$$I(k) = \sum_{j=k}^{K} s_{nj} a_j s_{nj}^* a_j^* \quad (21)$$

and the superscript $^*$ denotes complex conjugation.

We observe that $z_k$ is a Gaussian noise, with the same variance as $z$. Furthermore, we notice that because of the orthogonality of the spreading sequences in each set of users, the expression of the interference caused by the other users on the $k$th user reduces to:

$$I(k) = \sum_{j=k}^{K} s_{nj} a_j s_{nj}^* a_j^* \quad (22)$$

These expressions are used in the iterative interference cancellation presented in the next section.

B. Soft-Decision Interference Cancellation

First of all, the correlator outputs, i.e. the $v_k$’s, must be computed for each user ($k = 1, 2, ..., K$).

First iteration: To begin with, the $v_k$’s obtained for set-1 users ($k = 1, 2, ..., N$) are divided by $|a_k|^2$ and sent to the detector, which provides the first estimations $\hat{x}_k^{(1)}$ ($j = 1, 2, ..., N$) of set-1 symbols. Next, the estimated interference of set-1 users on each set-2 user is synthesized by substituting $\hat{x}_k^{(1)}$ for $x_k$ in (25). The estimated interference terms are subtracted from the values of $v_k$ associated to the set-2 users ($k = N+1, ..., K$). The resulting signals are then sent to the detector after division by $|a_k|^2$, giving symbol decisions $\hat{x}_k^{(1)}$ ($j = N+1, ..., K$) for the set-2 users. In an AWGN channel, we consider $a_k = 1, k \in \{1, ..., K\}$.

Iteration $i$ ($i > 1$): The symbol decisions made for set-2 users in the $(i-1)$th iteration ($\hat{x}_k^{(i-1)}$, $j = N+1, ..., K$) are used to synthesize the interference from these users on each set-1 user, by substituting for $\hat{x}_k^{(i-1)}$ for $x_k$ in (24). The estimated interferences are subtracted from the $v_k$’s obtained for set-1 users ($k = 1, 2, ..., N$). After dividing by $|a_k|^2$, improved decisions $\hat{x}_k^{(i)}$ ($j = 1, 2, ..., N$) are made for the symbols transmitted by the set-1 users. These decisions are next used to synthesize the interference from set-1 users on each set-2 user by using (28). The estimated interferences are subtracted from the $v_k$’s associated to the set-2 users ($k = N+1, ..., K$). After dividing by $|a_k|^2$, the symbol decisions corresponding to set-2 users at iteration $i$ ($\hat{x}_k^{(i)}$, $j = N+1, ..., K$) are computed.

To make soft decisions, we have used the piecewise linear function described in [7] as the nonlinearity involved in the decision device, except for the last iteration, where hard
decisions are made. The piecewise linear function is parameterized by $\theta$ and has the following expression:

$$\phi(x) = \begin{cases} \frac{x}{\theta} & |x| < \theta \\ \text{sgn}(x) & |x| \geq \theta \end{cases}$$

(25)

The value of $\theta$ is selected so as to minimize the bit error rate (BER) after 10 iterations. This non-linearity is used in the detector to provide the soft values corresponding to BPSK symbols.

For HDIC, the decision function is defined as

$$\phi(x) = \text{sgn}(x) = \begin{cases} -1 & x < 0 \\ +1 & x > 0 \end{cases}$$

(26)

In the last iteration of SDIC, we take hard decisions as given in (27). For HDIC scheme, in all iterations, we take hard decisions.

V. SIMULATION RESULTS

This section presents the Monte-Carlo simulation results of the proposed scheme with IMSD receiver. The simulation has been carried out in MAT-Lab to evaluate the BER performance of the proposed scheme on an AWGN and Rayleigh fading channels. The BER performance of soft decision interference cancellation (SDIC) has been evaluated. The value of the parameter $\theta$ is 0.5 for SDIC and it is fixed for all iterations.

For all simulations, the system performance is evaluated by means of critical capacity. We define the critical capacity or critical overload as the maximum number of users, so that the required SNR (=1/$\sigma^2$) for an average BER of 1e-5 is less than 0.35 dB with an interference cancelation receiver. It is a measure for the maximum number acceptable users or channel overload, so that the system performance is degraded slightly compared to the single user performance.

On a Rayleigh faded channel, critical capacity is defined at a BER of 5e-5 with a SNR penalty of less than 1 dB compared to single user bound on a Rayleigh fading channel.

In Fig. 1, the same SDIC receiver is simulated for N = 64 assuming perfect timing synchronization. It is observed that an overloading of 25%, i.e., 12 extra users can be obtained with an SNR degradation of about 0.35 dB at an average BER of 1e-5. Hence, the critical capacity is 25%, when spreading factor is 64. If we increase the overloading further to 28% or more, a BER error floor is reached.

In Fig. 2, the BER performance of SDIC scheme for N=64 at 19% overloading is shown for the timing error of 0.01Tc. We observe that the SNR degradation is about 0.35 dB and hence the critical capacity is 19% at 0.01Tc. Further, if we increase the timing error to 0.02Tc, it is observed that, we are able to load only up to 11% with an SNR degradation of about 0.35 dB at a BER of 1e-5. It is important to observe that a timing error of 0.02Tc reduces the capacity significantly from 25% (perfect timing) to 11%.

In Fig. 3, the timing error is increased to 0.05Tc. We observe that the for 11% overloading, the SNR degradation is about 2 dB. The achieved BER is 3e-5 at 11% overloading. Hence we conclude that timing synchronization is very critical in the case of overloaded DS-CDMA system.

Fig. 4, illustrates the BER performance on a Rayleigh fading channel with SDIC receiver. Here for an overloading of 25%, SNR degradation is about 1 dB as shown in Fig. 4. But when the overloading is further increased to 33%, the SNR degradation is more than 1 dB. So, the critical overload with timing error of 0.01Tc on a Rayleigh fading channel is 25% for OG/OG scheme.

In Fig. 5, when real scrambling is used with orthogonal Gold codes of two sets, it results in an s-OG/OG scheme. Although real scrambling does not increase the overloading on a Rayleigh fading channel, it gives more resistance to timing synchronization error. BER of s-OG/OG scheme is shown in Fig. 5 with a timing error of 0.01Tc. It is interesting to observe that, the SNR penalty is less than 1 dB at a BER of about 4e-4 for 33% overloading. Hence, overloading percentage increases to 33% from 25% with real scrambling and SDIC receiver.

In Fig. 6, we have compared the amount of overloading with and without timing error at 40% overloading. The timing error is fixed at 0.01Tc. We observe that without timing error, the SNR penalty is less than 1 dB. But with a timing error of 0.01Tc, the SNR penalty increases significantly. So, we can overload the channel with 40% overloading only if it is perfectly synchronized. So, timing synchronization is very important to get higher capacity on a Rayleigh faded channel.

VI. CONCLUSION

In this paper, BER performance of an O/O overloading scheme is evaluated using soft decision interference cancellation receiver with timing synchronization error. It is shown that this scheme with soft decision interference cancellation (SDIC) can overload the DS-CDMA systems by 25% i.e., 16 extra users can be supported at BER of 1e-5 for N=64, when there is no timing synchronization error. But with timing synchronization error of 0.02Tc, the overloading amount reduces to 11% only. On a Rayleigh fading channel, we can support 33% extra with real scrambling and SDIC receiver. So, timing control is an important requirement to increase the capacity or number of users for a given BER for overloaded DS-CDMA systems.
Fig. 1. BER performance comparison with Soft decision Interference cancellation (SDIC) with N=64 for the following overloading a) 30%; b) 28%; c) 25%; d) 22%; e) Single user performance on an AWGN channel with perfect timing synchronization

Fig. 2 BER performance of OG/OG scheme for N=64 with timing synchronization error of 0.01Tc for the following: a) Matched filter with 19% overloading; b) SDIC with 19% overloading and c) Single user performance on an AWGN channel with perfect timing synchronization

Fig. 3 BER performance of OG/OG scheme for N=64 with 0.05Tc synchronization error for the following: a) Matched filter with 11% overloading; b) SDIC with 11% overloading and c) Single user performance without timing error

Fig. 4 BER performance of OG/OG scheme for N=64 with timing synchronization error of 0.01Tc over a Rayleigh fading channel for the following: a) Matched filter with an overloading of 33%; b) SDIC with an overloading of 33%; c) SDIC with an overloading of 25% and d) Single user without timing error and overloading
Fig. 5 BER performance of scrambled OG/OG (s-OG/OG) scheme for N = 64 with a timing synchronization error of 0.01Tc over a Rayleigh fading channel for the following: a) Matched filter with an overloading of 40%; b) SDIC with an overloading of 40%; c) SDIC with an overloading of 33% and d) Single user without timing error

Fig. 6 Comparison of BER of OG/OG scheme for N = 64 at 40% overloading over a Rayleigh fading channel for the following: a) Matched filter with timing error of 0.01Tc; b) SDIC with timing error 0.01Tc; c) SDIC without timing error d) Single user without timing error and overloading

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