Analyzing of Noise inside a Simple Vehicle Cabin using Boundary Element Method

A. Soltani, M. Karimi Demneh

Abstract—In this paper, modeling of an acoustic enclosed vehicle cabin has been carried out by using boundary element method. Also, the second purpose of this study is analyzing of linear wave equation in an acoustic field. The results of this modeling consist of natural frequencies that have been compared with resultants derived from finite element method. By using numerical method (boundary element method) and after solution of wave equation inside an acoustic enclosed cabin, this method has been progressed to simulate noise inside a simple vehicle cabin.

Keywords—Boundary element method, natural frequency, noise, vehicle cabin.

I. INTRODUCTION

ODAYS, NVH (noise, vibration, harshness) is one of the most important and essential factor for vehicle industries to improve the quality of the products and to reduce interior noise in passenger vehicle cabin. It is important to note that interior noise in vehicles plays the main role in comfort of ride. High level of interior noise in vehicle cabin is a bad factor in assessment of vehicle quality. To reduce cost of designing and cost of manufacturing in automotive industries and to manage the time of conceptual design process, machine element designing, interior noise modeling has been carried out at the first step of conceptual and elements design. There is the booming noise of structure vibration in most passenger vehicles. So, Reduction of booming noise is very important in design process. To reduce the time of engineering analyzing, it is essential to investigate the booming noise before the manufacturing of primary prototype of a passenger vehicle [1].

Most of spectral noise energy is between 20 Hz and 200Hz. Also sound pressure is above 90 dB [2].

There are many applications of boundary element method (BEM) in acoustic problems. Control of noise is one of its applications. This numerical method (BEM) has capability to solve equations of acoustic wave for arbitrary enclosed shape numerically. The application of boundary element method in acoustic problems and noise control has been studied by cheng and seybert [3]. Their study showed that the noise of engine and other vibrating structures of vehicle diffuse wave to surround. Acoustic field of vehicle cabin has been modeled by Utsuno and Tanaka [4]. Boundary element method has been used to analyze the acoustic field in their studies. According to their study, the computing time and the cost of modeling will be reduced by using boundary element method.

In this research, simulation of a simple acoustic enclosed vehicle cabin has been carried out to solve the wave equation of acoustic field by using boundary element method. To evaluate the resultants of numerical solution, the amount of natural frequencies have been compared with results derived from finite element method. (Commercial engineering software: ANSYS)

II. BOUNDARY ELEMENT FORMULATION

Contribution of governed wave equation in acoustic surround is described by below wave equation [10, 11]:

\[ \nabla^2 u + \frac{\partial^2 u}{\partial t^2} = \frac{1}{C^2} \nabla^2 \nabla^2 u \]

(1)

In this relation \( C \) is sound velocity in acoustic surround and \( u \) is velocity potential that is function of position and time. So it can be written as below relation:

\[ u(x,t) = u(x) e^{i\omega t} \]

(2)

In relation (2), \( i \) is complex unit and \( \omega \) is angular frequency. By replacing equation (1) in equation (2), Helmholtz equation is obtained.

\[ \nabla^2 u + k^2 u = 0 \]

(3)

In this relation \( k = \frac{\omega}{C} \) is the wave number. Also,
acoustic pressure in each arbitrary point of acoustic wave is obtained from below relation:

$$P = -i \omega \rho \nabla u$$

(4)

In this relation, $$\rho$$ is the fluid density of acoustic surround at 20°C. It is possible to solve Helmholtz equation for complex geometry by using numerical methods. So, weighted residual method is used to solve this equation:

$$\int (\nabla^2 u + k^2 u) u^* d\Omega = 0$$

(5)

In above relation, $$u^*$$ is the main solution (Green function) of acoustic field, and $$\Omega$$ is acoustic domain.

$$u^* = \frac{e^{-i \omega r}}{4\pi r}, q^* = \frac{\partial u^*}{\partial r} = -\frac{1}{4\pi} \left(\frac{1}{r^2} + \frac{i \omega}{C r}\right) e^{-i \omega r}$$

(6)

In above relations, $$q^*$$ is the normal derivation of $$u^*$$ for a three-dimensional domain. $$r$$ is distance between two elements, $$n$$ is normal vector on element surface. Additionally, the equation of boundary integral equation is shown as below:

$$c^t u^t + \int \frac{\partial u^*}{\partial n} d\Gamma = \int \frac{\partial u}{\partial n} d\Gamma$$

(7)

In this relation, $$i$$ shows the inner points of boundary. Also, coefficient of $$c^t$$ is equal to 1 for inner points and it is equal to $$\frac{1}{2}$$ for points on boundary. For points out of boundary it is equal to zero. The boundary of enclosed cabin is divided in to N constant element. Therefore, to solve the boundary equation, relation (7) is rewritten as below relation:

$$\frac{1}{2} u^t + \sum_{m=1}^{N} H^{im} u^m = \sum_{j=1}^{N} G^{jq} q^j$$

(8)

In relation (8), $$N$$ is the number of nodes, $$u^m$$ and $$q^j$$ is potential and potential intensity at node $$m$$ and $$j$$ respectively. $$H^{im}$$ and $$G^{jq}$$ are effective coefficient that are calculated as below:

$$H^{im} = \int q^* d\Gamma$$

(9)

$$G^{jq} = \int u^* d\Gamma$$

(10)

$$H^{im} = H^{im} + \frac{1}{2}; i \neq m$$

(11)

$$H^{im} = \frac{1}{2}; i = m$$

According to relation (11), equation (8) can be re written as below:

$$Hu = Gq$$

(12)

$$H^{im}$$ and $$G^{jq}$$ are the elements of matrix coefficients of $$H$$ and $$G$$ by acting boundary conditions on equation (12). So, below relation is obtained:

$$AX = Y$$

(13)

In this relation, $$X$$ is an unknown amount vector on boundary that consists of unknown $$u$$ and $$q$$. Also, vector $$Y$$ is obtained from $$H$$ and $$G$$ by specific amount of $$u$$ and $$q$$.

By solving equation (13), points on boundary are specified and the amount of $$u$$ for inner points of enclosed cabin is obtained as below relation:

$$u^t = \sum_{j=1}^{N} G^{jq} q^j - \sum_{j=1}^{N} H^{jq} u^j$$

(14)

In this relation, $$u^t$$ and $$q^j$$ are potential and derivation of normal potential on boundary respectively. So, acoustic pressure is obtained in each arbitrary points of enclosed cabin.

III. MODELING OF ACOUSTIC ENCLOSED CABIN AND VIBRATING PANELS

The selected acoustic enclosed cabin is a sample vehicle cabin. It is assumed that the boundary of acoustic field is smooth and there is no curvature on boundary. Fig.1 shows a simple vehicle cabin and the selected mesh elements are constant and square shape.
divided into 12 elements. Table I presents the numbers of elements for each part of vehicle cabin.

<table>
<thead>
<tr>
<th>Part of Vehicle Cabin</th>
<th>Number of Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom of cabin</td>
<td>12 elements</td>
</tr>
<tr>
<td>Roof of cabin</td>
<td>9 elements</td>
</tr>
<tr>
<td>Rear of cabin</td>
<td>3 elements</td>
</tr>
<tr>
<td>Front of cabin</td>
<td>3 elements</td>
</tr>
<tr>
<td>Each other Shields</td>
<td>5 elements</td>
</tr>
<tr>
<td>Right side panel</td>
<td>8 elements</td>
</tr>
<tr>
<td>Left side panel</td>
<td>8 elements</td>
</tr>
<tr>
<td>Rear panel</td>
<td>6 elements</td>
</tr>
<tr>
<td>Front panel</td>
<td>6 elements</td>
</tr>
</tbody>
</table>

So, two dimensional arrays consist of coordinates and the numbers of all points of elements. The coordinate of centre point of each element has been calculated. So, to analyze properties of each element, centre node is sufficient.

The fluid inside the vehicle cabin is air and its density is $1.21 \frac{kg}{m^3}$ and the sound velocity in this fluid is $340 \frac{m}{s}$. The vibration of structure panels must be investigated. Since, the vibration influences on sound pressure level (SPL) inside the vehicle cabin. To investigate of this effect, the bottom panel of acoustic cabin must be vibrated and other sides of acoustic vehicle cabin are rigid.

Fig.2 shows the element of vibrating panel of vehicle cabin structure. The size of this panel is $1.35m \times 1.92m$ and the number of elements for bottom of cabin is 48 and it is shell type. The surface of plate has been reinforced by beam elements. Elasticity module, density and Poisson's ratio of panel are $207 \times 10^9 \frac{N}{m^2}$, $7800 \frac{kg}{m^3}$ and 0.3 respectively [8].

Table II shows the natural frequency of vibrating panel that has been calculated by modal analysis.

### Table II

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$ (Hz)</td>
<td>46.16</td>
<td>46.35</td>
<td>48.98</td>
<td>64.24</td>
</tr>
<tr>
<td>Mode</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>$f_n$ (Hz)</td>
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<td>73.85</td>
<td>96.4</td>
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<tr>
<td>Mode</td>
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<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>$f_n$ (Hz)</td>
<td>106.9</td>
<td>115.8</td>
<td>121.9</td>
<td>142.7</td>
</tr>
<tr>
<td>Mode</td>
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<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>$f_n$ (Hz)</td>
<td>161.75</td>
<td>166.6</td>
<td>185.9</td>
<td>187.8</td>
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<tr>
<td>Mode</td>
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<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>$f_n$ (Hz)</td>
<td>190.66</td>
<td>195.97</td>
<td>199.7</td>
<td>204.6</td>
</tr>
<tr>
<td>Mode</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>$f_n$ (Hz)</td>
<td>207.5</td>
<td>226.2</td>
<td>236</td>
<td>246</td>
</tr>
</tbody>
</table>

### IV. FREE VIBRATION AND NATURAL FREQUENCIES

The main solution of Helmholtz equation is non-linear and complex solution that is related to frequency of harmonic wave. Thus, elements of matrix $A$ in equation (13) are complex and these amounts are dependent to harmonic wave frequency. To obtain natural frequency of acoustic surround, equation (15) must be solved.

$$\det(A(f)) = 0$$  \hspace{1cm} (15)

The method for determining eigenvalues of matrix $A$ for acoustic problems is described as below relation. By changing the amount of parameter $f$ and substitution on equation (15), the amount of determinant of matrix $A$ has been calculated. This process is iterated in a band frequency. So, amount of $f$ that satisfy the equation (15) are natural frequencies of system.

To solve equation (15), the amount of frequency is assumed as complex number because equation (15) is a complex and non-linear equation. For acoustic vehicle cabin, the variation of absolute amount matrix determination has been shown in Fig.2. Table II represents the natural frequency and shape modes of vehicle cabin. These natural frequencies has been obtained by BEM and compared with resultant derived from finite element method (FEM) using ANSYS software.
In this relation the unit of SPL is dB and \( \langle p \rangle \) is amount of pressure in an arbitrary point. Base point for calculating SPL inside the vehicle cabin is a point with coordinate (0.4m, 1.8m, 0.8m). This point named point-1 and point-2 is at position (0.3m, 1.3m, 0.5m). The SPL in these two specific points compare with each other. In this condition there is no acoustic absorption material on walls of acoustic enclosed. Fig.3 shows SPL at point-1 for two conditions: without acoustic absorption materials and with acoustic absorption materials.

Fig.3 Comparison of SPL at point-1 and pont-2 without acoustic absorption materials

Fig.4 shows the SPL at point-1 with and without acoustic absorption materials on boundary. For instance, SPL in 148 Hz has been decreased by 43 dB.

Fig.4 Comparison of SPL at point-1 with and without acoustic absorption materials on boundary

Fig.5 presents the SPL at point-1 for two different thickness of vibrating panel. The thickness of vibrating panel is 2.25mm and 2.50mm respectively. There is no acoustic absorption material in this condition.

Fig.5 Comparison of SPL at point-1 with and without acoustic absorption materials on boundary

V. DETERMINATION OF SOUND PRESSURE LEVEL (SPL) IN SPECIFIC POINT INSIDE THE VEHICLE CABIN UNITS

To calculate the SPL in specific point inside an acoustic enclosed, boundary conditions of acoustic enclosed must be specified. According to BEM, parameters on boundary are calculated, and then sound pressure in each arbitrary point inside the acoustic cabin is obtained. The sound pressure in each point inside an acoustic enclosed is obtained from below relation:

\[
p(\alpha) = \frac{1}{4\pi} \int \int \frac{\partial p}{\partial n} \frac{e^{-ikr(\alpha, \zeta)}}{r(\alpha, \zeta)} ds - \frac{1}{4\pi} \int \int \frac{p(\zeta)}{\partial n} \left( \frac{e^{-ikr(\alpha, \zeta)}}{r(\alpha, \zeta)} \right) ds
\]

In this relation, \( \alpha \) is a point inside the acoustic field and \( \zeta \) is a point on boundary of acoustic enclosed. So, regarding to sound pressure of human audition (\( p_0 = 20 \mu p_a \)) SPL is calculated from below relation:

\[
SPL = 20 \log \frac{\langle p \rangle}{\langle p_0 \rangle} = 20 \log \langle p \rangle + 94
\]
This figure shows that, the numbers and the maximum amount of points are not equal in these two conditions. Since, shape modes of vibrating panels are not similar to each others. Accordingly to this figure, the effect of thickness variation is noticeable at 149 Hz and SPL has been reduced around 5dB.

Fig. 6 shows amount of SPL at point-1 for two different thickness of reinforced beam. The thicknesses of reinforced beams are 2mm and 3mm respectively. By increasing the thickness, SPL will be decreased in acoustic enclosed.

VI. CONCLUSION

Three dimensional acoustic field of a simple vehicle cabin has been investigated using boundary element method and natural frequency of this acoustic enclosed has been obtained and has been compared with resultants derived from finite element method. In this study, vibration of flexible panel has been studied and effect of thickness has been investigated using boundary element method.

Sound pressure inside the vehicle cabin due to vibrating panel has been calculated and effect of panel thickness, beam thickness and absorption material has been studied. So:

- The maximum amount of SPL graph and corresponding frequencies depend on position of points
- Effect of acoustic absorption material on enclosed wall has been studied. The results show that the reduction of SPL inside the vehicle cabin with absorption material is around 35 dB.
- Increasing of panel thickness and beam thickness changes the amount of sound pressure but this increase will not decreased SPL in all conditions and it depends on frequency and position of specific point.

REFERENCES