Nonlinear Dynamical Characterization of Heart Rate Variability Time Series of Meditation

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Abstract—Many recent electrophysiological studies have revealed the importance of investigating meditation state in order to achieve an increased understanding of autonomous control of cardiovascular functions. In this paper, we characterize heart rate variability (HRV) time series acquired during meditation using nonlinear dynamical parameters. We have computed minimum embedding dimension (MED), correlation dimension (CD), largest Lyapunov exponent (LLE), and nonlinearity scores (NLS) from HRV time series of eight Chi and four Kundalini meditation practitioners. The pre-meditation state has been used as a baseline (control) state to compare the estimated parameters. The chaotic nature of HRV during both pre-meditation and meditation is confirmed by MED. The meditation state showed a significant decrease in the value of CD and increase in the value of LLE of HRV, in comparison with pre-meditation state, indicating a less complex and less predictable nature of HRV. In addition, it was shown that the HRV of meditation state is having highest NLS than pre-meditation state. The study indicated highly nonlinear dynamic nature of cardiac states as revealed by HRV during meditation state, rather considering it as a quiescent state.

Keywords—Correlation dimension, Embedding dimension, Heart rate variability, Largest Lyapunov exponent, Meditation, Nonlinearity score.

I. INTRODUCTION

MEDITATION is an ancient spiritual practice with a potential benefit on health and well-being [1], [2]. It is a holistic system of mind-body practice for mental and physical health. This practice incorporates multiple techniques including breathing exercise, sustained concentration, physical posture, and many more. Indeed, meditation is considered as an altered state of consciousness different from ordinary wake and sleep states. Recently, the electrophysiological studies have revealed the importance of investigating the state changes related to meditation in order to achieve increased understanding of physiology in general. In this direction, many studies have focused on the physiological effect of different meditation techniques to gain insight into the physiological prerequisites responsible for the improvement of health [3]-[10].

Various studies carried out with meditation practitioners have suggested a number of physiological changes. It is said that during meditation, the body is under a hypometabolic state and controlled most of the time by parasympathetic nervous system [11], reduction or variability in heart rate [7], modifications in the concentrations of neurotransmitters [12], a drop in oxygen uptake as well as in carbon dioxide output [13], [11]. The use of meditation practice is becoming more popular to reduce stress, and effective as a complementary treatment for many disorders, such as hypertension, anxiety and insomnia [14], [15]. The meditation is associated with physiological signs of altered activation of autonomic and endocrine systems. This is evident from the studies of increased level of skin resistance and reduced levels of heart rate, blood-lactate level, cortisol, and respiration rate [16], [17], [12], [18].

The meditation practice has improved the self-rated quality of sleep in older persons compared to a group receiving an ayurveda poly-herbal preparation and another wait-list control group [19]. Meditation practice has shown to reduce stress and increase feelings of calm [20]. In normal volunteers, meditation has shown reduced psychophysiological arousal based on a decrease in oxygen consumption [21], [22], and changes in heart rate variability suggestive of a shift towards vagal dominance [23], decreased occupational stress levels and baseline autonomic arousal [24]. Studies have shown that meditation practice can improve mood [25], [26], increase resilience to chronic and acute stress [27], [28], enhanced performance on a variety of cognitive [29], [30], psychomotor [31], [30], and physical [25], [32] tasks.

Meditation is a complex physiological process which affects neural, psychological, behavioral, and autonomic functions, and is considered as an altered state of consciousness, differing from wakefulness, relaxation at rest, and sleep [33]-[35]. There are many evidences to the fact that meditation practice leads to functional changes in physiological states of humans [36]-[38], [12]. Most of the meditation techniques work by affecting the ANS, in turn regulating many organs and muscles, controlling functions such as the heart beat, sweating, breathing, and digestion. One possible way for meditation to act on autonomic activity is through respiration control. Respiration rate during meditation practice induces changes in the cardiovascular activity that corresponds to an increase in...
the activity of restorative parasympathetic system [41]. This increased parasympathetic activity has also been assessed through the slowing down of basal heart rate in meditators [42], and the increased synchronization, or respiratory sinus arrhythmia (RSA), between the breathing cycle and the heart beat during meditation [43], [44]. The RSA corresponds to high variability in heart rate as heart rate becomes faster during inhalation and slower during exhalation. Slow breathing has also been associated with increased baroreflex sensitivity [45], [46]. Decrease in blood pressure is often reported after meditation practice in both healthy subjects and hypertensive patients [47], [48]. Improved control of blood pressure is usually considered as a sign of balance between parasympathetic and sympathetic activity.

The analysis of heart rate variability (HRV), the variation of period between consecutive heart beats, provides valuable information to assess the autonomous nervous system (ANS). The HRV can be significantly affected by physiological state changes and many disease states. Hence, HRV analysis is becoming a major experimental and diagnostic tool. Its low cost, noninvasive nature and effectiveness encourages the development of new HRV analysis methods to broaden and improve its applications.

The analysis of HRV is not an easy task due to its nonstationarity [49]-[51] and nonlinearity [52]-[54] nature of time series. Traditional HRV analysis methods are based on linear methods in the time, frequency, or time-frequency domain. And these methods have been extensively used to reveal fundamental control activity of sympathetic and parasympathetic activity of ANS. The spectral analysis has led to the identification of three fairly distinct spectral peaks: high (0.15-0.5 Hz), low (0.05-0.15 Hz), and very low (0-0.05 Hz) frequency bands. The very low frequency (VLF) band has been associated with thermoregulation [55], low frequency (LF) spectral power reflects sympathetic and vagal influences on cardiac control via baroreceptor-mediated regulation of blood pressure [56]. The high frequency (HF) power is a function of respiratory modulation of vagal activity [41].

Many researchers have stressed on the importance of nonlinear techniques to study HRV [57]. This is because the cardiovascular system appears to be influenced by internal dynamics as well as from various external factors, which makes the system more dynamic and nonlinear. Generally, nonlinear dynamical analysis of time series involves estimation of dynamical invariants from the reconstructed attractor, such as, dimensions, Lyapunov exponents, and degree of nonlinearity. The classical nonlinear dynamical methods, such as correlation dimension [58], Lyapunov exponents [59], Poincare plots [60], various entropy measures [61]-[63], etc., have been used to quantify HRV dynamics. Several recent studies have used other nonlinear measures, including fractal dimension [64], approximate entropy [65], measures derived from symbolic dynamics and 1/f scaling [66], [67] to characterize HRV time series.

Many algorithms have been proposed in the literature to estimate nonlinear dynamical parameters from experimental time series data. However, most of these require long time series to obtain reliable estimates of nonlinear measures. In real biological systems, including while practicing meditation, acquiring long stationary time series may not be possible due to various reasons. Hence, the methods which give robust estimation results for shorter data lengths are very much desirable.

The nonlinear techniques, as mentioned above, have been widely used to study ANS in health as well as in various diseases. However, these methods have not been applied to study HRV during meditation. Even though there are some studies of HRV during meditation, those use linear spectral analysis techniques. These investigations have revealed that during meditation practices, there has been extremely prominent heart rate oscillations correlated with slow breathing [6], with amplitudes significantly greater than that measured before meditation, in the same individuals.

In this paper, we discuss application of nonlinear dynamical techniques to quantify HRV time series derived during meditation. Major emphasis is made on two types of meditation techniques; Chi meditation (Chinese style) [90] and Kundalini Yoga meditation (Indian style) [91]. We compute minimum embedding dimension, correlation dimension, largest Lyapunov exponent, and nonlinearity scores, from the HRV time series of both meditation and control (pre-meditation) state. We also aim to test the hypothesis that the nonlinear measures of HRV during meditation state would differ from those of control state.

II. MATERIALS AND METHODS

A. Minimum Embedding Dimension

The first step in nonlinear dynamical analysis is reconstruction of attractor in the phase space, from scalar time series. For this purpose, Taken’s embedding theorem is used, which ensures reconstructed attractor to preserve all topological properties of the original attractor. If \( x_i, i = 1, 2, ..., N \) is the time series, then the time delay vectors in phase space are formed as \( y = [y_1, y_2, ..., y_L]^T \), where the number of time delay vectors are \( L = N - (m-1)\tau \), and each time delay vector is expressed as \( y_i = (x_i, x_{i+\tau}, ..., x_{i+(m-1)\tau}), \quad i = 1, 2, ..., N - (m-1)\tau \), where \( m \) is the embedding dimension and \( \tau \) is the time delay.

Proper reconstruction of the attractor is guaranteed if the dimension of the phase space is sufficient to unfold the attractor. It is shown that an embedding dimension of \( m > 2d + 1 \) will achieve this [68], where \( d \) is the dimension of the attractor. In most cases of the observed time series analysis, one neither has knowledge of \( d \) or \( m \). There are many algorithms in the literature to estimate these quantities [69]-[73]. However, most of them have disadvantage of being too subjective or computationally intensive. The method...
proposed in [74] overcomes these difficulties and is suitable for short length time series also. More over, the method gives more reliable estimate of minimum embedding dimension (MED) even while the dimension of the system under consideration is sufficiently large.

The scalar time series is embedded in \( m \)-dimensional phase space, and the nearest neighbor for each phase space vector \( y_i(m) \) is found and its distance in both dimensions \( m \) and \( m + 1 \) are computed. The ratio of this distance is expressed as

\[
a(i,m) = \frac{\|y_i(m+1) - y_{n(i,m)}(m+1)\|}{\|y_i(m) - y_{n(i,m)}(m)\|},
\]

where \( i = 1,2,\ldots,N - m\tau \), and \( \| \| \) denotes some measurement of Euclidian distance. In this analysis we have used maximum norm which is defined as

\[
\|y_i(m) - y_j(m)\| = \max_{0 \leq j < m} |y_i(j) - y_j(j)|.
\]

The \( y_i(m+1) \) is the \( i^{th} \) reconstructed vector with embedding dimension \( m + 1 \). The \( n(i,m) \) is an integer in the range \( 1 \leq n(i,m) \leq N - m\tau \) such that \( y_{n(i,m)}(m) \) is the nearest neighbor of \( y_i(m) \) in the \( m \)-dimensional reconstructed phase space, in the sense of distance \( \| \| \). The \( n(i,m) \) depends on \( i \) and \( m \). If \( y_{n(i,m)}(m) = y_i(m) \), then the second nearest neighbor is taken.

If \( m \) is qualified as an embedding dimension according to the embedding theorem, then any two points which stay close in the \( m \)-dimensional reconstructed space will be still close in the \( m + 1 \) dimensional reconstructed space. Such a pair of points is called true neighbors, otherwise they are false neighbors. Perfect embedding means that no false neighbors exist. Then the following quantity which is the mean value of all \( a(i,m) \) is computed as

\[
E(m) = \frac{1}{N - m\tau} \sum_{i=1}^{N-m\tau} a(i,m).
\]

The \( E(m) \) is dependent only on the dimension \( m \) and the lag \( \tau \). To investigate its variation form \( m \) to \( m + 1 \), we compute

\[
E'(m) = \frac{E(m+1)}{E(m)}.
\]

If \( m \) is greater than some value \( m_0 \), if the time series comes from an attractor. Then \( m_0 + 1 \) is the minimum embedding dimension which accommodates the attractor completely. The saturation characteristics of \( E'(m) \) is one indicator of presence of chaos in the time series.

For random time series \( E'(m) \) will never attain a saturation value as \( m \) is increased. However, because of limited data samples and practical computations, it may be difficult to assess whether \( E'(m) \) is slowly changing or stopped changing. In such situations, another quantity which is useful in distinguishing deterministic signals from stochastic signals is as

\[
E^*(m) = \frac{1}{N-m\tau} \sum_{i=1}^{N-m\tau} |x_{i+m\tau} - x_{n(i,m)+m\tau}|.
\]

Its variation from \( m \) to \( m + 1 \) is computed as

\[
E^*(m+1) = \frac{E^{*(m+1)}}{E^*(m)}.
\]

Since for random data future values are independent of past values, \( E^*(m) \) will be equal to 1 for any \( m \), whereas for deterministic signals, there exist some \( m \) such that \( E^*(m) \neq 1 \). It is advisable to determine both \( E'(m) \) and \( E^*(m) \) to ensure presence of chaos. In the present study, we compute value of both of the quantities from epochs of HRV time series. The algorithm is particularly chosen because it is robust against length of time series, and gives reasonably good estimation even for high dimensional systems.

**B. Correlation Dimension**

The correlation dimension (CD) gives an estimate of system complexity. A dynamical system having strange attractor will have non-integer value for CD. Grassberger and Procaccia (GP) proposed an algorithm to compute CD from scalar time series [70], [75]. The disadvantage of GP algorithm is that it assumes the data is generated by a finite dimensional attractor and then seeks to determine its dimension. Hence, we almost always expect to get a finite fractional CD estimate from this algorithm. There are other methods to estimate correlation dimension based on kernel algorithms [76], [77].

Let \( \{y_j\}_{j=1}^{L} \) be an embedding of a time series \( \{x_i\}_{i=1}^{N} \) in \( \mathbb{R}^m \). The correlation function is defined by

\[
C_L(\varepsilon) = \left(\frac{L}{2}\right)^{-1} \sum_{0 \leq i < j \leq L} I\{y_i - y_j \leq \varepsilon\},
\]

where \( I(q) \) is the indicator function, which has a value of 1 if condition \( q \) is satisfied and 0 otherwise, and \( \| \| \) is distance function in \( \mathbb{R}^m \). The sum

\[
\sum_{i} I\{y_i - y_j \leq \varepsilon\}
\]

is the number of points within a distance \( \varepsilon \) of \( y_j \). If the points \( y_j \) are distributed uniformly within an object, this sum is proportional to volume of intersection of a sphere of radius \( \varepsilon \) with the object, and \( C_L(\varepsilon) \) is proportional to the average of such volumes. Then

\[
C_L(\varepsilon) \propto \varepsilon^{d_c},
\]

where \( d_c \) is the dimension of the object. Then the CD is defined as

\[
d_c = \lim_{\varepsilon \to 0} \lim_{L \to \infty} \frac{\log C_L(\varepsilon)}{\log \varepsilon}.
\]

Since the observation time series \( \{x_i\}_{i=1}^{N} \) are contaminated by noise, one cannot know \( \{y_j\}_{j=1}^{L} \) precisely. Therefore, computation
of $I(\|y_i - y_j\| < \varepsilon)$ is actually somewhat fuzzy. One can replace the hard indicator function with a continuous one, such as Gaussian basis functions $\exp\left(-\frac{\|y_i - y_j\|^2}{4\varepsilon^2}\right)$. The method is called Gaussian kernel algorithm. The generalization of correlation integral is as follows:

$$T^d_L(h) = \frac{1}{L} \sum_{\theta \in \text{rand}} \left(\frac{1}{L-1} \sum_{\theta \in \text{rand}} \exp\left(-\frac{\|y_i - y_j\|^2}{4h^2}\right)\right)^d,$$

where $h$ is analogous to $\varepsilon$, $d_e$ is the embedding dimension. It is possible to use any other functions $\varphi(.)$ as kernel function, provided they should have finite (bounded) support. For any such function, it can be shown that the following correlation dimension scaling law holds $T^d_L(h) \propto h^d$. By using $\varphi(q) = \exp(-q^2 / 4)$, we get

$$T^d_L(h) \approx \varphi\left(\frac{h^2}{h^2 + \sigma^2}\right)^d e^{-Kd} \left(\frac{h^2 + \sigma^2}{d_e}\right)^{d/2},$$

when $h^2 + \sigma^2 \to 0$ and $d_e \to \infty$. In the above equation $K$ is the correlation entropy, $\tau$ is the time delay. The noise level $\sigma = \sigma_n / \sigma_s$, where $\sigma_n$ is the standard deviation of the additive Gaussian noise in the signal, and $\sigma_s$ is the standard deviation of the observed signal (including the noise component). If $\sigma > 0$, $\sqrt{h^2 + \sigma^2} \to 0$ does not hold. By estimating $T^d_L(h)$ for a range of embedding dimensions $d_e$, one can estimate each of the parameters $d_e$, $K$, and $\sigma$, simultaneously. This is relatively robust technique which correctly accounts for noise when the Gaussian is additive. This method is although technically more complex, in practice, more reliable and less prone to misinterpretation. We have computed CDS of epochs of HRV time series for various values of embedding dimension, and compared the values for pre-mediation and meditation states.

C. Measure of Predictability

Lyapunov exponents (LE) are used to characterize attractor of a dynamical system. The LEs quantify the sensitivity of the system to initial conditions and are related to average rate of divergence or convergence of nearby trajectories in phase space. An $m$-dimensional dynamical system has $m$ LEs, of which some of them may be positive. The set of all LEs is called Lyapunov spectrum, and presence of positive LE is an indication of chaos. However, a completely predictable system (such as periodic signals, etc) has zero LE, whereas chaotic systems have at least one positive LE. In most of the applications, it is sufficient to estimate only largest LE (LLE) instead of Lyapunov spectrum. The LLE gives an idea about prediction zone of the time series under consideration.

Computing Lyapunov spectrum or LLE is straightforward when differential equations governing the system are known. However, in experimental set up, the governing equations of the system are not known, and one has only observation time series of the experiment. Wolf et al [78] proposed a method to estimate LLE. However, the method has a deficiency of orientational problem, since one has to successively replace nearby orbits, minimizing the orientational change. The method proposed by Sato et al [79] and is further improved by Rosenstein et al [80, overcome some of these problems.

After reconstruction of the attractor dynamics in a phase space, the number of phase space points is $L = N - (m - 1)\tau$. The algorithm locates the nearest neighbor of each point on the trajectory. The nearest neighbor, $y_j$, is found by searching for the point that minimizes the distance to the particular reference point, $y_i$. That is $d_j(0) = \min_{y_j} \|y_i - y_j\|$, where $d_j(0)$ is the initial distance from the $j^{th}$ point to its nearest neighbor, and $\|\|$ denotes the Euclidian norm.

An additional constraint is imposed that nearest neighbors have a temporal separation greater than the mean period of the time series. That is $|j - \hat{j}| > \text{mean period}$. The mean period can be computed from the power spectrum as the reciprocal of mean frequency. Each pair of neighbors is considered to be nearby initial conditions for different trajectories. Then the LLE is estimated as mean rate of separation of the nearest neighbors.

$$\hat{\lambda}_c(i) = \frac{1}{\Delta t} \frac{1}{L - i} \sum_{j=1}^{L-i} \frac{d_j(i)}{d_j(0)},$$

where $\Delta t$ is the sampling period of the time series, $d_j(i)$ is the distance between $j^{th}$ pair of nearest neighbors after $i$ discrete time steps ($i\Delta t$ sec), and $L$ is the number of phase space points. Since the equation converges slowly, then an alternate form is proposed.

$$\lambda_c(i,k) = \frac{1}{k\Delta t} \frac{1}{L - k} \sum_{i=1}^{L-k} \ln \frac{d_j(i+k)}{d_j(i)}, \quad k \text{ is held constant},$$

$\hat{\lambda}_c$ is estimated by looking the plateau region of $\lambda_c(i,k)$ with respect to $i$. This location of plateau is some times problematic and very subjective and hence estimated $\hat{\lambda}_c$ is unreliable many times. This difficulty is due to the normalization of $d_j(i)$.

In general, the LLE can be defined using the equation $d(t) = Ce^{\lambda t}$, where $d(t)$ is the average divergence at time
The $j^{th}$ pair of neighbors diverge approximately at a rate given by the largest LE, then $d_j(i) \approx C e^{\lambda_j(i\Delta t)}$, where $C$ is the initial separation. By taking log of both sides of this equation, we get $\ln d_j(i) \approx \ln C_j + \lambda_j(i\Delta t)$. This equation represents a set of approximately parallel lines (for $j = 1, 2, ..., L$), each with slope approximately proportional to $\lambda_j$. The average line can be defined by $z(i) = \frac{1}{\Delta t} \langle \ln d_j(i) \rangle$, where $\langle \rangle$ denotes average over all values of $j$. The LLE is estimated using a least-square fit to this average line. This process of averaging helps to calculate accurate values of $\lambda_1$ using small, noisy time series. The LLE $\lambda_j$ is extracted from a least square fit to the longest possible linear region of $z(i)$ versus $i$ plot. The presence of smooth linear region indicates positive LE.

The method is easy to implement, and computationally fast because it uses a simple measure of exponential divergence. This method gives more accurate values even for small data sets because it takes advantage of all the available data. The time evolution of logarithmic divergence is calculated and slope of the scaling region (LLE) is computed from epochs of HRV time series for both pre-meditation and meditation states.

**D. Measure of Nonlinearity**

Detection of nonlinearity in a time series needs a check for the presence of nonlinear time correlations among the time series values. The algorithm used in this study is based on the analysis of the extrema in time series. The theoretical and numerical results suggest that the sequence of extrema of a time series contains dynamical information on the process responsible for its generation.

It has been shown that a polar singularity corresponds to each local maximum or local minimum of the real time solution. Regular distribution of singularities reflects the corresponding periodic behavior of the real time solution [81]. The second condition arises from the general property of a stochastic process, which states that given a mean square differentiable stochastic process $w(t)$, the expected number of its extrema for unit time is contained in the joint density function of $w(t)$, $w'(t)$, and $w''(t)$ [82]. If the system is in a chaotic regime, then the corresponding sequence of singularities in the complex time plane associated with the local extrema becomes very irregular. It was also shown that the distances of these singularities from the real time axis, that is the position of these extrema are related to real values of the solution [83], [84].

The two algorithms proposed by Di Garbo et al extract these features such as number of extrema and length of broken line connecting these extrema as suitable statistics. The pattern of singularities in the complex time plane (PSC) algorithm uses length of broken line connecting these extrema. The number of extrema for unit time (NET) algorithm uses number of extrema in the unit time as statistics. Then, surrogate test is performed based on these statistics on extrema to account for Gaussian linear stochastic process. The significance of the test gives an estimation of its deviation from linearity. The procedure of nonlinearity test involves measurement of the above mentioned statistics for both original and surrogate data followed by statistical discrimination between them.

A larger value of nonlinear score means system is more nonlinear or it is deviating from a linear process which share many properties of system under consideration like mean, standard deviation, autocorrelation, and power spectrum, by a greater extent. We make use of methods proposed by [85].

Two types of surrogates are used in this analysis, Gaussian scaled (GS) and phase randomized (PR) surrogates [86]. The GS surrogates preserve histogram of amplitudes and approximately, the linear time correlations of the original time series. The PR surrogates preserve autocorrelation function and hence power spectrum. The steps involved in generating GS surrogates of time series are: (i) histogram transformation, (ii) Fast Fourier transform (FFT), (iii) phase randomization, (iv) inverse FFT, (v) inverse histogram transformation. However, in generating PR surrogates, only steps (ii) to (iv) are used.

In the PSC algorithm, the local maxima of the given time series are located and length of the line connecting these maxima is computed. The significance is computed as

$$S_{psc} = \frac{L - L_o}{\sigma_s},$$

where $L$ is the broken length corresponding to original and $L_o$ is the mean of the broken line lengths $L_i(i)$, $i = 1, 2, ..., M$ corresponding to $M$ surrogates, and $\sigma_s$ is the standard deviation of lengths $L_i(i)$. In the NET algorithm, number of extrema for unit time in both original and surrogate time series is determined, and the statistic is computed as

$$S_{net} = \frac{|N_o - N_s|}{\sigma_s},$$

where $N_o$ is the number of extrema in the original signal and $N_s$ is the mean of $N_i(i)$, $i = 1, 2, ..., M$ corresponding to number of extrema in the surrogate set, and $\sigma_s$ is the standard deviation of $N_i(i)$. The values of $S_{psc}$ and $S_{net}$ for both GS and PR surrogates are computed. Both PSC and NET algorithms are used to calculate NLS of epochs of HRV time series, considering both Gaussian scaled and phase randomized surrogates. This has been done for pre-meditation and meditation states of both Chi and Kundalini systems of meditation.
E. Subjects and Meditation Protocols

In this study, two meditation techniques have been studied; (i) Chinese Chi meditation and (ii) Kundalini Yoga meditation. The meditation practitioners of both groups were in good general health, and they did not follow any specific exercise routines. From the 8 Chi meditators (5 females and 3 males, age range 26-35 years, mean age 29 years), 10 hours of Holter ECG recordings have been obtained. Each of the practitioners practiced one hour of meditation. During this session, they sat quietly, listening to the taped guidance of the Yoga Master. They were instructed to breathe spontaneously, while visualizing the opening and closing of a perfect lotus in the stomach. The meditation session lasted for about one hour. The 4 Kundalini Yoga meditators (2 females and 2 males, age range 20-52 years, mean age 33 years), wore Holter monitor for one and a half hours. 15 minute of base line quiet breathing were recorded before the one hour of meditation. The meditation protocol consisted of a sequence of breathing and chanting exercises, performed while seated in a cross legged posture.

Practitioners, in good general health condition, have been selected and ECG signals are recorded. The Holter recordings have been manually verified, and outliers deleted. The data is grouped into pre-meditation control and meditation, and is available online in the PhysioNet data archives [6]. Instantaneous heart rate time series are derived by taking the inverse of each successive inter beat intervals.

Prior to the analysis, all HRV time series are made uniformly sampled to have 4 samples per second, using cubic spline interpolation technique, and then digitally filtered with a band pass filter of 0.01-0.57 Hz, in order to remove high frequency noise which is out of the band of interest. The pre-processed HRV time series are then segmented into 3 minute epochs (3x60x4 = 720 samples), and then each of the epochs are subjected to nonlinear dynamical analysis. The nonlinear dynamical nature of HRV epochs is verified using Cao’s method, and then nonlinear dynamical measures, such as correlation dimension, LLE, and nonlinearity scores are estimated for each of the epochs and averaged over the entire length of HRV time series. The extracted parameter values are group tested for statistical significance. Normal distribution of the values of nonlinear measures is assessed using the Shapiro-Wilks test for normality. Since many of the parameter values are non-normally distributed, comparison of nonlinear parameter values between meditation and pre-meditation.
control is carried out using Kruskal-Wallis test, and a probability value of 0.05 is accepted as significant, and marked as ‘a’ in the tables.

III. RESULTS

The parameters $E_1$ and $E_2$ are computed from the epochs of HRV time series and averaged across the subjects, for each of the meditation cases. The variation of parameters $E_1$ and $E_2$ with respect to embedding dimension are shown in Fig. 1 and 2, respectively, for Chi and Kundalini meditation. It is clear from the figures that the parameter values attain saturation for higher values of embedding dimension. From the Fig. 1(a) and 2(a), it is clear that irrespective of the meditation type, the MED value reaches a constant value which can be taken 5, for both pre-meditation and meditation states. From the Fig. 1(b) and 2(b), $E_2$ is not constant for all embedding dimension, and there are many embedding dimensions for which the value of $E_2$ is not equal to one, indicating the chaotic nature of HRV time series under consideration.

Table 1 summarizes the CD values (mean, SD), for pre-meditation and meditation states and the p-values of Kruskal-Wallis test, for Chi meditation. The same is depicted in the Table 2 for the case of Kundalini meditation. The average value of CD is plotted for various embedding dimensions in Figure 3. The pre-meditation state has lower CD values than the meditation state, for embedding dimensions greater than 4, with significant differences between the two states. The result suggests that the HRV during meditation state is less complex than during control state.

In order to estimate LLE, the time evolution of log of divergence is plotted as shown in Fig. 4. The box plot of estimated LLEs is shown in Fig. 5, and results are summarized in the Table 3. There is a significant increase in the value of LLE for meditation state compared to pre-meditation state, which states that the HRV time series during meditation is less predictable than that during pre-meditation state.

The nonlinearity scores (NLS) estimated from HRV time series using PSC and NET algorithms, considering both GS and PR surrogates, are shown in Table 4 and 5 respectively for Chi and Kundalini meditation. It is observed that the NLS are higher for meditation state than that for pre-meditation state. The results are significant in NET algorithm ($p < 0.05$). Even though, the NLS using Gaussian scaled PSC algorithm has not
IV. DISCUSSION

In this study, we have analyzed HRV time series, derived while performing two types of meditation practices, using nonlinear dynamic techniques. The meditation practices considered here are Chi meditation and Kundalini Yoga meditation. The nonlinear dynamic parameters such as NLS, MED, CD, and LLE are derived from the reconstructed attractor of scalar HRV time series. To compute these nonlinear dynamical parameters, we have chosen the methods, which are explained in the methods section, that give more accurate estimation results even for short length of time series.

The MED parameter is measured to check the nonlinear dynamical nature of HRV during meditation, which quantifies the interaction of dynamical parameters. In each of the meditation states, the attractor of the system can be reconstructed from time series with smaller embedding dimension compared to pre-meditation state. The MED values can be taken in the range 8-15 to completely unfold the dynamics. Even though there are not much difference in the MED values of HRV between meditation and pre-meditation states, the results indicate high dimensional chaotic nature of HRV time series in both meditation and pre-meditation states.

The HRV during meditation has shown a higher value of NLS compared to pre-meditation state. Higher NLS have suggested an increased nonlinearity of HRV, especially in meditation state. The higher NLS probably reflect possible increase in sympathetic function during meditation. The differences in NLS between the two states are also statistically significant. The NLS measures used in this work are based on local extrema of time series.

The CD gives a measure of complexity of systems measured as degree of freedom or number of state variables of the dynamical system. It is found that during meditation state the HRV time series have significantly lower CD values than pre-meditation state. From this it is inferred that the HRV during meditation is more rhythmic. The reduction of irregularity of HRV could be explained by a decrease of dynamical complexity of cardiovascular system. The statistical analysis has showed that the CDs are best differentiable between the two groups.
The predictability of the system is quantified by LLE, which also gives the presence of chaos. The LLE describes the rate of exponential divergence of trajectories and sensitive dependence on the initial condition. During meditation state the HRV showed higher LLE which confirms decreased predictability in this state compared to pre-meditation state. The decrease in predictability (increase in LLE) probably reflects a high degree of chaos during meditation state compared to pre-meditation state. This can be due to increased nonlinear interaction of variables in meditation state, and may be related to increased sympathetic activity and changes in peripheral vascular mechanisms. It is also found that HRV time series are chaotic both before and during meditation, as suggested by the positive LLEs in either state.

The intense stimulation of either the sympathetic or parasympathetic system could ultimately result in simultaneous discharge of both systems [87], [97]. Several studies have demonstrated influence of autonomic activity during meditation associated with decreased heart rate and blood pressure, decreased respiratory rate, and decreased oxygen metabolism [88], [3], [10], [92], [93]. However, a recent study of meditative techniques suggested a mutual activation of parasympathetic and sympathetic systems by demonstrating an increase in the variability of heart rate during meditation [6]. The increased variation in heart rate was hypothesized to reflect activation of both arms of ANS. This notion is consistent with recent developments in the study of autonomic interactions [89] [94]-[96], and also fits the characteristic description of meditative states in which there is a sense of overwhelming calmness as well as significant alertness.

The present study of HRV time series using nonlinear techniques has shown significant differences between the pre-meditation and meditation states, and thus could give additional insight into underlying dynamics of HRV and in the investigation of cardiac autonomic function during meditation. However, it should be noted that the present study has been considered on a smaller sample size of data and further
investigation is required on a larger sample of data size to substantiate the present work.

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