Micropolar Fluids Effects on the Dynamic Characteristics of Four-lobe Journal Bearing

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Abstract—Dynamic characteristics of a four-lobe journal bearing of micropolar fluids are presented. Lubricating oil containing additives and contaminants is modelled as micropolar fluid. The modified Reynolds equation is obtained using the micropolar lubrication theory and solving it by using finite difference technique. The dynamic characteristics in terms of stiffness, damping coefficients, the critical mass and whirl ratio are determined for various values of size of material characteristic length and the coupling number. The results show compared with Newtonian fluids, that micropolar fluid exhibits better stability.

Keywords—Four-lobe bearings, dynamic characteristics, stability analysis, micropolar fluid

I. INTRODUCTION

Rotating machines carrying large rotor loads and working at high speeds encounter the problem of instability. Non-circular bearings have the advantage of being more stable than the conventional circular bearings [1-3]. The new type of bearing, namely, four-lobe bearing was studied by Bhushan [4]. Advances in technology and in many practical lubrication applications necessitate the development of improved lubricants where the Newtonian fluids constitutive approximation is not a satisfactory engineering approach to lubrication problems. The experimental results support the achievement of better lubricating effectiveness on blending small amount of ling-chained additives with the Newtonian lubricants. Micropolar fluids obtained from the general micropolar fluids by imposing the assumption of the skew microcontinuum have been developed to explain the behavior of these fluids as polymeric fluids [6]. Huang et al.[7] presented the dynamic characteristics of finite-width journal bearings lubricated with micropolar fluids. The effects of micropolar lubricants and three-dimensional irregularities in hydrodynamic journal bearings were studied by Lin [8]. The dynamic characteristics of journal bearings lubricated with micropolar fluids were presented by Das et al.[9]. Prabhakaran Nair et al.[10] presented an analysis of the deformation effect of the bearing liner on the static and dynamic characteristics of an elliptical journal bearing with a micropolar lubricant. In recent investigation, the static characteristics of a noncircular journal bearing (two-lobe, three-lobe and four-lobe) lubricated with a micropolar fluids were studied by Rahmatabadi et al.[11]. In the present work, dynamic characteristics in terms of stiffness, damping coefficients, the critical mass and whirl ratio are determined for various values of size of material characteristic length and the coupling number for a four-lobe journal bearing lubricated with a micropolar fluids.

II. MODIFIED REYNOLDS EQUATION

Under the usual assumptions made for the lubrication film, the assumptions of the absence of body forces, body couples and constancy of characteristic coefficients across the film of the micropolar fluid, the modified Reynolds equation in dimensionless form is written in the following form [7]:

$$\frac{\partial}{\partial \theta} \left( \frac{G}{R} \frac{\partial \bar{p}}{\partial \theta} \right) + \left( \frac{R}{B} \right)^{\frac{1}{2}} \frac{\partial}{\partial z} \left( \frac{G}{R} \frac{\partial \bar{p}}{\partial z} \right) = \frac{\partial \bar{h}}{\partial \theta} + 12 \frac{\partial \bar{h}}{\partial t}$$

(1)

Where

$$\bar{G}(N, \bar{h}, L) = \frac{h}{h} - 2 N \frac{2h}{L} \coth \left( \frac{NL}{2} \right)$$

(2)

and

$$\mu = \mu + \frac{1}{2} k_v, \quad \ell = \left( \frac{\nu}{4 \mu} \right)^{1/2}, \quad N = \left( \frac{k_v}{2 \mu + k_v} \right)^{1/2}$$

(3)

Where $\theta = \frac{x}{R}, \quad z = \frac{z}{L}, \quad e = \frac{e}{c}, \quad \bar{p} = \frac{p c^2}{\mu UR}, \quad \bar{h} = \frac{h}{c}, \quad \bar{L} = \frac{L}{\ell}$

$N$ and $\ell$ are two parameters distinguishing a micropolar fluid from a Newtonian fluid. $N$ is a dimensionless parameter called the coupling number which couples the linear and angular momentum equations arising due to the microrotational effect of the suspended particles in the fluid. $\ell$ represents the interaction between the micropolar fluid and the film gap and is termed as the characteristics length of the micropolar fluid.

A. Bearing Geometry and Boundary Conditions

The configuration of the four-lobe bearing is shown in Fig.1. The non-dimensional fluid film thickness for each lobe is given by Bhushan [4]:

$$\bar{h}_i = 1 + \bar{\varepsilon}_i \cos(\theta - \phi_i), i=1,2,3,4$$

(4)

where $\bar{h} = \frac{h}{c}, \quad \bar{\varepsilon} = \varepsilon(1 - \delta)$

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The eccentricity ratios and the attitude angles of each lobe for the bearing are given by:

\[
\begin{align*}
\varepsilon_1^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos \frac{\pi}{4} \\
\varepsilon_2^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \sin \frac{\pi}{4} \\
\varepsilon_3^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \sin \frac{\pi}{4} + \phi \\
\varepsilon_4^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos \frac{\pi}{4} + \phi \\
\phi_1 &= \frac{5\pi}{4} + \sin^{-1} \left[ \frac{\varepsilon_1 \sin \left( \frac{\pi}{4} - \phi \right)}{\varepsilon} \right] \\
\phi_2 &= \frac{2\pi}{4} - \sin^{-1} \left[ \frac{\varepsilon_2 \cos \left( \frac{\pi}{4} + \phi \right)}{\varepsilon} \right] \\
\phi_3 &= \frac{\pi}{4} - \sin^{-1} \left[ \frac{\varepsilon_3 \cos \left( \frac{\pi}{4} + \phi \right)}{\varepsilon} \right] \\
\phi_4 &= \frac{3\pi}{4} - \sin^{-1} \left[ \frac{\varepsilon_4 \sin \left( \frac{\pi}{4} + \phi \right)}{\varepsilon} \right]
\end{align*}
\]

The pressure boundary conditions in dimensionless form are:

\[
\mathbf{P} = 0 \quad \text{at} \quad z = 0, \quad z = 1, \quad \theta = \theta_1, \quad \theta = \theta_2, \quad \theta = \theta_3, \quad \text{and} \quad \theta = \theta_4
\]

\[
\frac{\partial \mathbf{P}}{\partial \theta} = 0 \quad \text{for} \quad \theta = \theta_{e1}, \quad \theta = \theta_{e2}, \quad \theta = \theta_{e3}, \quad \text{and} \quad \theta = \theta_{e4}
\]

\[
\mathbf{P} = 0 \quad \text{for} \quad \theta = \theta_{e1}, \quad \theta = \theta_{e2}, \quad \theta = \theta_{e3}, \quad \text{and} \quad \theta = \theta_{e4}
\]

\[
\theta_{e4} \geq \theta_{e3} \geq \theta_{e2} \geq \theta_{e1}
\]

(7a)

(7b)

(7c)

Eq. (7a) is result from the fact that the ends of the bearing are exposed to the ambient pressure, while Eqs. (7b) and (7c) are the Reynolds (Swift-Stieber) conditions.

B. Stability Analysis

The linearized equations of the disturbed motion of the journal centre are [3]:

\[
\begin{align*}
M \ddot{x} + K_{xx} x + C_{xx} \dot{x} + K_{xy} y + C_{xy} \dot{y} &= 0 \\
M \ddot{y} + K_{yx} x + C_{yx} \dot{x} + K_{yy} y + C_{yy} \dot{y} &= 0
\end{align*}
\]

(8)

Where the fluid film stiffness and damping coefficients are respectively given by

\[
\begin{pmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{pmatrix} = -\left( \frac{\partial}{\partial x} \right) \begin{pmatrix} W_x & W_y \end{pmatrix}
\]

(9)

Equations (8) are used to study the stability of the bearing system. Harmonic solution of the type:

\[
x = x_0 e^{i\lambda t}, \quad y = y_0 e^{i\lambda t}
\]

(11)

will be assumed [3] where \( \lambda = \eta + iv \) is a complex frequency. The sign of the real part \( \eta \) allows the system stability to be defined. If \( \eta < 0 \) the system is stable and vice versa. On the threshold of stability \( \eta = 0 \), \( x \) and \( y \) are pure harmonic motions with a frequency \( \lambda = iv \). Thus equations can be written as:
\[ \begin{bmatrix} K_{xx} - M \nu^2 + i \nu C_{xx} & K_{xy} + i \nu C_{xy} \\ K_{yx} + i \nu C_{yx} & K_{yy} - M \nu^2 + i \nu C_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \] (12)

For a nontrivial solution the determinant must vanish and equating the real and imaginary parts to zero gives:

\[ M \gamma^2 = \frac{C_{xx} K_{yy} + C_{yy} K_{xx} - C_{xy} K_{yx} - C_{yx} K_{xy}}{C_{xx} + C_{yy}} \] (13)

\[ \gamma^2 = \frac{(K_{xx} - M \gamma^2)(K_{yy} - M \gamma^2) - K_{xy} K_{yx}}{C_{xx} C_{yy} - C_{xy} C_{yx}} \] (14)

where

\[ C_y = C_y \frac{c \omega_2}{W}, \quad \bar{K}_y = K_y \frac{c}{W}, \quad \bar{M} = \frac{Mc \omega^2}{W}, \quad \gamma = \frac{\nu}{\omega} \]

From equations (13) and (14), the critical mass and the whirl ratio \( \gamma \) are calculated. \( M_c \) is the critical mass parameter above which the bearing is unstable.

C. Solution Procedure

The modified Reynolds equation (1) is solved using the finite difference technique with Gauss Seidel method under the boundary conditions (7). Integration of the pressure, the horizontal and vertical components of the load are calculated for the four lobes of the bearing. By giving small values for \( x \) and \( y \) around the equilibrium position, the stiffness and damping coefficients can be calculated.

Applying the central finite-difference scheme to equation (1), the value of any pressure is given by:

\[ A_0 \bar{P}_{i,j} + A_1 \bar{P}_{i+1,j} + A_2 \bar{P}_{i-1,j} + A_3 \bar{P}_{i,j+1} + A_4 \bar{P}_{i,j-1} = B_{i,j} \] (15)

Where the values of the constants \( A_i \) can be defined by comparing this equation with equation (1)

III. RESULTS AND DISCUSSION

The dynamic characteristics for a four-lobe journal bearing are computed for an ellipticity ratio of 0.5 and \( R/L = 1 \). The ellipticity ratio used in this study is 0.5. The micropolar effects become insignificant and the fluid converts to Newtonian fluid as \( L \rightarrow \infty \) or \( N \rightarrow 0 \). To establish the validity of the present analysis, the results in terms of the stiffness and damping coefficients for a four-lobe journal bearing lubricated with a Newtonian fluid are compared with the results published [5]. The results agree very well. The effect of non-dimensional characteristics length of the micropolar fluid and the coupling number on the critical mass and the whirl ratio are shown in figures 2 and 3. From fig.2, it is observed that from the Newtonian fluid, the critical mass increases with an increasing of \( N \) and it converges to that of Newtonian fluid as \( L \rightarrow \infty \). In Fig.3, it can be observed that the whirl ratio decreases with an increase of the parameter \( N \) and at high values of \( L \), the whirl ratio tends the Newtonian fluid. The variation of critical mass with non-dimensional characteristic length \( L \) for various values of eccentricity ratio at \( N^2 = 0.6 \) is shown in Fig.4. This figure shows that the critical mass decreases with an increase of the eccentricity ratio initially then increases after \( \varepsilon = 0.15 \). Fig.5 shows the variation of the whirl ratio with \( L \) for various of eccentricity ratio at \( N^2 = 0.6 \). It can be observed that as the eccentricity ratio increases the whirl ratio decreases.
Based on the results presented in this paper, the following conclusions can be made:

1) The critical mass increases while the whirl ratio for the four-lobe journal bearing decreases with an increase of the parameter N.
2) At high values of L, the critical mass and the whirl ratio for this journal bearing converge to that for a Newtonian fluid.
3) The stability of the four-lobe journal bearing is improved by using a micropolar fluid compared to a Newtonian fluid.

**IV. CONCLUSIONS**

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2) At high values of $L$, the critical mass and the whirl ratio for this journal bearing converge to that for a Newtonian fluid.
3) The stability of the four-lobe journal bearing is improved by using a micropolar fluid compared to a Newtonian fluid.

**REFERENCES**


**NOMENCLATURE**

- $c$: major clearance
- $C_{ij}$: dimensionless damping coefficients
- $e$: eccentricity
- $e_p$: $c - c_m$, ellipticity
- $h$: oil film thickness
- $K_{ij}$: dimensionless stiffness coefficients
- $L$: bearing ratio
- $N$: coupling number
- $l$: material length
- $M$: mass of journal
- $P$: pressure
- $R$: journal radius
- $W$: bearing load
- $M^w = W(c/R)^2/\mu_i RL$, non-dimensional bearing load
- $\delta$: $e_p/c$, ellipticity ratio
- $\varepsilon$: $e/c$, eccentricity ratio based on major clearance
- $\varepsilon_m$: eccentricity ratio based on minor clearance
- $\theta$: angular coordinate
- $\theta_1, \theta_2, \theta_3, \theta_4$: angular coordinates at the end of bearing pads
- $\theta_1, \theta_2, \theta_3, \theta_4$: angular coordinates at the start of bearing pads
- $\theta_1, \theta_2, \theta_3, \theta_4$: angular coordinates at the trailing edges
- $\mu$: lubricant Viscosity
- $k_v$: spin viscosity
- $\mu_i$: viscosity coefficient for Newtonian fluid
- $\nu$: whirl frequency
- $\gamma$: whirl ratio
- $\phi$: attitude angle
- $\omega$: angular velocity of the journal